

# Patterns of Calcium Signaling based on HH Model

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**Abstract**— In this paper we represent the HH model. The study of HH model mechanism generates the action potential in axon and also gives mathematical equations. In this model we used different ions like Na and K but we do not considered the  $Ca^{2+}$  ion but in this paper we show how calcium channel signal is used for signaling in neuron network. Calcium channels are mostly used for providing the communication between neuron networks.

**Keywords**— Action potential, Mathematical, Calcium Channels, Communication, Neuron Network.

## 1. INTRODUCTION

For neuronal development there are different aspects to regulate the neuronal oscillations like neuronal migration, neuronal differentiation and connection pattern. The spontaneous synchronized  $Ca^{2+}$  ions are the one type of the oscillation which are observed by neuron without using any stimuli. The result is came through excitatory synaptic transmission. [1]. There are two different type of modes of transmission. The study of small antibiotic gramicidin Hladky and Haydon define the existence of the different ions channels. In 1952 Hodgkin and Huxley's analyse the electric activity of the two different ions  $Na^{2+}$  and  $K^{2+}$ . They suggest that ionic pathway of the channels take 20 times more year than Hladky and Haydon define the existence of the different ions channels.[2].

First time ELF magnetic field is planned which is based on the pervasion of the calcium channels in which stimulation is applied by the magnetic field. It supposed that the probability of the open changes in the opening channels in magnetic field then it increase the number of the opening channel with magnetic field.[4]. The reason behind the increment in number of opened channels may be the increment in the intra cellular calcium concentration. According to the Brownian dynamics model shows the curve against time, displacement and average velocity of calcium ions which showing the ELF magnetic field.[4,5]. The capacity of calcium channels also based on the inter cellular based wave model which is basically called calcium signaling which is used by the widespread range of the different organism with different types of cells. Calcium ion( $Ca^{2+}$ ) is most important for the calcium cellular signaling. In  $Ca^{2+}$  signaling there is a movement between intracellular and extracellular compartment who alters the action potential. In humans we believe that the oscillation is acts as a path for the long range of signaling and information transport in the brain.[6,7]. The inter-cellular calcium ion  $Ca^{2+}$  passing alternans free from the sarcoplasmic reticulum (SR). It shows that the inter-cellular of calcium ion ( $Ca^{2+}$ ) alternans was generated through the transmitting waves of calcium ions

release and continuous through the interchange of SR of  $Ca^{2+}$  content that has a hard relationship with the  $Ca^{2+}$  transient. Through this we can easily understand the  $Ca^{2+}$  alternans with out any need of refractoriness of L-type calcium ion. The effect of the  $Ca^{2+}$  is that it generate the diffusion wave concordant and dis-concordant  $Ca^{2+}$  alternans in the cell was also analyzed.[6]. In calcium signaling  $Ca^{2+}$  channels involves the VGCC, NMDA receptors, AMPA receptors, TRP channels, and depot controlled channels.

In universe calcium ion( $Ca^{2+}$ ) is second most important messenger to spread the information in tissues for the calcium signaling. When calcium waves transmits there is gap between channels which is formed by the cell membrane. To utilize the property of calcium signaling we use the model called cluster based network model. Which is composed of different cell and neurosensors[9]. Calcium signaling is also helpful to investigate the errors. Mean it also important for error control which is based on molecular communications[8]. Calcium signaling network consist of different nodes -

- i) Receptors and Ligands,
- ii) Soluble second messengers
- iii) Selective/non-selective ion channels

The  $Ca^{2+}$  signaling network act as a toolkit. It is used to assemble the signals through system with very different temporal and spatial dynamics.[10].

## 2. HH Model

Calcium ions( $Ca^{2+}$ ) is universal signaling ion that regulates the variation of neuronal function. By binding to and modifying the state of effector proteins. Using mathematical or computational modeling we can easily understand the concept of calcium signaling.

