Parametric Study of Thin Cylindrical Shells

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Abstract: **Reinforced concrete shells and folded plates are the systems widely used for its effectiveness and for its appealing appearance. These structures are unique in character and are termed as stressed skin structures. It carries load by its shape rather than material strength.Shell structures are in curved shape in which thickness is small compared to radius and other dimensions. Because of their geometry and small flexural rigidity of the skin, tend to carry loads primarily by direct stresses acting in their plane.Due to the incompatibility in the method of analysis of shell structures and the insufficient knowledge led to the structures having thickness of 25mm to 30mm. The accelerated development of material, system and software facilities with analytical tools like finite element analysis, there is massive scope for the advancement of shell structures. Shells can be singly or doubly curved with positive or negative curvature. Each type of shell has its own advantageous in respect of the application.Cylindrical shells are under the category of singly curved shells which are the simplest form of shell. Due to their excellent behaviour under compression, the cylindrical shell has been proposed as precast shell roof. The present study is proposed to analytically investigate in arrive an optimum geometry of cylindrical shells for efficiency. The parametric study is carried out on thin cylindrical shell structureof 1mx1m with the thickness of 10mm with40mm depth edge rib beams on all four sides,with and without shell reinforcements. Closed form solutions are carried out withmembrane analysis is used to determine the stresses acting on the shell.A nonlinear finite element analysis has been carried out with both material and geometrical nonlinearity. The results from the nonlinear finite element analysis and the close form solution are compared and found that the results have discrepancies and the close form solutions are more conservative.**

Keywords: Shell structures,Concrete, Material Nonlinearity, Damaged Plasticity Model, Cylindrical Shell, FEA.

INTRODUCTION:

Shell structures have been present in nature since ancient times. The shape of a turtle hood protects the tender body of the turtle.Furthermore similar examples led to the investigation of shell structures and development of theory of shells. Igloo is the perfect example of man-made shell structure. In the early 20th century the shell structures werepredominant. The Zeiss planetarium in Germany was the first thin concrete shell of the modern era. Thin shell structures spanned and flourished from 1950 to 1970.The structures constructed in that span had thickness of the structure to be more between 25 to 35mm. Due to the high cost of construction, intricacy of the mould to be prepared and difficulty in the method of analysis, shell structures became scarce. Theory of shells and the shell dimensions were limited to a certain ratio in the earlier times. Advancements and recent trends in civil engineering have made the thin shell structures to be revisited. The advancement brought about the shells to be viewed as potential precast building roofs.

Shells are curved structures with small thickness compared to radius and other dimensions.Shells belong to the class of stressed skin structures which by virtue of their geometry and small flexural rigidity tend to carry loads primarily by direct stresses lying in their plane accompanied by little or no bending. They are classified as thick shells and thin shells. The ratio of thickness to the radius of curvature is less than or equal to 1/20 is termed as thin shell. Shells are further classified into singly curved and doubly curved. The Gauss curvature is zero for singly curved shells and either positive or negative values for doubly curved shells.

Closed form solutions are the methods of analyzing the shells to be investigated. Among which membrane analysis is used for analyzing the structure. Linear finite element analysis is carried out with the help of ABAQUS/CAE finite element module. The model created and the section properties of concrete have been applied. Models with and without reinforcement along the edge ribs have been created. The stresses acting onboth the shell under uniform pressure is studied with the help of this software.The shell structures are predominant in compression and therefore concrete is best suited for the purpose. Reduction of materials is the most influential factor for the shell roofs. Owing to the reduction in thickness of the member and capacity to span over large area material reduction is achieved. The shell action provides the structural integrity for the shells. They are also used when large space, column free space is required. The aesthetic appearance of the shell roofs are also their main advantage.

Literature Review:

