International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 1 Issue 6, August - 2012

PARTIALLY ORDERED Γ-SEMIGROUPS

VB Subrahmanyeswara Rao Seetamraju[1], A. Anjaneyulu[2], D. Madhusudana Rao[3].

[1]*Dept. of Mathematics, V K R, V N B & A G K College Of Engineering, Gudivada, A.P. India.*

[2]*Dept. of Mathematics, V S R & N V R College, Tenali, A.P. India.*

[3]*Dept. of Mathematics, V S R & N V R College, Tenali, A.P. India.*

Abstract

In this paper, the notion of an ordered Γ-semigroup is introduced and some examples are given. Further the terms commutative ordered Γ-semigroup, quasi commutative ordered Γ-semigroup, normal ordered Γ-semigroup, left pseudo commutative ordered Γ-semigroup, right pseudo commutative ordered Γ-semigroup are introduced. It is proved that (1) if S is a commutative ordered Γ-semigroup then S is a quasi commutative ordered Γ-semigroup, (2) if S is a quasi commutative ordered Γ-semigroup then S is a normal ordered Γ-semigroup, (3) if S is a commutative ordered Γ-semigroup, then S is both a left pseudo commutative and a right pseudo commutative ordered Γ-semigroup. Further the terms; left identity, right identity, identity, left zero, right zero, zero of an ordered Γ-semigroup are introduced. It is proved that if *a* is a left identity and *b* is a right identity of an ordered Γ-semigroup S, then *a* = *b*. It is also proved that any ordered Γ-semigroup S has at most one identity. It is proved that if *a* is a left zero and *b* is a right zero of an ordered Γ-semigroup S, then *a* = *b* and it is also proved that any ordered Γ-semigroup S has at most one zero element. The terms; ordered Γ-subsemigroup, ordered Γ-subsemigroup generated by a subset, α-idempotent, Γ-idempotent, strongly idempotent, midunit, *r*-element, regular element, left regular element, right regular element, completely regular element, (α, β)-inverse of an element, semisimple element and intra regular element in an ordered Γ-semigroup are introduced. Further the terms idempotent ordered Γ-semigroup and generalized commutative ordered Γ-semigroup are introduced. It is proved that every *α*-idempotent element of an ordered Γ-semigroup is regular. It is also proved that, in an ordered Γ-semigroup, *a* is a regular element if and only if *a* has an (*α*, *β*)-inverse. It is proved that, (1) if *a* is a completely regular element of an ordered Γ-semigroup S, then *a* is both left regular and right regular, (2) if '*a*' is a completely regular element of an ordered Γ-semigroup S, then *a* is regular and semisimple, (3) if '*a*' is a left regular element of an ordered Γ-semigroup S, then *a* is semisimple, (4) if '*a*' is a right regular element of an ordered Γ-semigroup S, then *a* is semisimple, (5) if '*a*' is a regular element of an ordered Γ- semigroup S, then *a* is semisimple and (6) if '*a*' is a intra regular element of an ordered Γ-semigroup S, then *a* is semisimple. The term separative ordered
Γ-semigroup is introduced and it is proved that, in a separative ordered Γ-semigroup S, for any *x, y, a, b* ∈ S, the statements (i) $x\Gamma a \leq x\Gamma b$ if and only if $a\Gamma x \leq b\Gamma x$, (ii) $x\Gamma\, x\Gamma a \leq x\Gamma x\Gamma b$ implies $x\Gamma a \leq x\Gamma b$,(iii) $x\Gamma y\Gamma a \leq x\Gamma y\Gamma b$ implies $y\Gamma x\Gamma a \leq y\Gamma x\Gamma b$ hold.

1. Introduction

Γ- semigroup was introduced by Sen and Saha [15] as a generalization of semigroup. Anjaneyulu. A [1], [2] and [3] initiated the study of ideals and radicals in semigroups. Many classical notions of Γ-semigroups heve been extended to semigroups to Γ-semigroups by Madhusudhana Rao, Anjaneyulu and Gangadhara Rao [11]. The concept of partially ordered Γ-semigroup was introduced by Y. I. Kwon and S. K. Lee [10] in 1996, and it has been studied by several authors. In this paper we introduce the notions of ordered Γ-semigroups and characterize ordered Γ-semigroups.

ISSN: 2278-0181
Vol. 1 Issue 6, August - 2012

2. PRELIMINARIES :

DEFINITION 2.1: Let S and Γ be any two non-empty sets. S is called a ***Γ-semigroup*** if there exist a mapping from $S \times \Gamma \times S$ to S which maps $(a, \alpha, b) \rightarrow a\alpha b$ satisfying the condition : $(a\gamma b)\mu c = a\gamma(b\mu c)$ for all $a,b,c \in M$ and $\gamma,\mu \in \Gamma$.

NOTE 2.2: Let S be a Γ-semigroup. If A and B are subsets of S, we shall denote the set $\{ a\alpha b : a \in A , b \in B \text{ and } \alpha \in \Gamma \}$ by AΓB.

DEFINITION 2.3: A Γ-semigroup S is said to be ***commutative*** provided $a\gamma b = b\gamma a$ for all $a,b \in S$ and $\gamma \in \Gamma$.

NOTE 2.4: If S is a commutative Γ-semigroup, then $a\Gamma b = b\Gamma a$ for all $a, b \in S$.

DEFINITION 2.5 : A Γ-semigroup S is said to be ***quasi commutative*** provided for each $a,b \in S$, there exists a natural number n such that $a\gamma b = (b\gamma)^n a \ \forall \gamma \in \Gamma$.

NOTE 2.6 : If a Γ-semigroup S is ***quasi commutative*** then for each $a,b \in S$, there exists a natural number n such that, $a\Gamma b = (b\,\Gamma)^n a$.

THEOREM 2.7: If S is a commutative Γ-semigroup then S is a quasi commutative Γ-semigroup.

DEFINITION 2.8 : A Γ-semigroup S is said to be ***normal*** provided $a\alpha S = S\alpha a \ \forall \alpha \in \Gamma$ and $\forall a \in S$.

NOTE 2.9 : If a Γ-semigroup S is ***normal*** then $a\Gamma S = S\Gamma a$ for all $a \in S$.

THEOREM 2.10 : If S is a quasi commutative Γ-semigroup then S is a normal Γ-semigroup.

COROLLARY 2.11 : Every commutative Γ-semigroup is a normal Γ-semigroup.

DEFINITION 2.12 : A Γ-semigroup S is said to be ***left pseudo commutative*** provided $a\Gamma b\Gamma c = b\Gamma a\Gamma c$ for all $a,b,c \in S$.