In 1952 Alan Lloyd Hodgkin and Andrew Huxley gives the experimental sets of equation to fit the data in axon. In calcium ion channels there are two different divisions i.e. LVA(lower-voltage activation) and HVA(high-voltage activation). HVA channel found in the outside the presynaptic and owing high threshold. LVA often called T-channels. Hodgkin and Huxley model is also known as "modeling schema" rather than model. The basic HH circuit model further more extended to gives the basic details of the physiology of the neurons. The HH model is most famous and complex model till date. The HH model is a mathematical model which is "conductance based model" which describe the action potential in neuron which are

initiated and propagated. It is a set of nonlinear equation which describe the electrical characteristics of different neuron cells. The HH model works on three ion channels  $K^+$  (potassium),  $Na^+$  (sodium) and leak current that consist of  $Cl^-$  ions.  $K^+$  and  $Na^+$  ions channels they both are used to control the flow of ion channels in cell membrane and they both are voltage dependent ion channels. And the leak current channel is mainly responsible for the inactive membrane potential. The Hodgkin and Huxley model is beneficial for the squid. Squid is the large diameter of the giant axons. It is beneficial because it manipulates the non technical small axons which is used in the biophysical studies. In 1963 Hodgkin and Huxley model is awarded with the noble prize in Medicines and Physiology [1,3,4].-

2.1 Electrical equivalent circuit

In terms of mathematical expression, HH model based on parallel circuit having resistors, capacitor and batteries. In fig.1 capacitor is used to store the charge capacity of the cell membrane. Resistors are the various types of ion channels which are fixed in membranes and batteries represent the intracellular and extracellular ions [12].

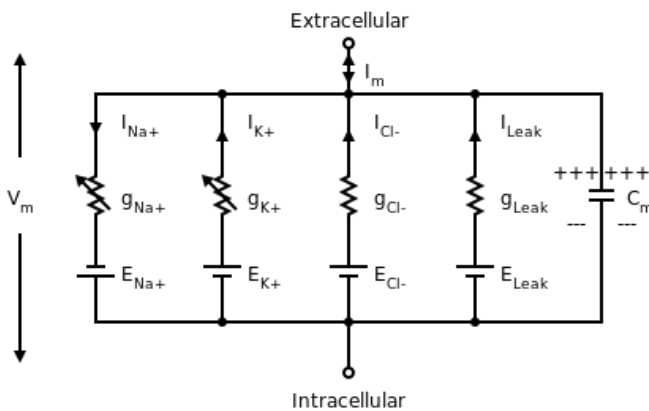


FIGURE 1. Electrical circuit of HH model

The leak current or leakage channel is determined by the voltage independent conductance.

$$g = 1/R \tag{2.1}$$

On the other hand conductance is voltage and time dependent. These channels describe some different variables  $m$ ,  $n$  and  $h$ . Where  $m$  and  $h$  controls by the  $Na^+$  ions and  $n$  controls by  $K^+$  ion channels. These variables are also called gating variables. They represented as  $gNa$ ,  $gK$  and  $gL$ .

$$m = \alpha_m(V)(1 - m) - \beta_m(V)m \tag{2.2}$$

$$n = \alpha_n(V)(1 - n) - \beta_n(V)n \tag{2.3}$$

$$h = \alpha_h(V)(1 - h) - \beta_h(V)h \tag{2.4}$$

TABLE -1

X	$E_x$ (mv)	$g_x$ (ms/cm <sup>2</sup> )
Na	55.17	1.2
K	-72.14	0.36
L	-49.42	0.003

We have some equations in which we have various function  $\alpha$  and  $\beta$  to determine the empirical function of  $a$  determine by the Hodgkin and Huxley to fit the data in axon of the squid.  $E_{Na}$ ,  $E_K$  and  $E_l$  these parameters are known as Reversal potentials.

TABLE -2

x	$\alpha_x$
n	$0.01 * (v + 50) / (1 - \exp(-(v + 50) / 10))$
m	$0.1 * (v + 35) / (1 - \exp(-(v + 35) / 10))$
h	$0.07 * \exp(0.05 * (v + 60))$

TABLE-3

x	$\beta_x$
n	$0.125 * \exp(-(v + 60) / 80)$
m	$4.0 * \exp(-0.0556 * (v + 60))$
h	$1 / (1 + \exp(-(0.1) * (v + 30)))$

Geometrically in synaptic integration neuron plays a significant role. The effect of synaptic input on membrane potential depends on the location of synapse on the dendrites tree. The result is depending on the passive dendrites which is usually necessary for the complex geometry and active ions channels.