Shell structures are built using concrete for their load carrying capacity and their aesthetic appearance. Under external loads cylindrical shells are prone to buckling rather than strength failure. Buckling investigation using analytical model, numerical and semi empirical models were done by Himayat Ullah and Sagheer Ahmad (2007). The buckling results obtained using the Analytical shell model was done based on the Kirchoff-Love hypothesis. Buckling coefficients were included in the Semi empirical model for correcting the discrepancies between theory and test data. In Numerical buckling analysis, the model was analysed by both linear and non linear method to predict the buckling strength. They concluded that linear theory can only be used for determining the mode shapes. Computer aided analysis of multiple cylindrical shells were done using SAP2000 software by Ravindra Rai and Umesh Pendharkar (2012). The numerical model was analysed on different parameters changing their thickness and radius of curvature. They considered all model having different radius and same thickness and the portion of maximum forces lies at that portion where two shells are joined with each other. For models having same radius with different thickness it is found that the portion of maximum moment, maximum forces and maximum stresses is remaining same but due to reduction in thickness all moment,forces and stresses are reduced.Mahmoud Shariati in the year 2010 published paper on numerical and experimental study on buckling of cylindrical panels subjected to axial load. The paper deals about the effective of length, sector angle and different boundary conditions on the buckling load and post buckling of CK20 cylindrical panel. ABAQUS software has been used for the numerical analysis and the results concluded that increase in length decreases the buckling load and sector angle changes increases the buckling load.

The precast techniques are more popular due to its uniqueness and quality. Shell structures are more appropriate for the precast roofing due to its light weight. In views of this, it is necessary to investigate the shells for arriving more efficient shell system with light weight. The parametric study of thin cylindrical shells is proposed for precast shell roof. Thin shells are an example of strength through form as opposed to strength through mass. The effort in design is to make the shell as thin as practical requirementswill permit so that dead weight is reduced and the structure functions as a membrane free from large bending stresses. By this means, a minimum of materials is used to the maximum structural advantage.Shell roofs provide the aesthetic appearance and they employ to provide large column free space. Cylindrical shells are singly curved shells in which the generatix is a straight line. The surface is generated by moving a straight line against a plane curve. Projection of arc of the shell is known as chord width. Cylindrical shell used in this project is supported on both ends by an end diaphragm and on all four sides by edge ribs.

Membrane Analysis:

Closed form solutions involve various methods to determine the stresses acting on the shell surface. The three most common method of analysis of shell structures are Membrane theory, Beam method and Analytical method. Among which 'Membrane theory' is the best method to analyse for short, thin cylindrical structures.

Membrane theory was propounded by Lame and Clapeyron in the year 1828. In the membrane analysis it is assumed that no bending and twisting moments are developed in the shell and only the forces lying in the shell surface develop giving direct stresses. The shell is regarded as a perfectly flexible membrane which is infinite in extent and is assumed to carry loads by means of forces in its plane only. Thin shells acts partially as an arch and partially as a beam. The internal forces acting on the shell are longitudinal normal force N_x, a transverse normal force N_ϕ and shear forces $N_{x\phi}$ and $N_{\phi x}$. Equations of the shell are maintained by the in-plane stresses. External bending moment computed at any section treating the shell as a simply supported beam is restricted by resultant forces of longitudinal forces, axial forces at the edge members and vertical component of bending at the edge member.

The forces acting on the cylindrical shell obtained after integration are formulated separately for dead load and live load. The forces based upon the dead load are shown in equation.1 to equation.3:

$$Nx = \frac{q.\cos\emptyset}{R}\left(x^2 - \frac{L^2}{4}\right) \ldots\ldots\ldots 1$$

$$N_\emptyset = -q.R.\cos\emptyset \ldots\ldots\ldots\ldots.. 2$$

$$N_{x\emptyset} = -2q.\sin\emptyset.x \ldots\ldots\ldots\ldots 3$$

The equations for the live load acting on the shell surface are given in equations from equation.4 to equation.6 :

$$Nx = \frac{1.5\,p^0}{R}\left(x^2 - \frac{L^2}{4}\right)\cos 2\emptyset \ldots\ldots\ldots\ldots 4$$

$$N_\emptyset = -p^0.R.\cos^2\emptyset \ldots\ldots\ldots\ldots\ldots\ldots.. 5$$

$$N_{x\emptyset} = -1.5\,p^0.\sin 2\emptyset.x \ldots\ldots\ldots\ldots\ldots 6$$

Where,
q = Dead load of the shell
R = Radius of the shell

L = Length of the shell
p^0 = Live load of the shell
x = Point at which the stresses are considered.

These are the equations for membrane analysis of a shell with circular directrix. The dead load of the shell is taken as 24 KN/m^3. The stresses are calculated at the edge and also at the semi-central angle. The shear stresses are calculated at the middle of the edge member.

The dimensions of the thin cylindrical shell are tabulated in table.1.

Table 1 – Dimensions of cylindrical shell

1.	Span	1m
2.	Chord Length	1m
3.	Thickness	0.015m
4.	Radius of Curvature	1.3m
5.	Semi-central Angle	22.5º

Since the shells are investigated as roof for buildings, the live load of the shell is taken as 0.75kN/m^2 as per National Building Code(2005).