DEFINITION 2.13 : A Γ-semigroup S is said to be ***right pseudo commutative*** provided $a\Gamma b\Gamma c = a\Gamma c\Gamma b$ for all $a,b,c \in S$.

THEOREM 2.14 : If S is a commutative Γ-semigroup, then S is both a left pseudo commutative Γ-semigroup and a right pseudo commutative Γ-semigroup.

DEFINITION 2.15 : An element a of a Γ-semigroup S is said to be a ***left identity*** of S provided $a\alpha s = s$ for all $s \in S$ and $\alpha \in \Gamma$.

DEFINITION 2.16 : An element 'a' of a Γ-semigroup S is said to be a ***right identity*** of S provided $s\alpha a = s$ for all $s \in S$ and $\alpha \in \Gamma$.

DEFINITION 2.17 : An element 'a' of a Γ-semigroup S is said to be a ***two sided identity*** or an ***identity*** provided it is both a left identity and a right identity of S.

THEOREM 2.18 : **If a is a left identity and b is a right identity of a Γ-semigroup S, then $a = b$.**

DEFINITION 2.19 : An element a of a Γ-semigroup S is said to be a ***left zero*** of S provided $a\alpha s = a$ for all $s \in S$ and $\alpha \in \Gamma$.

DEFINITION 2.20 : An element a of a Γ-semigroup S is said to be a ***right zero*** of S provided $s\alpha a = a$ for all $s \in S$ and $\alpha \in \Gamma$.

DEFINITION 2.21 : An element a of a Γ-semigroup S is said to be a ***two sided zero*** or ***zero*** provided it is both a left zero and a right zero of S.

DEFINITION 2.22: An element a of Γ- semigroup S is said to be a ***Γ- idempotent*** provided $a\alpha a = a$ for all $\alpha \in \Gamma$.

NOTE 2.23: If an element a of Γ- semigroup S is ***a Γ- idempotent,*** then $a\Gamma a = a$.

DEFINITION 2.24: A Γ- Semigroup S is said to be an ***idempotent Γ-semigroup*** or a ***band*** provided every element in S is a Γ- idempotent.

DEFINITION 2.25 : An element a of Γ-semigroup S is said to be a ***midunit*** provided $x\Gamma a\Gamma y = x\Gamma y$ for all $x, y \in S$.

NOTE 2.26 : Identity of a Γ- semigroup S is a midunit.

DEFINITION 2.27 : An element 'a' of Γ- semigroup S is said to be ***an r-element*** provided $a\Gamma s = s\Gamma a$ for all $s \in S$ and if $x, y \in$ S, then $a\Gamma x\Gamma y = b\Gamma y\Gamma x$ for some $b \in$ S.

DEFINITION 2.28 : A Γ- semigroup S with identity 1 is said to be *a* ***generalized commutative* Γ- *semigroup*** provided 1 is an r-element in S.

DEFINITION 2.29 : An element a of a Γ-semigroup S is said to be ***regular*** provided $a = a\alpha x\beta a$, for some $x \in$ S and $\alpha, \beta \in \Gamma$. i.e, $a \in a\Gamma S\Gamma a$.

DEFINITION 2.30 : A Γ-ideal A of a Γ-semigroup S is said to be ***regular*** if every element of A is regular in A.

DEFINITION 2.31 : A Γ- semigroup S is said to be *a* ***regular* Γ- *semigroup*** provided every element is regular.

DEFINITION 2.32 : An element a of a Γ-semigroup S is said to be ***left regular*** provided $a = a\alpha a\beta x$, for some $x \in$ S and $\alpha, \beta \in \Gamma$. i.e, $a \in a\Gamma a\Gamma S$.

DEFINITION 2.33 : An element a of a Γ-semigroup S is said to be ***right regular*** provided $a = x\alpha a\beta a$, for some $x \in$ S and $\alpha, \beta \in \Gamma$. i.e, $a \in S\Gamma a\Gamma a$.

DEFINITION 2.34 : An element a of a Γ-semigroup S is said to be ***completely regular*** provided, there exists an element $x \in$ S such that $a = a\alpha x\beta a$ for some $\alpha, \beta \in \Gamma$and $a\alpha x = x\beta a$ i.e., $a \in a\Gamma x\Gamma a$ and $a\Gamma x = x\Gamma a$.

DEFINITION 2.35 : A Γ-semigroup S is said to be ***completely regular* Γ-*semigroup*** provided every element of S is completely regular.

DEFINITION 2.36 : An element a of Γ- semigroup S is said to be ***semisimple*** provided $a \in \langle a \rangle \Gamma \langle a \rangle$, that is, $\langle a \rangle \Gamma \langle a \rangle = \langle a \rangle$.

DEFINITION 2.37 : A Γ- semigroup S is said to be ***semisimple* Γ- *semigroup*** provided every element of S is a semisimple element.

DEFINITION 2.38 : An element a of a Γ-semigroup S is said to be ***intra regular*** provided $a = x\alpha a\beta a\gamma y$ for some $x, y \in$S and $\alpha, \beta, \gamma \in \Gamma$.

THEOREM 2.39 : If '*a*' is a completely regular element of a Γ- semigroup S, then *a* is regular and semisimple.

THEOREM 2.40 : If '*a*' is a intra regular element of a Γ- semigroup S, then *a* is semisimple.

3. ORDERED Γ-SEMIGROUP :

DEFINITION 3.1 : A Γ-semigroup S is said to a partially ordered Γ-semigroup if S is partially ordered set such that $a \le b \Rightarrow a\gamma c \le b\gamma c$ and $c\gamma a \le c\gamma b \ \forall a, b, c \in S$ and $\gamma \in \Gamma$.

NOTE 3.2 : A partially ordered Γ-semigroup simply called po-Γ-semigroup or ordered Γ-semigroup.

EXAMPLE 3.3 : Let S = { ∅, {a}, {b}, {c}, {a, b}, {b, c}, {a, c}, {a, b, c}} and Γ = {∅, {a}, {a, b, c}}. If for all A, C ∈ S and B ∈ Γ, ABC = A ∩ B ∩ C and A ≤ C ⇔ A ⊆ C, then S is an ordered Γ-semigroup.

EXAMPLE 3.4: **1.1.8** : Let S = {∅, {a}, {b}, {c}, {a, b}, {b, c}, {a, c}, {a, b, c}} and Γ = {{a, b, c}}. If for all A, C ∈ S and B ∈ Γ, ABC = A ∩ B ∩ C and A ≤ C ⇔ A ⊆ C, then S is an ordered Γ-semigroup.

