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Parametric Amplification of Optically Excited Coherent Collective Mode in a Semiconductor Magneto-Plasma

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***Abstract*—In the present analytical investigation, using hydrodynamic model for one component plasma, we have explored the threshold and gain characteristics of parametric amplification of optically excited coherent collective mode in a semiconductor magneto-plasma. Propagation of intense pump beam in semiconductor plasma has been investigated by considering induced second order optical nonlinearity. Coupled mode theory has been applied to analyze coupling of electron and longitudinal optical phonons and further pump, polaron and signal modes in the medium. Theoretical model has been developed with the help of rotating wave approximation method. Numerical estimates have been made by considering n-InSb crystal illuminated by CO_2 laser in the presence of an external magnetostatic field under Voigt geometry. The results indicate that the second order optical susceptibility can be tailored by varying magnitudes of external magnetostatic field and doping concentration. It has been found that the proper selection of magnetostatic field and doping concentration reduces the threshold pump field for the onset of parametric amplification and enhances the parametric gain coefficient. The results strongly suggest the potential of n-InSb-CO_2 laser system for the fabrication of parametric amplifier and design of optical switches.**

Keywords*—Parametric amplification, Fröhlich interaction, Magneto-plasma excitations, Semiconductor plasmas.

I. INTRODUCTION

Understanding and controlling the light-matter interactions are of fundamental importance owing to their manifold technological applications in science and engineering. More than 100 years ago, puzzles over blackbody radiation and atomic line spectra give rise the birth of quantum mechanics and since there have been countless developments, not least the laser. Since the development of the laser, a major area of research, known as optoelectronics emerged. It has been devoted to the development of optoelectronic devices capable of providing tunable coherent radiation at optical frequencies which are not directly available from laser sources [1]. Pulsed lasers have been used in optical tweezers for biomedical applications [2]. Surface plasmon polaritons have being developed as bio-molecule sensors for detection of chemical and biological species [3]. On the other hand, optical bistable studies in nonlinear surface-plasmon polaritonic crystals explore possible ways to control light with light [4].

Parametric amplification in the radio as well as microwave frequency regime was well studied before the development of lasers [5]. The same process was also expected at optical frequencies, but could be demonstrated only after the development of laser [6]. Optical parametric oscillation (OPO), an important extension of optical parametric amplification (OPA) was first demonstrated in 1965 [7]. The progress in OPA/OPO has been the subject of review papers by Brunner and Paul [8], Peng [9], Ciriolo et.al. [10]. OPA/OPO provide an attractive way of generating coherent infrared sources which could be tuned over a wide range of frequencies in the spectral range where lasers simply do not exist – for the need of spectroscopy, medical applications, remote sensing etc. This fundamental but interesting process in a solid-state plasma also plays an important role in fabrication of low-noise amplifiers [11] and oscillators [12]. This process is widely used for the low-loss optical switching in semiconductors [13].

In OPA, a weak signal is made to interact with a strong, higher frequency pump and both the generated difference frequency (known as the idler) and the original signal are amplified. In a polar material, the electric field interacts with both electrons and optical phonons. The scattered radiation consists generally of two parts: (i) a single particle portion, and (ii) a collective part. The single particle scattering is due to individually moving electrons in the plasma and is nearly elastic. This portion of the spectrum can be used to determine the electron velocity distribution and presents a great interest in transport theory. The collective mode of scattering is caused by plasma waves (plasmons) in the electron gas. In solid-state plasma, the collective modes interact with phonons and as a result, a number of plasmon-phonon interactions arise [14], such as (i) deformation potential interaction with both acoustical phonons (APs) as well as optical phonons (OPs); (ii) piezoelectric interaction with APs; and (iii) Fröhlich interaction with LOPs.

In noncentrosymmetric crystals, the electrostrictive strain gives rise to an electric field via piezoelectric effect. APs may be active via piezoelectric effect in certain directions. The piezoelectric plasmon-AP interaction affects to a large extent the nonlinear optical properties of semiconductor crystals for improving the performance of optoelectronic devices [15]. The Fröhlich interaction is a Coulomb interaction between itinerant electron and the longitudinal electric field produced by the LOPs. It is considered to be the strongest exciton–phonon interaction in polar semiconductor crystallites for incoming photon energy in resonance with excitonic states [16].