The stress values of dead load and live load are given in table 2 below:

Table 2 – Final stress component values using membrane theory

Stress	$\Phi = 0^o$	$\Phi = 22.5^o$
N_x	-1443 N/m^2	-1264.58 N/m^2
N_ϕ	-285 N/m^2	-216.90 N/m^2
$N_{x\phi}$	0	-1070.83 N/m^2

Finite Element Analysis

ABAQUS/CAE is a general purpose; finite element module which provides a large number of capabilities for analyzing many different types of problems, including many nonstructural applications.The program is designed for ease of use on complex problems and has a simple input language, with comprehensive data checking as well as a wide range of preprocessing and post processing output display options. ABAQUS/CAE can be used to create application-specific systems. They use a neutral database files that are machine independent. The user interface consists of integrating modeling, analysis, job management and results evaluation. CAE stands for Complete Abaqus Environment which provides the most complete interface with ABAQUS solver programs available. The model created in the software is eligible for editing and regenerating which are very useful in the case of parametric studies. After meshing the model the software can generate plots that highlight the elements whose aspect ratios, maximum and minimum angles, and shape factors exceed specified limits.

The cylindrical shell used in this project has a span of 1mX1m. The thickness of the shell is taken as 10mm; considering short thin cylindrical shells. The shell modelcreated using solid, homogeneous material is shown in figure1 and the wire model of cylindrical shell is shown in figure 2.

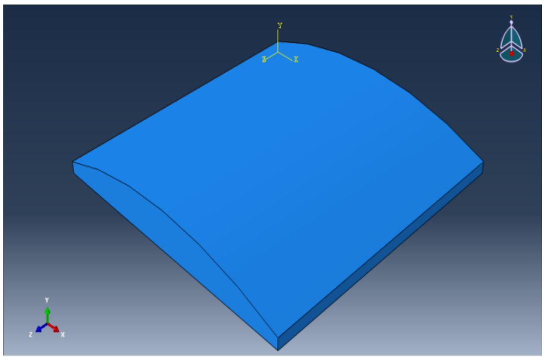

Fig 1 – Cylindrical shell model

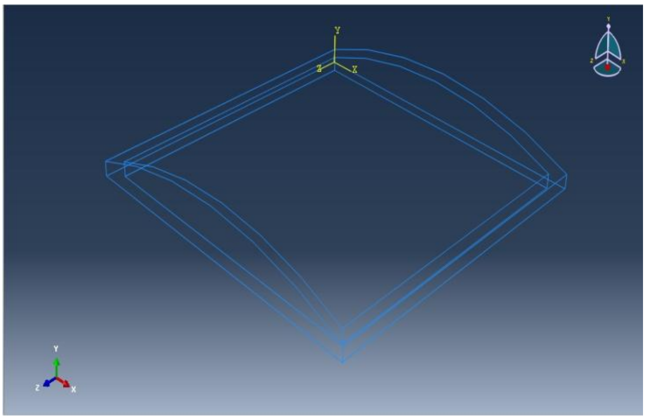

Fig 2 – Wire model of cylindrical shell without reinforcement

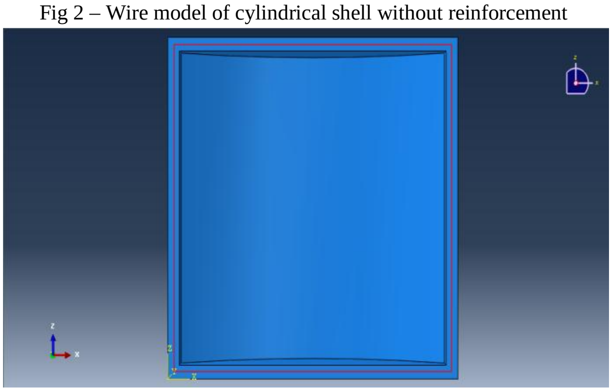

Fig – 3 Cylindrical shell model with reinforcements

Another model with same span and same thickness was generated with steel reinforcements along the edge ribs of the shell are shown in figure 3. The steel is assumed to be 10mm diameter bar. The placement of the reinforcement was placed with the cover of 20mm along the edges and 10mm from the bottom.