NOTE 3.5: In general, let P(X) be the power set of any nonempty set X and Γ a topology on X. If we define for all A, C ∈ S and B ∈ Γ, ABC = A ∩ B ∩ C and A ≤ C ⇔ A ⊆ C, then P(X) is an ordered Γ-semigroup.

EXAMPLE: **3.6** : Let S be the set of all 2 × 3 matrices over Q, the set of positive integers and Γ be the set of all 3 × 2 matrices over the same set. Define AαB = usual matrix product of A, α, B; for all A, B ∈ S and for all α ∈ Γ. Then S is a Γ-semigroup. Note that S is not a semigroup. Also S and Γ are posets with respect to "≤" defined by $(a_{ik}) \le (b_{ik})$ if and only if $a_{ik} \le b_{ik}$ for all i, k. Then S is an ordered Γ-semigroup.

EXAMPLE: **3.7** : Let S be the set of all integers of the form $4n+1$ where n is an integer and Γ denote the set of all integers of the form $4n+3$. If $a\gamma b$ is $a+\gamma+b$ (usual sum of the integers) and $a \le b$ means a is less than or equal to b, for all $a, b \in$ S and $\gamma \in \Gamma$, then S is an ordered Γ-semigroup.

EXAMPLE 3.8 : Let M be the set of all isotone mappings from a poset P into another poset Q and Γ the set of all isotone mappings from the poset Q into the poset P. Let $f, g \in$ M and α ∈ Γ. Define by *fαg,* the usual mapping composition of *f*, α and *g*. Then M is a Γ-semigroup. We define a relation ≤ on M by $f \leq g$ if and only if $af \leq ag$, for all $a \in$ P. This relation is a partial order on M and as such M is a poset. We also define a relation ≤ on Γ by $\alpha \leq \beta$ if and only if $x\alpha \leq x\beta$, for all $x \in$ Q. For this relation Γ is a poset. Then M is an ordered Γ-semigroup.

EXAMPLE 3.9 : For *a, b* ∈ [0,1], let M = [0, *a*] and Γ = [0, *b*]. Then M is an ordered Γ-semigroup under usual multiplication and usual partial order relation.

EXAMPLE 3.10 : Fix $m \in$ Z, and let M be the set of all integers of the form mn +1 and Γ denotes the set of all integers of the form $mn + m - 1$ where n is an integer. Then M is a ordered Γ-semigroup under usual addition and usual partial order relation.

In the following we introduce the notion of a commutative ordered Γ-semigroup.

DEFINITION 3.11 : An ordered Γ-semigroup S is said to be ***ordered commutative Γ-semigroup*** provided S is a commutative Γ-semigroup.

NOTE 3.12 : An ordered Γ-semigroup S is said to be ordered commutative Γ-semigroup provided $a\gamma b = b\gamma a$ for all $a,b \in S$ and $\gamma \in \Gamma$.

NOTE 3.13 : If S is an ordered commutative Γ-semigroup then $a\Gamma b = b\Gamma a$ for all $a, b \in S$.

NOTE 3.14 : Let S be a Γ-semigroup and *a, b* ∈ S and *α* ∈ Γ. Then $a\alpha a\alpha b$ is denoted by $(a\alpha)^2 b$ and consequently *a α a α a α*…..(*n terms*)*b* is denoted by $(a\alpha)^n b$.

In the following we introduce a quasi commutative ordered Γ-semigroup.

DEFINITION 3.15 : An ordered Γ-semigroup S is said to be ***ordered quasi commutative Γ-semigroup*** provided S is a quasi commutative Γ-semigroup.

NOTE 3.16 : An ordered Γ-semigroup S is said to be ordered quasi commutative Γ-semigroup provided for each *a,b* ∈ S, there exists a natural number *n* such that $a\gamma b = (b\gamma)^n a \ \forall \gamma \in \Gamma$.

NOTE 3.17 : If an ordered Γ-semigroup S is ordered quasi commutative Γ-semigroup then for each *a,b* ∈ S , there exists a natural number *n* such that, $a\Gamma b = (b\,\Gamma)^n a$.

THEOREM 3.18 : If S is an ordered commutative Γ-semigroup then S is an ordered quasi commutative Γ-semigroup.

***Proof*:** Suppose that S is an ordered commutative Γ-semigroup.

Then S commutative Γ-semigroup. Let $a, b \in S$.

Now $a\alpha b = b\alpha a \Rightarrow a\alpha b = (b\ \alpha)^1 a$. Therefore S is a quasi commutative Γ-semigroup.

Therefore S is an ordered quasi commutative Γ-semigroup.

In the following we introduce the notion of an ordered normal Γ-semigroup.

DEFINITION 3.19 : An ordered Γ-semigroup S is said to be ***ordered normal Γ-semigroup*** provided S is normal Γ-semigroup.

NOTE 3.20 : An ordered Γ-semigroup S is said to be ordered normal Γ-semigroup provided $a\alpha S = S\alpha a \ \forall \alpha \in \Gamma$ and $\forall a \in S$.

NOTE 3.21 : If an ordered Γ-semigroup S is ordered normal Γ-semigroup then $a\Gamma S = S\Gamma a$ for all $a \in$ S.

THEOREM 3.22 : If S is an ordered quasi commutative Γ-semigroup then S is a normal ordered Γ-semigroup.

***Proof*:** Suppose that S is an ordered quasi commutative Γ- semigroup.

Then S is quasi commutative Γ-semigroup.

Let $a \in$ S and $\alpha \in \Gamma$

Let $x \in a\,\alpha$ S. Then $x = a\,\alpha$ b, where $b \in$ S

Since S is quasi commutative, $a\,\alpha$ b = $(b\alpha)^n a$ for some $n \in$ N

Therefore, $x = a\,\alpha$ b = $(b\alpha)^n a = (b\alpha)^{n-1} b\alpha a \in S\,\alpha\,a$

$\therefore a\,\alpha\,S \subseteq S\,\alpha\,a$ ----(1)

Let $x \in S\,\alpha\,a$. Then $x = t\,\alpha\,a$, for some t ∈ S

Since S is quasi commutative, $t\,\alpha\,a = (a\alpha)^n t$ for some $n \in$ N

Now, $x = t\,\alpha\,a = (a\alpha)^n t = a\,\alpha\,(a\alpha)^{n-1} t \in a\,\alpha\,S$

$\therefore S\,\alpha\,a \subseteq a\,\alpha\,S$ ----(2)

From (1) and (2), $a\,\alpha\,S = S\,\alpha\,a$ for all $\alpha \in \Gamma$ and for all $b \in S$

Hence S is a normal Γ-semigroup. Therefore S is an ordered normal Γ-semigroup.