The interaction of free carrier collective excitations with LOPs by macroscopic longitudinal electric fields in polar semiconductors has been treated theoretically [17]. This coupled wave approach can also be generalized to include waves other than electromagnetic. However, in noncentrosymmetric crystals, the idler electromagnetic wave can be replaced by an optically excited coherent collective mode (viz. acoustical, optical, polaron, polariton modes etc.), and a strong tunable electromagnetic Stokes wave can be obtained as a signal wave at the expense of the pump wave using the coupled mode approach. A polaron is a quasi-particle the origin of which lies in the self-induced polarization of the conduction electron (or hole) in an ionic crystal or in a polar semiconductor [18, 19]. This polarization is local in character and arises due to the displacements of ions from the lattice positions caused by the field produced by the electron density perturbations. The resulting interaction with the polarized lattice changes the energy as well as mass of the electron [20].

Earlier studies on polaronic effects arising due to Fröhlich interaction were made by Lee and Pines [21], Feynman [22] and Landau [23] on theoretical grounds recognized as pioneering work in this field. Polaronic effects based on Fröhlich interaction between electrons and LOP of a polar semiconductor have been the subject of a number of research groups [24-28] and have emerged as an explosive area of research. The electron-phonon interaction based linear and nonlinear optical properties of semiconductor microcrystallites have been studied by Rink et al. [29]. It was observed by them that the strength of the electron phonon coupling increases with decreasing particle size. Blocking of polaron effect and spin-split cyclotron resonance in a two-dimensional electron gas has been studied by Wu et.al. [30]. They reported that due to electron-phonon interaction large corrections are produced on cyclotron resonance frequency not only in the resonant region, but also in the off-resonant region. The polaron effect well above the LOP energy was studied through cyclotron resonance measurements in compound semiconductor in presence of ultra-high magnetic fields by Miura and Imanaka [31]. They reported that the resonant polaron effect manifests itself when the cyclotron frequency approaches the LOP energy at sufficiently high magnetic fields. The concept of Fröhlich interaction and polaron mass was tested using infrared magneto absorption measurements in doped GaAs quantum well structures by Faugeras et.al. [32]. A study of polar optical phonon modes and Fröhlich electron-phonon interaction Hamiltonians in four layers $Ga_{1-x}Al_xAs$/GaAs coaxial cylindrical quantum cables yield that the high frequency phonon modes have more significant contributions to the coupling of the electron-phonon interaction [33]. Fröhlich interaction based electron-phonon interaction in a single modulation-doped GaInAs has been studied by Orlita et.al. [34]. Resonant enhancement of Raman modes of nano structured copper oxide films showed the existence of strong electron–phonon coupling mediated by Fröhlich interaction [35]. Fröhlich interaction based parametric oscillations in magnetized semiconductor plasmas have been studied by Dubey and Ghosh [36]. They reported the feasibility of achieving a polaron-induced single resonant parametric oscillator through an n-InSb-CO_2 laser system with a transverse magnetostatic field. Recently, acoustical phonon and polaron mode-induced OPA in transversely magnetized III-V semiconductors have been studied by Jangra et.al. [37]. They reported the significance of Fröhlich interaction in heavily doped transversely magnetized III-V semiconductors and possibility of fabrication of optical parametric amplifier using n-InSb-CO_2 laser system.

Here, it should be worth pointing out that Fröhlich interaction remains absent in covalent materials such as Ge and Si, but it significantly affect the mobility of III-V semiconductors. For this type of studies III-V semiconductors happens to be obvious choice because of the possibility of rendering them p-type or n-type conductors through doping. At low doping levels, the interaction remains unscreened. With increasing doping levels, plasmons and phonons will no longer remain decoupled, rather the system will exhibit oscillations at coupled phonon-plasmon modes with frequency

$$\Omega_{\pm}^2 = \frac{1}{2}\left[(\Omega_p^2 + \Omega_{LO}^2) \pm \{(\Omega_p^2 + \Omega_{LO}^2)^2 - 4\Omega_p^2\Omega_{TO}^2\}^{1/2}\right], \tag{1}$$

where $\Omega_p = \left(\frac{n_0 e^2}{m\varepsilon}\right)^{1/2}$ is the electron-plasma frequency.