Some authors give a best review on neuron which is related to HH model. In 1995 *Nelson and Rinzel* gives a best review of HH model in the book in which they include some historical part based on neuron. In 1987 *Cronin* discuss about the monograph of Cronin using the mathematical equation of the HH model. The book of *Christ of Koch* in 1999 *Koch* gives the comprehensive and readable introduction of single neuron. In 1992 in *B.Hille's* book we found the detailed information of the ion channels and the effect of the membrane on 'Ionic channels of excitable membranes'. In 1995 *Bower and Beeman* gives the practical guidelines related to the simulation of the single neuron model.

2. What HH model actually did?

The working of HH model consists of three stages:

- 1) The model assumptions:

In this the basic model consists of independent channels. These independent channels contain the gates that follow the first order of kinetics. And the currents that carried by the ions moving down electrochemically gradient. This is simple by state. But since there are many alternative models have been proposed like Integrate and Fire model, Fitzhugh-Nagumo model.

2) *Obtaining parameters for the model:*

The equations of membrane potential it has to be found to fill in the different unknown parameters. There were 3 levels where details are required which had to be determined: a) the macro characteristics of channel types, b) the number of activation and inactivation gates in the channels, and c) to describe the quantitative voltage dependency of  $\alpha$  and  $\beta$  for each gate in each channel.

*Number of Gates:-* In HH model there is a large number of microscopic ionic channels which arise from the combined effect embedded in the membrane. In transient channel it consists of two gates:- i) activation (open probability increases with depolarization) and ii) Deactivation (open probability decreases with depolarization). During the depolarization step HH observed that the conductance change had a *sigmoid* shape, and during repolarization step conductance change had an exponent

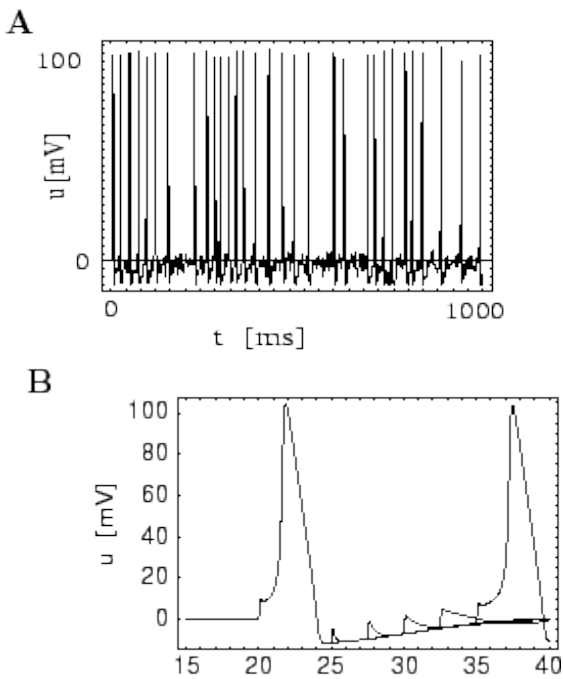


Figure 2

The properties of an excitable cell the Hodgkin and Huxley developed the four set of equations. These equations are:-

$$I = C_m \frac{dV_m}{dt} + g_k(V_m - V_k) + g_{Na}(V_m - V_{Na}) + g_l(V_m - V_l) \quad 2.6$$

Mathematical equation for the current following through lipid bilier:-

$$I_c = C_m \frac{dV_m}{dT} \quad 2.7$$

the current following through a given ion,

$$I_i = g_i (V_m - V_i) \quad 2.8$$

Voltage - gated ion channels,

$$I_m = C_m \frac{dV_m}{dT} + \bar{g}_K n^4 (V_m - V_k) + \bar{g}_{Na} m^3 h (V_m - V_{Na}) + \bar{g}_l (V_m - V_l) \quad 2.9$$

$$\frac{dn}{dt} = \alpha_n(V_m)(1 - n) - \beta_n(V_m)n \quad 2.10$$

$$\frac{dm}{dt} = \alpha_m(V_m)(1 - m) - \beta_m(V_m)m \quad 2.11$$

$$\frac{dh}{dt} = \alpha_h(V_m)(1 - h) - \beta_h(V_m)h \quad 2.12$$

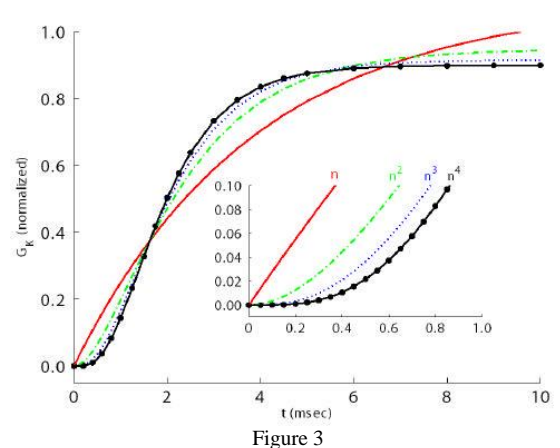


Figure 3

3) Reconstructed the spike :