COROLLARY 3.23 : Every ordered commutative Γ-semigroup is a normal ordered Γ-semigroup.

Proof : Let S be an ordered commutative Γ-semigroup. By theorem 3.18, S is an ordered quasi commutative Γ-semigroup. By theorem 3.22, S is an ordered normal Γ-semigroup. Therefore every ordered commutative Γ-semigroup is an ordered normal Γ-semigroup.

In the following we are introducing ordered left pseudo commutative Γ-semigroup and ordered right pseudo commutative Γ-semigroup.

DEFINITION 3.24 : An ordered Γ-semigroup S is said to be ***ordered left pseudo commutative Γ-semigroup*** provided S is left pseudo commutative Γ-semigroup.

NOTE 3.25: An ordered Γ-semigroup S is said to be ordered left pseudo commutative provided $a\Gamma b\Gamma c = b\Gamma a\Gamma c$ for all $a,b,c \in S$.

DEFINITION 3.26 : An ordered Γ-semigroup S is said to be ***ordered right pseudo commutative Γ-semigroup*** provided S is right pseudo commutative Γ-semigroup.

NOTE 3.27 : An ordered Γ-semigroup S is said to be ***ordered right pseudo commutative Γ-semigroup*** provided $a\Gamma b\Gamma c = a\Gamma c\Gamma b$ for all $a,b,c \in S$.

THEOREM 3.28 : If S is an ordered commutative Γ-semigroup, then S is both an ordered left pseudo commutative Γ-semigroup and an ordered right pseudo commutative Γ-semigroup.

Proof : Suppose that S is ordered commutative Γ-semigroup.
Then S is commutative Γ-semigroup.
Then $a\Gamma b\Gamma c = (a\Gamma b)\Gamma c = (b\Gamma a)\Gamma c = b\Gamma a\Gamma c$.
Therefore S is left pseudo commutative Γ-semigroup.
Therefore S is an ordered left pseudo commutative Γ-semigroup.
Again $a\Gamma b\Gamma c = a\Gamma(b\Gamma c) = a\Gamma(c\Gamma b) = a\Gamma c\Gamma b$.
Therefore S is a right pseudo commutative Γ-semigroup.
Therefore S is an ordered right pseudo commutative Γ-semigroup.

NOTE 3.29 : The converse of the above theorem is not true. i.e., if S is an ordered left and right pseudo commutative Γ-semigroup then S need not be an ordered commutative Γ-semigroup.

EXAMPLE 3.30 : Let $S = \{a, b, c\}$ and $\Gamma = \{x, y, z\}$ Define a binary operation '.' in S as shown in the following table:

.	*a*	*b*	*c*
a	*a*	*a*	*a*
b	*a*	*a*	*a*
c	*a*	*b*	*c*

Define a mapping $S \times \Gamma \times S \to S$ by $a\alpha b = a.b$ for all $a, b \in S$ and $\alpha \in \Gamma$. It is easy to see that S is a ordered Γ-semigroup. Now S is an ordered left and right pseudo commutative Γ-semigroup. But S is not an ordered commutative Γ-semigroup.

In the following we are introducing left identity, right identity and identity of a ordered Γ-semigroup.

DEFINITION 3.31 : An element ***a*** of an ordered Γ-semigroup S is said to be a ***left identity*** of S provided $a\alpha s = s$ and $s \le a$ for all $s \in S$ and $\alpha \in \Gamma$.

DEFINITION 3.32 : An element ***a*** of an ordered Γ-semigroup S is said to be a ***right identity*** of S provided $s\alpha a = s$ and $s \le a$ for all $s \in S$ and $\alpha \in \Gamma$.

DEFINITION 3.40 : An element '***a***' of an ordered Γ-semigroup S is said to be a ***two sided identity*** or an ***identity*** provided it is both a left identity and a right identity of S.

NOTE 3.41 : An element '***a***' of an ordered Γ-semigroup S is said to be a ***two sided identity*** or an ***identity*** provided $s\alpha a = a\alpha s = s$ and $s \le a$ for all $s \in S$ and $\alpha \in \Gamma$.

EXAMPLE 3.42 : **1.1.7** : Let S = { ∅, {*a*}, {*b*}, {*c*}, {*a*, *b*}, {*b*, *c*}, {*a*, *c*}, {*a*, *b*, *c*}} andΓ = {∅}. If for all A, C ∈ S and B ∈ Γ, ABC = A ∪ B ∪ C and A ≤ C ⇔ A ⊆ C, then S is a partially ordered Γ-semigroup with identity.

EXAMPLE: **3.43** : Let S = {∅, {*a*}, {*b*}, {*c*}, {*a*, *b*}, {*b*, *c*}, {*a*, *c*}, {*a*, *b*, *c*}} and Γ = {{*a*, *b*, *c*}}. If for all A, C ∈ S and B ∈ Γ, ABC = A ∩ B ∩ C and A ≤ C ⇔ A ⊆ C, then S is a partially ordered Γ-semigroup with identity.

THEOREM 3.44 : **If *a* is a left identity element and *b* is a right identity element of an ordered Γ-semigroup S, then *a* = *b*.**

Proof: Since *a* is a left identity of S, $a\alpha s = s$ and $s \leq a$ for all $s \in$ S and $\alpha \in \Gamma$ and hence $a\alpha b = b$ and $b \leq a$ for all $\alpha \in \Gamma$. Since *b* is a right identity of S, $s\alpha b = s$ and $s \leq b$ for all $s \in$ S and $\alpha \in \Gamma$ and hence $a\alpha b = a$ and $a \leq b$ for all $\alpha \in \Gamma$. Now $b \leq a$ and $a \leq b$ $\Rightarrow a = b$.

THEOREM 3.45 : Any ordered Γ-semigroup S has at most one identity.

Proof **:** Let *a*, *b* be two identity elements of the ordered Γ-semigroup S. Now *a* can be considered as a left identity and *b* can be considered as a right identity of S. By theorem 3.44, *a* = *b*. Then S has at most one identity.

NOTE 3.46 : The identity (if exists) of an ordered Γ-semigroup is usually denoted by 1 or *e*.

DEFINITION 3.47 : A partially ordered Γ-semigroup S with identity is called a partially ordered Γ-***monoid***.