$\varepsilon = \varepsilon_0\varepsilon_\infty$ is the dielectric constant, in which ε_∞ is the high frequency dielectric constant of the medium. Ω_{LO} and Ω_{TO} represent the longitudinal optical phonon frequency and transverse optical phonon frequency, respectively.

Hence, it would be interesting to find out the doping levels favorable for plasmon-LOP coupling in a degenerate polar semiconductor plasma.

Since semiconductors may be regarded as universally recognized materials with high optical nonlinearities which can be easily controlled by externally applied electric and magnetic fields [38, 39]. In the presence of externally applied magnetic field, the collective cyclotron excitations – LOPs coupling, via the macroscopic longitudinal electric field give rise to modified normal modes. Under Voigt geometry, modified normal modes frequencies at the center of Brillouin zone (zero wave vector mode) is given by [40]

$$\bar{\Omega}_{\pm}^2 = \frac{1}{2}\left[(\Omega_p^2 + \Omega_c^2 + \Omega_{LO}^2) \pm \{(\Omega_p^2 + \Omega_c^2 + \Omega_{LO}^2)^2 - 4(\Omega_p^2\Omega_{TO}^2 - \Omega_c^2\Omega_{LO}^2\}^{1/2}\right] \tag{2}$$

where $\Omega_c = \frac{e}{m}B_0$ is the electron-cyclotron frequency in the presence of magnetostatic field B_0.

$\bar{\Omega}_{\pm}^2$ represent the modified normal mode frequency corresponding to the magneto-plasma excitations. It is clear that presence of magneto-plasma excitations modifies the coupling of electron-LOP quiet distinctively.

Hence, motivated by the present state of art, a study on the effect of magneto-plasma excitations via Fröhlich interaction has been carried out in compound semiconductor medium. Study of inter-dependence of coupled plasmon-phonon modes and magneto-plasma excitations during Fröhlich interaction in the presence of an intense laser beam is the main focus of our study. Nonlinear response of the medium leads to excitation of coherent collective mode as a result of coupling between pump wave and these optically excited coherent collective modes (viz. acoustical, optical, polaron mode etc.). This three wave mixing process has been considered as OPA process. Expressions for the nonlinear polarization and second-order susceptibility arising from the nonlinear induced current density have been obtained. Threshold pump field for the onset of OPA and parametric gain coefficient have been obtained. Data of III-V direct band gap n-InSb semiconductor subjected to pulsed CO_2 laser have been used for the numerical estimations.

II. THEORETICAL FORMULATIONS

In this section, using the hydrodynamic model of the homogeneous one component (viz., n-type) semiconducting-plasma, a theoretical depiction for evaluating expressions for second-order optical susceptibility $\chi^{(2)}$, threshold pump amplitude $\zeta_{0,th}$ for the onset of OPA and parametric gain coefficient g_{para} has been developed. We consider the propagation of an intense pump wave $\vec{\xi}_0 = \hat{x}\xi_0 \exp(-i\Omega_0 t)$ applied along x-direction in homogeneous semiconductor medium subjected to an external magnetostatic field $\vec{B}_0 = \hat{z}B_0$ along the z-axis. This considered field geometry is known as Voigt geometry [41]. We also consider that the transfer of energy and momentum among the pump (Ω_0, k_0), polaron (Ω_{pl}, k_{pl}) and signal waves (Ω_1, k_1) satisfy phase matching conditions:

$$\hbar\Omega_1 = \hbar\Omega_0 - \hbar\Omega_{pl}\text{, and } \hbar\vec{k}_1 = \hbar\vec{k}_0 - \hbar\vec{k}_{pl}.$$

We assume spatially uniform pump field ($|\vec{k}_0| \approx 0$) so that $|\vec{k}_1| \approx |\vec{k}_{pl}| = k$ (say).

The optical phonons are active only in infrared regime [42], thus we consider here semiconductor plasma to be irradiated by an infrared pulsed laser with pulse duration much larger than the acoustic damping time, and thus the interaction may be treated as a steady-state process.