The material property of the shell structure has been assigned as concrete with the density of 24kN/m^3, Young's modulus value of 31622.74 and Poisson's ratio of 0.18. The concrete damage plasiticity has been considered for the non-linearity behaviour of concrete with dilation angle of 34 degrees and 1.16 for ratio between equi-biaxial compressive stress to uni-axial compressive stress. The tensile behaviour of concrete has been tabulated as for 2.5 yield stress 0 raking strain and for 0 yield stress, cracking strain of 0.001. the concrete tension damage has the damage parameter of 0.9 for which the cracking strain is assumed as 0.9. The compressive behaviour of concrete is taken as tabulated in table3:

Table 3 – Compressive behaviour of concrete values

Yield Stress	Inelastic Strain
7.52	0
15	0.0008
20	0.001
25	0.0015
30	0.0025

The material property of the reinforcement which is included in the second model was given the density of 77kN/m^3. The elastic property of the steel was given 2E05N/mm^2 with the Poisson's ratio of 0.3. The plastic behaviour of the steel is given as mentioned in table 4:

Table 4 – Plastic behaviour of steel values

Yield Stress	Plastic Strain
332	0
352	0.0001
373	0.0003
394	0.001
415	0.002
350	0.003

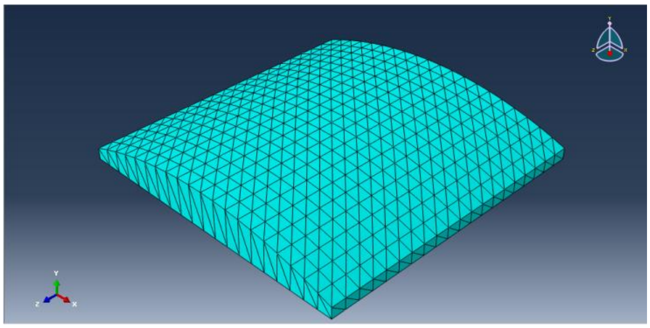

Fig 4 – Meshed model of cylindrical shell

The model was created by modeling outer and innerdimensions of the shell and cutting them. The shell has been meshed using a 3D stress element of C3D4 which is a 4noded tetrahedron element. The approximate size of the global element has been given 50. The meshed model of the structure is shown in figure 4.

For the assembly of the cylindrical shell with reinforcements, two instances were created with first being the shell alone and the second being reinforcement alone. The Embeddded constraint was used to embed the steel into the concreteshell.

Loading of the shell was given as uniform pressure along the top surface of the shell as shown in figure 5 and also the load given was calculated earlier for membrane analysis. The load was given as 1.11E-03.

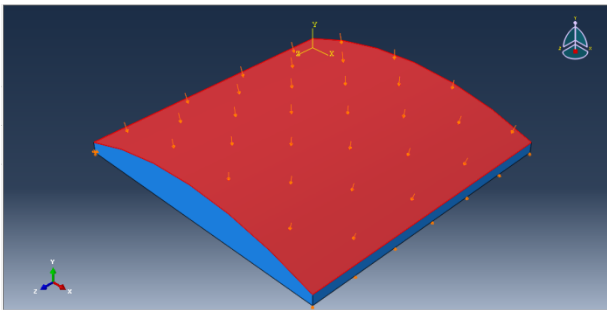

Fig 5 – Loading on the shell

The boundary conditions of the shell are takes on the opposite edges of the edge members as seen in figure 6. Along one side the vertical displacement is arrested and on the other side both vertical and horizontal displacements are arrested.

The static riks method of analysis is carried out inorder to overcome the instability occurring in the static general method of analysis.

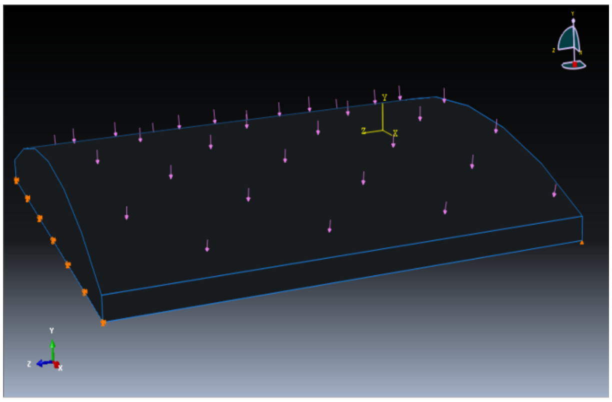

Fig 6 – Boundary conditions on the shell

Results and Discussions:

The stress values based upon the loading conditions were obtained and are shown below in figure 7 and figure 8. The stress values along xy direction were taken and presented.