In the following we are introducing left zero, right zero and zero of an ordered Γ-semigroup.

DEFINITION 3.48: An element *a* of an ordered Γ-semigroup S is said to be a ***left zero*** of S provided $a\alpha s = a$ and $a \leq s$ for all $s \in$ S and $\alpha \in \Gamma$.

DEFINITION 1.1.36 : An element *a* of an ordered Γ-semigroup S is said to be a ***right zero*** of S provided $s\alpha a = a$ and $a \leq s$ for all $s \in$ S and $\alpha \in \Gamma$.

DEFINITION 3.49 : An element *a* of an ordered Γ-semigroup S is said to be a ***two sided zero*** or ***zero*** provided it is both a left zero and a right zero of S.

NOTE 3.50 : An element *a* of an ordered Γ-semigroup S is said to be a ***two sided zero*** or ***zero*** provided $a\alpha s = s\alpha a = a$ and $a \leq s$ for all $s \in$ S and $\alpha \in \Gamma$.

EXAMPLE3.51 : Let S = {∅, {*a*}, {*b*}, {*c*}, {*a*, *b*}, {*b*, *c*}, {*a*, *c*}, {*a*, *b*, *c*}} and Γ = {{*a*, *b*, *c*}, ∅, {*a*}}. If for all A, C ∈ S and B ∈ Γ, ABC = A ∩ B ∩ C and A ≤ C ⇔ A ⊆ C, then S is a partially ordered Γ-semigroup with zero.

EXAMPLE 3.52 : Let S = {∅, {*a*}, {*b*}, {*c*}, {*a*, *b*}, {*b*, *c*}, {*a*, *c*}, {*a*, *b*, *c*}} andΓ = {{*a*, *b*, *c*}}. If for all A, C ∈ S and B ∈ Γ, ABC = A ∩ B ∩ C andA ≤ C ⇔ A ⊆ C, then S is a partially ordered Γ-semigroup with zero and identity.

NOTE 3.53 : In general, let P(X) be the power set of any non-empty set X and Γ a topology on X. If we define for all A, C ∈ S and B ∈ Γ, ABC = A ∩ B ∩ C andA ≤ C ⇔ A ⊆ C, then P(X) is a partially ordered Γ-semigroup with zero.

We are now introduce left zero ordered Γ-semigroup, right zero ordered Γ-semigroup and zero ordered Γ-semigroup.

DEFINITION 3.54 : An ordered Γ-semigroup in which every element is a left zero is called a ***left zero ordered Γ-semigroup***.

DEFINITION 3.55 : An ordered Γ-semigroup in which every element is a right zero is called a ***right zero ordered Γ-semigroup***.

DEFINITION 3.56 : An ordered Γ-semigroup S with 0 in which the product of any two elements equals to 0 and $0 \leq s$ for all $s \in$ S is called a ***zero ordered* Γ*-semigroup*** or a ***null ordered Γ-semigroup***.

THEOREM 3.57 : If *a* is a left zero element and *b* is a right zero element of an ordered Γ-semigroup S, then *a* = *b*.

Proof: Since *a* is a left zero of S, $a\alpha s = a$ and $a \leq s$ for all s ∈ S, $\alpha \in \Gamma$ and hence $a\alpha b = a$ and $a \leq b$ for all α ∈ Γ. Since *b* is a right zero of S, $s\alpha b = b$ and *b*

$\leq s$ for all s ∈ S and α ∈ Γ and hence $a\alpha b = b$ and $b \leq a$ for all α ∈ Γ. Now $a \leq b$ and $b \leq a \Rightarrow a = b$.

THEOREM 3.58 : Any ordered Γ-semigroup S has at most one zero element.

Proof **:** Let *a*, *b* be two zeros of the Γ-semigroup S. Now *a* can be considered as a left zero and *b* can be considered as a right zero. By theorem 3.57, $a = b$. Thus S has at most one zero.

NOTE 3.59 : The zero (if exists) of an ordered Γ-semigroup is usually denoted by 0.

NOTATION 3.60 : Let S be an ordered Γ-semigroup. If S has an identity, let $S^1 = S$ and if S does not have an identity, let S^1 be the ordered Γ-semigroup S with an identity adjoined usually denoted by the symbol 1. Similarly if S has a zero, let $S^0 = S$ and if S does not have a zero, let S^0 be the ordered Γ-semigroup S with zero adjoined usually denoted by the symbol 0.

NOTATION 3.61 : Let S be an ordered Γ-semigroup and T is a nonempty subset of S. If H is a nonempty subset of T, we denote $\{t \in T : t \leq h$ for some $h \in H\}$ by $(H]_T$. $\{t \in T : h \leq t$ for some $h \in H\}$ by $[H)_T$. $(H]_s$ and $[H)_s$ are simply denoted by (H] and [H) respectively.

THEOREM 3.62 : Let S be an ordered Γ-semigroup and A ⊆ S, B ⊆ S. Then (i) A ⊆ (A], (ii) ((A]] = (A], (iii) (A]Γ(B] ⊆ (AΓB] and (iv) A ⊆ (B] for A ⊆ B, (v) (A] ⊆ (B] for A ⊆ B .

Proof **:** (i) Let $x \in A$. $x \in A \Rightarrow x \in S$ and $x \leq x \Rightarrow x \in (A]$. Therefore A ⊆ (A].

(ii) Let $x \in ((A]] \Rightarrow x \leq y$ for some $y \in (A] \Rightarrow y \leq z$ for some $z \in A$. $x \leq y,\ y \leq z \Rightarrow x \leq z$. Thus $x \in (A]$. Therefore ((A]] ⊆ (A] and from (i) A ⊆ (A] ⇒ (A] = ((A]] and hence ((A]] = (A].

(iii) Let $x \in (A]\,\Gamma\,(B] \Rightarrow x \leq a\alpha b$ for some $a \in A$, $\alpha \in \Gamma$, $b \in B \Rightarrow x \leq a\alpha b$ for some $a\alpha b \in A\Gamma B \Rightarrow x \in (A\Gamma B]$. Therefore (A] Γ (B] ⊆ (AΓB].

(iv) From (i) B ⊆ (B] ⇒ A ⊆ B ⊆ (B].

(v) A ⊆ B ⇒ A ⊆ (B] ⇒ (A] ⊆ ((B]] = (B] Therefore (A] ⊆ (B].

DEFINITION 3.63 : Let S be an ordered Γ-semigroup. A nonempty subset T of S is said to be an ***ordered*** **Γ*-subsemigroup*** of S if $a\gamma b \in T$, for all $a, b \in T$ and $\gamma \in \Gamma$ and $t \in T, s \in S, s \leq t \Rightarrow s \in T$.