The basic equations governing parametric interactions of a pump with the medium are as follows:

$$\frac{\partial n_1}{\partial t} + n_0\frac{\partial v_1}{\partial x} + n_1\frac{\partial v_0}{\partial x} + v_0\frac{\partial n_1}{\partial x} = 0 \tag{3}$$

$$\frac{\partial \xi_{pl}}{\partial x} = -\frac{n_1 e}{\varepsilon_0} + \left(\frac{Nq}{\varepsilon_0}\right)\frac{\partial R}{\partial x}. \tag{4}$$

Conservation of charge is represented by continuity equation (Eq. (3)), in which n_0 and n_1 are the equilibrium and perturbed electron densities, respectively. v_0 and v_1 are the oscillatory fluid velocities of electrons of effective mass m. Effective polaron electrostatic field ζ_{pl} arising due to the induced electronic and lattice polarizations can be deduced from Poisson's equation (Eq. (4)) in which ε_0 is the permittivity of free space.

The induced depolarizing field strongly couples the longitudinal and transverse degrees of freedom of the medium and shifts the natural frequency away from the cyclotron frequency and hence induces the collective cyclotron excitation with the resonance frequency $\Omega_{cc} = (\Omega_p^2 + \Omega_c^2)^{1/2}$.

We proceed with the following equations of motion for electron and polaron mode under one-dimensional configuration (along x-axis) for n-type moderately doped semiconductor magneto-plasma:

$$\frac{\partial^2 \vec{r}}{\partial t^2} + (\Omega_p^2 + \Omega_c^2)\vec{r} + 2\Gamma_e\frac{\partial \vec{r}}{\partial t} = -\frac{e}{m}\left[\vec{\xi}_0 + \left(\frac{\partial \vec{r}}{\partial t} \times \vec{B}_0\right)\right] \tag{5}$$

$$\frac{\partial^2 \vec{R}}{\partial t^2} + (\Omega_p^2 + \Omega_c^2)\vec{R} + 2\Gamma_{pl}\frac{\partial \vec{R}}{\partial t} = \frac{q}{M_{pl}}\vec{\xi}_{pl}. \tag{6}$$

Here, $\Gamma_{pl} = \Gamma_e + \Gamma_{ph}$, in which Γ_e represents electron-electron collision frequency and Γ_{ph} ($=10^{-2}\,\bar{\Omega}_+$) takes into account the optical phonon decay constant. These loss parameters have been introduced phenomenologically and do not vary with the pump and the external magnetostatic fields. q is the effective charge given by

$$q = \omega_L\left[\frac{M}{N}\varepsilon_0\left(\frac{1}{\varepsilon} - \frac{1}{\varepsilon_s}\right)\right]^{1/2}, \tag{7}$$

where M and N ($= a^{-3}$) represent the reduced mass of the diatomic molecule and number of unit cells per unit volume, respectively, and a is the lattice constant of the crystal.

M_{pl} stands for polaron mass. If the coupling between electron and phonon in the semiconductor plasma is strong enough to form polarons and/or bi-polarons, one will expect a substantial isotope effect on effective mass of charge carriers. In a semiconductor plasma, moving electron drags the lattice distortion with it thereby by creating a larger inertia. This results in slight increment in polaron mass. For weak coupling limit the polarization may be regarded as a small perturbation [43] and quantum mechanical perturbation theory yields polaron mass [44] as

$$M_{pl} \approx m\left(1 + \frac{\alpha}{6}\right)\text{, for } \alpha \ll 1. \tag{8}$$

Here α is Fröhlich coupling constant which is always positive. Half of α gives the average number of 'virtual phonons' carried along by electron [45].

The components of oscillatory electron fluid velocity in presence of pump and magnetostatic fields may be obtained from Eq. (5) as

$$v_{0x} = \frac{\bar{\xi}}{2\Gamma_e - i\Omega_0} \text{ and } v_{0y} = \frac{\Omega_c\bar{\xi}}{\Omega_0^2},$$

where $\bar{\xi} = -\dfrac{e}{m}\dfrac{\Omega_0^2}{(\Omega_0^2 - \Omega_c^2)}\xi_0$.