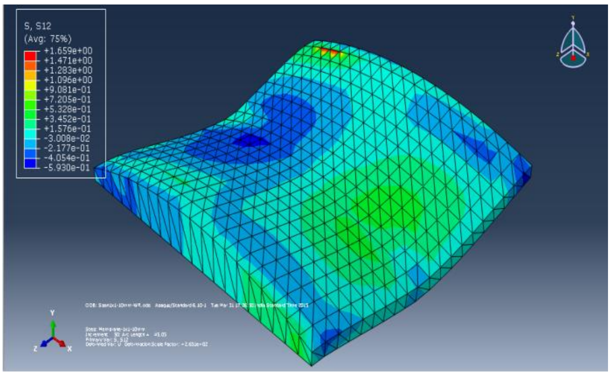

Fig 7 – Stress value of cylindrical shell with reinforcement

The value obtained along the diaphragm of the shell is -3E-02. The stress value is obtained at the crown of the shell i.e., the semi-central angle of the shell.

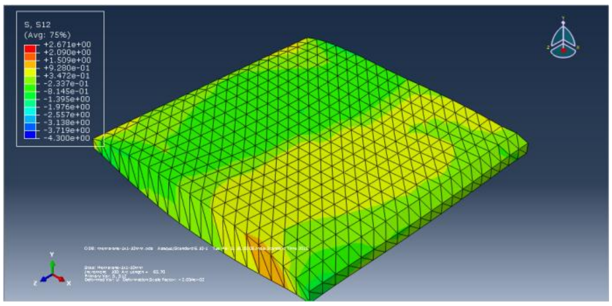

Fig 8 – Stress value of cylindrical shell without reinforcement

The deformation of the structure according to the given uniform loading has been obtained. The load deformation graph for both shells with and without reinforcement is shown figure 9 and figure 10.

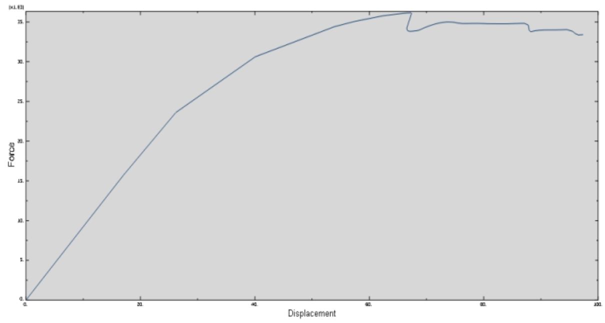

Fig 9 – Load deformation graph of structure with reinforcement

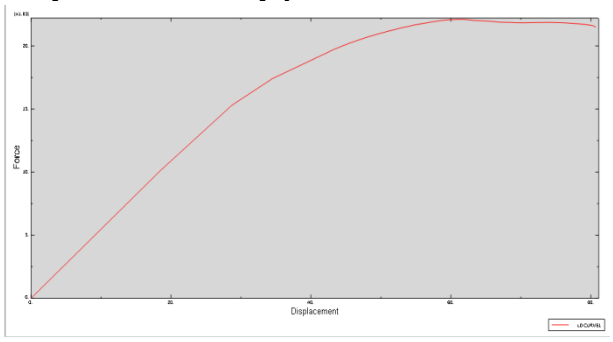

Fig 10 – Load deformation graph of structure without reinforcement

The maximum stress value obtained at the semi-central angle at the crown of the shell is obtained as -2.337E-1.

The final stress values obtained by classical method, using finite element method of both cylindrical shells with reinforcement and without reinforcement are tabulated in table 5.

Table 5 – Stress values comparison

Method/Type	Compressive Stress N/mm^2
Membrane Theory	1264
ABAQUS - Without Reinforcement	2337
ABAQUS – With Reinforcement	3008

CONCLUSION

The investigation of thin cylindrical shell between shell with reinforcement and without reinforcements have been studied and reported. The stress values acting along the edges of the structure has been calculated and found out. The following conclusions were obtained:

- The stress component value obtained through membrane analysis is less compared with the finite element analysis.
- There were 85% increase in the stress values between classical solution and ABAQUS shell without reinforcement.
- There was 135% increase in the stress values between the membrane solution and shell model created through FEM.

The discrepancies in the stress component values of the thin cylindrical shell are mainly due to the method of analysis. The membrane analysis proved to be more conservative and therefore discrepancies arise.

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