EXAMPLE 3.64 : Let S = [0,1] and Γ = { 1/*n* : *n* is a positive integer}. Then S is an ordered Γ-semigroup under the usual multiplication and usual order relation. Let T = [0, 1/2]. Now T is a nonempty subset of S and $a\gamma b \in T$, for all $a, b \in T$ and $\gamma \in \Gamma$. Then T is an ordered **Γ**-subsemigroup of S.

THEOREM 3.65 : A nonempty subset T of an ordered Γ-semigroup S is an ordered Γ-subsemigroup of S iff (1) TΓT ⊆ T, (2) (T] ⊆ T.

THEOREM 3.66 : The nonempty intersection of two ordered Γ-subsemigroups of an ordered Γ-semigroup S is an ordered Γ-subsemigroup of S.

Proof : Let T_1, T_2 be two ordered Γ-subsemigroups of S. Let $a, b \in T_1 \cap T_2$ and $\gamma \in \Gamma$.

$a, b \in T_1 \cap T_2 \Rightarrow a, b \in T_1$ and $a, b \in T_2$

$a, b \in T_1$, $\gamma \in \Gamma$, T_1 is an ordered Γ-subsemigroup of S $\Rightarrow a\gamma b \in T_1$ and $(T_1] \subseteq T_1$.

$a, b \in T_2$, $\gamma \in \Gamma$, T_2 is a Γ-subsemigroup of S $\Rightarrow a\gamma b \in T_2$ and $(T_2] \subseteq T_2$.

$a\gamma b \in T_1$, $a\gamma b \in T_2 \Rightarrow a\gamma b \in T_1 \cap T_2$ and $T_1 \cap T_2 \subseteq T_1$, $T_1 \cap T_2 \subseteq T_2$

$\Rightarrow (T_1 \cap T_2] \subseteq (T_1] = T_1$ and $(T_1 \cap T_2] \subseteq (T_2] = T_2 \Rightarrow (T_1 \cap T_2] \subseteq T_1 \cap T_2 \Rightarrow (T_1 \cap T_2] = T_1 \cap T_2$. and hence $T_1 \cap T_2$ is an ordered Γ-subsemigroup of S.

THEOREM 3.67 : The nonempty intersection of any family of ordered Γ-subsemigroups of an ordered Γ-semigroup S is an ordered Γ-subsemigroup of S.

In the following we are introducing an ordered Γ-subsemigroup which is generated by a subset and a ***cyclic ordered Γ-subsemigroup*** of an ordered Γ-semigroup.

DEFINITION 3.68 : Let S be an ordered Γ-semigroup and A be a nonempty subset of S. The smallest ordered Γ-subsemigroup of S containing A is called an ***ordered*** **Γ*-subsemigroup of S generated by A***. It is denoted by < A >.

THEOREM 3.69 : Let S be an ordered Γ-semigroup and A be a nonempty subset of S. Then < A > = the

intersection of all ordered Γ-subsemigroups of S containing A.

DEFINITION 3.70 : Let S be an ordered Γ-semigroup. An ordered Γ-subsemigroup T of S is said to be ***cyclic ordered Γ-subsemigroup*** of S if T is generated by a single element subset of S.

DEFINITION 3.71 : An ordered Γ-semigroup S is said to be a ***cyclic ordered Γ-semigroup*** if S is a cyclic ordered Γ-subsemigroup of S itself.

4. SPECIAL ELEMENTS OF AN ORDERED Γ-SEMIGROUP

We now introduce α-idempotent element and Γ-idempotent element in an ordered Γ-semigroup.

DEFINITION 4.1 : An element a of an ordered Γ-semigroup S is said to be a ***α-idempotent*** provided $a \leq a\alpha a$.

NOTE 4.2 : The set of all α-idempotent elements in an ordered Γ- semigroup S is denoted by E_S^{α} .

THEOREM 4.3 : For any ordered Γ-semigroup S, E_S^{α} with the binary relation defined by $a \leq b$ iff $a = a\alpha b = b\alpha a$ is a patially ordered set.

DEFINITION 4.4 : An element a of an ordered Γ-semigroup S is said to be an ***idempotent*** or **Γ-*idempotent*** if $a \leq a\alpha a$ for all $\alpha \in \Gamma$.

NOTE 4.5: An element a of an ordered Γ- semigroup S is said to be an ***idempotent*** or **Γ-*idempotent*** if $a \in (a\Gamma a]$

NOTE 4.6 : In an ordered Γ-semigroup S, ***a*** is an idempotent iff a is an α-idempotent for all $\alpha \in \Gamma$.

We now introduce an idempotent ordered Γ-semigroup and a strongly idempotent ordered Γ-semigroup.

DEFINITION 4.7 : An ordered Γ-semigroup S is said to be an ***ordered idempotent*** **Γ-*semigroup*** provided every element of S is α –idempotent for some $\alpha \in \Gamma$.

DEFINITION 4.8 : An ordered Γ-semigroup S is said to be an ***ordered strongly idempotent Γ- semigroup*** provided every element in S is an idempotent.

We now introduce a special element which is known as midunit in an ordered Γ-semigroup.

DEFINITION 4.9 : An element ***a*** of an ordered Γ-semigroup S is said to be a ***midunit*** provided $(x\Gamma a\Gamma y] = (x\Gamma y]$ for all $x, y \in S$.

NOTE 4.10 : Identity of an ordered Γ- semigroup S is a midunit.

We now introduce an *r*-element in an ordered Γ-semigroup and also a generalized commutative ordered Γ-semigroup.

DEFINITION 4.11 : An element 'a' of ordered Γ-semigroup S is said to be ***an r-element*** provided $(a\Gamma s] = (s\Gamma a]$ for all $s \in S$ and if $x, y \in$ S, then $(a\Gamma x\Gamma y] = (b\Gamma y\Gamma x]$ for some $b \in$ S.

DEFINITION 4.12 : An ordered Γ- semigroup S with identity 1 is said to be *an* ***ordered generalized commutative Γ- semigroup*** provided 1 is an *r*-element in S.

We now introduce a regular element in an ordered Γ-semigroup and regular ordered Γ-semigroup.

DEFINITION 4.13 : An element a of an ordered Γ-semigroup S is said to be ***regular*** provided $a \leq a\alpha x\beta a$, for some $x \in$ S and $\alpha, \beta \in \Gamma$.