The above equations describe and determined the appropriate numerical parameters by experiment. The reconstruction of the stimulus from the neuron shows the non- linearity of neuron dynamics. This model gives a numerical solution rather than the analytical one to reconstruct the stimuli. According to *M. Sarangdhar and C. Kambhampati* the analytical solution does not exist for reconstruction for retrieve the neural dynamics. [11]

3. CONCLUSION

In this paper we focused on the HH model. The HH model has been successful in describing and predicting the property of the large number of neurons. The extension of the voltage - dependent channels pair has been widely used in research area throughout the world. This model also analyzed the signal process through the sensory neurons. In this we also represent the calcium signaling process how  $Ca^{2+}$  ions regulate in the neuron. In this paper we also assumed the increase in magnetic field using calcium channels.

## REFERENCES

- [1] Robert J. French and Gerald W. Zamponi "Voltage- Gated Sodium and Calcium Channels in Nerve, Muscle, and Heart" VOL. 4, No.1, March 2005.
- [2] Jun He, Wei Zhou, Shaoqun Zeng, Qingming Luo "L-type calcium channels mediate synchronized spontaneous Ca<sup>2+</sup> spikes in cultured cortical networks" 0-7803-8740-6, September 2005.
- [3] ZHANG Yu-Hong, ZHAN Yong, ZHAO Tong-Jun, HAN Ying-Rong, Liu Hui "Mechanism of Permeation in Calcium Channels Activation by Applied Magnetic Fields" 1-4244-0788-5, August 2007.
- [4] Zhang Yuhong, Zhan Yong, Zhao Tongjun, Han Yingrong, Liu Hui "Monte Carlo Simulation of the permeability of calcium channels exposure to electromagnetic fields" 1-4244-1120-3, November 2007.
- [5] H Zhang, T Tao, SC O'Neil "Approaching the Mechanistic Insights towards the Genesis of Intracellular Calcium Transient Alternans – a Simulation Study" Computers in Cardiology 2008; 35:425–428.
- [6] Gale L. Craviso, Paroma Chatterjee, Gabriel Maalouf, Alex Cerjanic, Jihwan Yoon, Indira Chatterjee, P. Thomas Vernier "Nanosecond Electric Pulse-induced Increase in Intracellular Calcium in Adrenal Chromaffin Cells Triggers Calcium-dependent Catecholamine Release" 1070-9878, IEEE March 2009.
- [7] Fei Liu, Monika Heiner "Multiscale modelling of coupled Ca<sup>2+</sup> channels using coloured stochastic Petri nets" Vol. 7, No: 4, IEEE March 2013.
- [8] Akif Cem Heren, M. S, "ukr'u Kuran, H. Birkan Yilmaz and Tuna Tugcu "Channel Capacity of Calcium Signalling Based on Inter-cellular Calcium Waves in Astrocytes" 978-1-4673-5753-1, IEEE International Conference on Communications 2013.
- [9] Yiqun Yang, Chai Kiat Yeo "Design and Analysis of a Cluster-based Calcium Signaling Network Model" The 10th Annual IEEE CCNC-Wireless Networking Track, 978-1-4673-3, 2013 IEEE.
- [10] Kaufman, R. Tindjong, D. G. Luchinsky, P. V. E. McClintock, R. S. Eisenberg "Resonant Multi-Ion Conduction in a Simple Model of Calcium Channels" ICNF2013 978-1-4799-0671, 2013 IEEE.
- [11] Dr. W. J. Heitler "The Hodgkin-Huxley Model for the Generation of Action Potentials" J. Physiol. (Lond.) 117, 500-544, July 2007.
- [12] William Aristizabal Botero, Alvaro H. Salas and Silvia Janeth Gonzalez Colorado "The Hodgkin-Huxley neuron model on the fast phase plane" Vol. 8(20), pp. 1049-1057, May, 2013.