The plasmon-phonon modes and magneto-plasma excitations during Fröhlich interaction produces density perturbation with in the semiconductor plasma, which can be obtained by following the procedure adopted by one of the present authors [46]. Differentiating Eq. (3) with respect to time and using Eqs. (4), (5) and (6), we obtain

$$\frac{\partial^2 n_1}{\partial t^2}+2\Gamma_e\frac{\partial n_1}{\partial t}+\frac{qm}{eM}\delta_1\bar{\Omega}_p^2 n_1-\frac{\partial R}{\partial x}\frac{Nq^2}{M_{pl}\varepsilon_0}\delta_1 n_0 -2\Gamma_{ph}\left(n_1\frac{\partial v_0}{\partial x}+v_0\frac{\partial n_1}{\partial x}\right)=-ikn_1\delta_2\bar{\xi}\,, \quad (9)$$

where $\delta_1=\frac{\bar{\Omega}_+^2}{\bar{\Omega}_+^2-\Omega_p^2-\Omega_c^2}$,

$\delta_2=\frac{\Omega_0^2}{\Omega_0^2-\Omega_p^2-\Omega_c^2}$, and

$\bar{\Omega}_p^2=\frac{qm}{eM}\Omega_p^2$.

The perturbed electron concentration n_1 may be expressed as: $n_1 = n_{1f}(\Omega_1) + n_{1s}(\Omega_{pl})$, where the slow component $n_{1s}(\Omega_{pl})$ is associated with polaron mode, while the fast component $n_{1f}(\Omega_1)$ oscillates at electromagnetic wave frequencies $\Omega_0 \pm p\Omega_{pl}$, where p = 1, 2, 3, …. The higher-order terms at frequencies $\Omega_1(=\Omega_0 \pm p\Omega_{pl})$, for p = 2, 3, … being off-resonant, have been neglected. In the forthcoming formulation, we shall consider only the first-order Stokes component of scattered electromagnetic wave.

Under rotating wave approximation (RWA), Eq. (9) leads to following coupled equations in terms of density perturbations:

$$\frac{\partial^2 n_{1f}}{\partial t^2}+2\Gamma_e\frac{\partial n_{1f}}{\partial t}+\frac{qm}{eM}\delta_1\bar{\Omega}_p^2 n_{1f}-\frac{\partial R}{\partial x}\frac{Nq^2}{M_{pl}\varepsilon_0}\delta_1 n_0 -2\Gamma_{ph}\left(n_{1f}\frac{\partial v_0}{\partial x}+v_0\frac{\partial n_{1f}}{\partial x}\right)=-ikn_{1s}^*\delta_2\bar{\xi} \quad (10a)$$

and

$$\frac{\partial^2 n_{1s}}{\partial t^2}+2\Gamma_e\frac{\partial n_{1s}}{\partial t}+\frac{qm}{eM}\delta_1\bar{\Omega}_p^2 n_{1s}-\frac{\partial R}{\partial x}\frac{Nq^2}{M_{pl}\varepsilon_0}\delta_1 n_0=-ikn_{1f}^*\delta_2\bar{\xi}\,. \quad (10b)$$

Eqs. (10a) and (10b) reveal that the slow and fast components (n_{1s}, n_{1f})) of density perturbations are coupled to each other via pump electric field $\bar{\xi}$. That it is clear that for OPA, the presence of pump field is the fundamental necessity.

Mathematical simplification of Eqs. (10a) and (10b) yield the slow component of density perturbation as

$$n_s=\frac{ikn_0Nq^3\delta_1\xi_{pl}}{M_{pl}^2\varepsilon_0[(\bar{\Omega}_+^2+\Omega_{cc}^2)+2i\Gamma_{pl}\bar{\Omega}_+]}\times\left[(\Delta_2^2-2i\Gamma_e\bar{\Omega}_+)-\frac{k^2\delta_2^2|\xi|^2}{(\Delta_1^2+2i\Gamma_e\Omega_1)}\right]^{-1}, \quad (11)$$

where $\Delta_1^2=\delta_1\bar{\Omega}_p^2-\Omega_1^2$ and $\Delta_2^2=\delta_1\bar{\Omega}_p^2-\bar{\Omega}_+^2$.