NOTE 4.14 : An element a of an ordered Γ-semigroup S is said to be regular provided $a \in (a\Gamma S\Gamma a]$.

DEFINITION 4.15 : An ordered Γ- semigroup S is said to be an ***ordered regular*** **Γ- *semigroup*** provided every element is regular.

EXAMPLE 4.16: Let S = {0, a, b} and Γ be any nonempty set. If we define a binary operation on S as the following Cayley table, then S is a semigroup.

.	0	a	b
0	0	0	0
a	0	a	a
b	0	b	b

Define a mapping from S × Γ × S to S as $a\alpha b = ab$ for all $a, b \in$ S and $\alpha \in \Gamma$. Then S is ordered regular Γ-semigroup under usual partial order relation.

THEOREM 4.17 : Every α-idempotent element in an ordered Γ-semigroup is regular

Proof **:** Let a be an α-idempotent element in an ordered Γ-semigroup S.
then $a \leq a\alpha a$ for some $\alpha \in \Gamma$. Hence $a \leq a\alpha a\alpha a$. Therefore a is a regular element.

We now introduce left regular element, right regular element, completely regular element in an ordered Γ-semigroup and completely regular ordered Γ-semigroup.

DEFINITION 4.18 : An element a of an ordered Γ-semigroup S is said to be ***left regular*** provided $a \leq a\alpha a\beta x$, for some $x \in$ S and $\alpha, \beta \in \Gamma$. i.e, $a \in (a\Gamma a\Gamma S]$.

DEFINITION 4.19 : An element a of an ordered Γ-semigroup S is said to be ***right regular*** provided $a \leq x\alpha a\beta a$, for some $x \in$ S and $\alpha, \beta \in \Gamma$. i.e, $a \in (S\Gamma a\Gamma a]$.

DEFINITION 4.20 : An element a of an ordered Γ-semigroup S is said to be ***completely regular*** provided, there exists an element $x \in$ S such that $a \leq a\alpha x\beta a$ for some $\alpha, \beta \in \Gamma$ and $a\alpha x = x\beta a$.

NOTE 4.21 : An element a of an ordered Γ-semigroup S is said to be ***completely regular*** provided, there exists an element $x \in$ S such that $a \in (a\Gamma x\Gamma a\ ]$ and $a\Gamma x = x\Gamma a$.

DEFINITION 4.22 : An ordered Γ-semigroup S is said to be ***ordered completely regular* Γ*-semigroup*** provided every element of S is completely regular.

DEFINITION 4.23 : Let S be an ordered Γ-semigroup, $a \in$ S and $\alpha, \beta \in \Gamma$. An element $b \in$ S is said to be an **(*α*, *β*)*-inverse*** of a if $a \leq a\alpha b\beta a$ and $b \leq b\beta a\alpha b$.

THEOREM 4.24 : Let S be an ordered Γ-semigroup and *a* ∈ S. If *a* has an (*α*, *β*)-inverse then *a* is a regular element.

Proof : Suppose that b is an (α, β)-inverse of a.
Then $a \leq a\alpha b\beta a$ and $b \leq b\beta a\alpha b$. Therefore $a \leq a\alpha b\beta a$ and hence a is regular.

We now introduce a semisimple element of a Γ-semigroup and a semisimple Γ-semigroup.

DEFINITION 4.25 : An element a of an ordered Γ-semigroup S is said to be ***semisimple*** provided $a \in (< a > \Gamma < a >]$, that is, $(< a >] = (< a > \Gamma < a >]$.

DEFINITION 4.26 : A Γ- semigroup S is said to be ***ordered semisimple Γ- semigroup*** provided every element of S is a semisimple element.

We now introduce an intra regular element of an ordered Γ-semigroup.

DEFINITION 4.27 : An element a of a Γ-semigroup S is said to be ***intra regular*** provided $a \leq x\alpha a\beta a\gamma y$ for some $x, y \in$S and $\alpha, \beta, \gamma \in \Gamma$.

NOTE 4.28 : An element a of a Γ-semigroup S is said to be intra regular provided $a \in (S\Gamma a\Gamma a\Gamma S]$.

EXAMPLE 4.29 : The Γ-semigroup given in example 4.16 is an intra regular ordered Γ-semigroup.

THEOREM 4.30: If '*a*' is a completely regular element of an ordered Γ- semigroup S, then *a* is regular and semisimple.

Proof **:** Since a is a completely regular element in the Γ- smigroup S, $a = a\alpha x\beta a$ for some α, β ∈ Γ and $x \in$ S. Therefore a is regular.
Now $a \leq a\alpha x\beta a \in a\Gamma x\Gamma a \subseteq < a > \Gamma < a > \Rightarrow a \in (< a > \Gamma < a >]$. Therefore a is semisimple.

THEOREM 4.31 : If '*a*' is a completely regular element of an ordered Γ- semigroup S, then *a* is both a left regular element and a right regular element.

Proof **:** Suppose that a is completely regular. Then $a \in (a\Gamma S\Gamma a]$ and $a\Gamma S = S\Gamma a$.
Now $a \in (a\Gamma S\Gamma a] = (a\Gamma a\Gamma S]$. Therefore a is left regular.
Also $a \in (a\Gamma S\Gamma a] = (S\Gamma a\Gamma a]$. Therefore a is right regular.

THEOREM 4.32 : If '*a*' is a left regular element of a Γ-semigroup S, then *a* is semisimple.

Proof: Suppose that a is left regular. Then $a \in (a\Gamma a\Gamma x]$ and hence $a \in (< a > \Gamma < a >]$. Therefore a is semisimple.

THEOREM 4.33 : If '*a*' is a right regular element of a Γ-semigroup S, then *a* is semisimple.

Proof: Suppose that a is right regular. Then $a \in (a\Gamma a\Gamma x]$ and hence $a \in (< a > \Gamma < a >]$. Therefore a is semisimple.

THEOREM 4.34 : If '*a*' is a regular element of a Γ-semigroup S, then *a* is semisimple.

Proof: Suppose that a is regular element of Γ-semigroup S.
Then $a \leq a\alpha x\beta a$, for some $x \in S$, $\alpha, \beta \in \Gamma \Rightarrow a \in (a\Gamma x\Gamma a]$ and hence $a \in (< a > \Gamma < a >]$.
Therefore a is semisimple.

THEOREM 4.35 : If '*a*' is a intra regular element of a Γ- semigroup S, then *a* is semisimple.

Proof: Suppose that a is intra regular. Then $a \in (x\Gamma a\Gamma a\Gamma y]$ for $x, y \in S$ and hence $a \in (< a > \Gamma < a >]$ Therefore a is semisimple.

DEFINITION 4.36 : An element a of an ordered Γ-semigroup S is said to be ***left α-cancellative*** *provided* for $\alpha \in \Gamma$, $a\alpha b \leq a\alpha c$ implies $b \leq c$.

DEFINITION 4.37 : An element a of an ordered Γ-semigroup S is said to be ***right α-cancellative*** provided for $\alpha \in \Gamma$, $b\alpha a \leq c\alpha a$ implies $b \leq c$.

DEFINITION 4.38 : An element a of an ordered Γ-semigroup S is said to be ***α-cancellative*** provided a is both a left α-cancellative element and a right α-cancellative element.

DEFINITION 4.39 : An element a of an ordered Γ-semigroup S is said to be ***left*** **Γ**-***cancellative*** provided a is left α-cancellative for all $\alpha \in \Gamma$.

DEFINITION 4.40 : An element a of an ordered Γ-semigroup S is said to be ***right*** **Γ**-***cancellative*** provided a is right α-cancellative for all $\alpha \in \Gamma$.

DEFINITION 4.41 : An element a of an ordered Γ-semigroup S is said to be **Γ**-***cancellative*** provided a is a both left Γ-cancellative and right Γ-cancellative.

NOTE 4.42 : $a\Gamma b \leq c\Gamma d$ if and only if $a\Gamma b \subseteq c\Gamma d$.

DEFINITION 4.43 : An element a of an ordered Γ-semigroup S is said to be ***strongly left*** **Γ**-***cancellative*** provided $a\Gamma b \leq a\Gamma c$ implies $b \leq c$.

NOTE 4.44 : An element a of an ordered Γ-semigroup S is said to be ***strongly left*** **Γ**-***cancellative*** provided $a\alpha b \leq a\beta c$, $\alpha, \beta \in \Gamma \Rightarrow b \leq c$.

DEFINITION 4.45 : An element a of an ordered Γ-semigroup S is said to be ***strongly right*** **Γ**-***cancellative*** provided $b\Gamma a \leq c\Gamma a$ implies $b \leq c$.

NOTE 4.46 : An element a of an ordered Γ-semigroup S is said to be ***strongly right*** **Γ**-***cancellative*** provided $b\alpha a \leq c\beta a$, $\alpha, \beta \in \Gamma \Rightarrow b \leq c$.

DEFINITION 4.47 : An element a of an ordered Γ-semigroup S is said to be ***strongly*** **Γ**-***cancellative*** provided a is a both strongly left **Γ**-cancellative and strongly right **Γ**-cancellative.

Now we introduce a weakened version of strongly cancellative ordered Γ-semigroup is provided by the following notion.

DEFINITION 4.48 : An ordered Γ-semigroup S is said to be ***separative*** if for any $x, y \in S$,

(1) $x\Gamma x \leq x\Gamma y$ and $y\Gamma y \leq y\Gamma x$ imply $x = y$,
(2) $x\Gamma x \leq y\Gamma x$ and $y\Gamma y \leq x\Gamma y$ imply $x = y$.

THEOREM 4.52 : In a separative ordered Γ-semigroup S, for any *x, y, a, b* ∈ S, the following statements hold.

(i) $x\Gamma a \leq x\Gamma b$ if and only if $a\Gamma x \leq b\Gamma x$,
(ii) $x\Gamma x\Gamma a \leq x\Gamma x\Gamma b$ implies $x\Gamma a \leq x\Gamma b$,
(iii) $x\Gamma y\Gamma a \leq x\Gamma y\Gamma b$ implies $y\Gamma x\Gamma a \leq y\Gamma x\Gamma b$.

Proof **:** (i) If $x\Gamma a \leq x\Gamma b$, then $a\Gamma(x\Gamma a)\Gamma x \leq a\Gamma(x\Gamma b)\Gamma x$ and $b\Gamma(x\Gamma a)\Gamma x \leq b\Gamma(x\Gamma b)\Gamma x$ so that $(a\Gamma x)\Gamma(a\Gamma x) \leq (a\Gamma x)\Gamma(b\Gamma x)$ and $(b\Gamma x)\Gamma(a\Gamma x) \leq (b\Gamma x)\Gamma(b\Gamma x)$ which by separativity implies $(a\Gamma x) \leq (b\Gamma x)$. The opposite implication follows by symmetry.

(ii) If $x\Gamma x\Gamma a \leq x\Gamma x\Gamma b$, then by part (i), $x\Gamma a\Gamma x \leq x\Gamma b\Gamma x$ $\Rightarrow$ $(a\Gamma x)\Gamma(a\Gamma x) \leq (a\Gamma x)\Gamma(b\Gamma x)$ and $(b\Gamma x)\Gamma(a\Gamma x) \leq (b\Gamma x)\Gamma(b\Gamma x)$ and thus by separativity $(a\Gamma x) \leq (b\Gamma x)$. But then by part (i) $x\Gamma a \leq x\Gamma b$.

(iii) Let $x\Gamma y\Gamma a \leq x\Gamma y\Gamma b$. Then $x\Gamma y\Gamma a\Gamma y \leq x\Gamma y\Gamma b\Gamma y$, and thus by part (i), $y\Gamma a\Gamma y\Gamma x \leq y\Gamma b\Gamma y\Gamma x$. Multiplying by suitable elements on the right and using part (i), we obtain the following strings of inequalities:

$y\Gamma a\Gamma y\Gamma x\Gamma a \leq y\Gamma b\Gamma y\Gamma x\Gamma a$ $y\Gamma a\Gamma y\Gamma x\Gamma b \leq y\Gamma b\Gamma y\Gamma x\Gamma b$

$a\Gamma y\Gamma x\Gamma a\Gamma y \leq b\Gamma y\Gamma x\Gamma a\Gamma y$ $a\Gamma y\Gamma x\Gamma b\Gamma y \leq b\Gamma y\Gamma x\Gamma b\Gamma y$

$(a\Gamma y\Gamma x)\Gamma(a\Gamma y\Gamma x) \leq (b\Gamma y\Gamma x)\Gamma(a\Gamma y\Gamma x)$

$(a\Gamma y\Gamma x)\Gamma(b\Gamma y\Gamma x) \leq (b\Gamma y\Gamma x)\Gamma(b\Gamma y\Gamma x)$

Which by separativity implies $(a\Gamma y\Gamma x) \leq (b\Gamma y\Gamma x)$. But then by part (i), we have $y\Gamma x\Gamma a \leq y\Gamma x\Gamma b$.

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