In deriving Eq. (11), we have neglected the effect of transition dipole moment with an aim to concentrate only to the contribution of nonlinear current density on the induced polarization of the medium. The induced nonlinear current density is given by

$$J(\Omega_1)=-ev_0n_s^*\,, \quad (12)$$

where * represents the complex conjugate of the quantity. Substitutions of corresponding expressions in Eq. (12) yields

$$J(\Omega_1)=\frac{-kq^3N\Omega_0\Omega_p^2\delta_1\xi_0\xi_{pl}^*}{M_{pl}^2(\Omega_0^2-\Omega_c^2)[(\bar{\Omega}_+^2+\Omega_{cc}^2)-2i\Gamma_{pl}\bar{\Omega}_+]}\times\left[(\Delta_2^2+2i\Gamma_e\bar{\Omega}_+)-\frac{k^2\delta_2^2|\xi|^2}{(\Delta_1^2-2i\Gamma_e\Omega_1)}\right]^{-1}. \quad (13)$$

The time integral of Eq. (13) yields an expression for nonlinear induced polarization due to the perturbed carrier density as

$$P_{nl}(\Omega_1)=\int J(\Omega_1)dt\,. \quad (14)$$

By using Eq. (13) in the above equation, the expression of polarization is obtained as

$$P_{nl}(\Omega_1)=\frac{ikq^3N\Omega_0\Omega_p^2\delta_1\xi_0\xi_{pl}^*}{M_{pl}^2\Omega_1(\Omega_0^2-\Omega_c^2)[(\bar{\Omega}_+^2+\Omega_{cc}^2)-2i\Gamma_{pl}\bar{\Omega}_+]}\times\left[(\Delta_2^2+2i\Gamma_e\bar{\Omega}_+)-\frac{k^2\delta_2^2|\xi|^2}{(\Delta_1^2-2i\Gamma_e\Omega_1)}\right]^{-1}. \quad (15)$$

Nonlinear induced polarization is given by

$$P_{nl}(\Omega_1)=\varepsilon_0\chi^{(2)}\xi_0\xi_{pl}^*\,. \quad (16)$$

On comparing with Eq. (54), we get second-order optical susceptibility as

$$\chi^{(2)}=\frac{ikq^3N\Omega_0\Omega_p^2\delta_1}{\varepsilon_0M_{pl}^2\Omega_1(\Omega_0^2-\Omega_c^2)[(\bar{\Omega}_+^2+\Omega_{cc}^2)-2i\Gamma_{pl}\bar{\Omega}_+]}\times\left[(\Delta_2^2+2i\Gamma_e\bar{\Omega}_+)-\frac{k^2\delta_2^2|\xi|^2}{(\Delta_1^2-2i\Gamma_e\Omega_1)}\right]^{-1}=(\chi^{(2)})_r+i(\chi^{(2)})_i\,. \quad (17)$$

In Eq. (17), $(\chi^{(2)})_r$ and $(\chi^{(2)})_i$ represent the real and imaginary parts of complex $\chi^{(2)}$, given by

$$(\chi^{(2)})_r=\frac{ikq^3N\Omega_0\Omega_p^2\delta_1(\bar{\Omega}_+^2+\Omega_{cc}^2)}{\varepsilon_0M_{pl}^2\Omega_1(\Omega_0^2-\Omega_c^2)[(\bar{\Omega}_+^2+\Omega_{cc}^2)^2+4\Gamma_{pl}^2\bar{\Omega}_+^2]}\times\left[(\Delta_2^2+2i\Gamma_e\bar{\Omega}_+)-\frac{k^2\delta_2^2|\xi|^2}{(\Delta_1^2-2i\Gamma_e\Omega_1)}\right]^{-1} \quad (17a)$$

and

$$(\chi^{(2)})_i=\frac{2ikq^3N\Omega_0\Omega_p^2\delta_1\Gamma_{pl}\bar{\Omega}_+}{\varepsilon_0M_{pl}^2\Omega_1(\Omega_0^2-\Omega_c^2)[(\bar{\Omega}_+^2+\Omega_{cc}^2)^2+4\Gamma_{pl}^2\bar{\Omega}_+^2]}$$

$$\times\left[(\Delta_2^2+2i\Gamma_e\bar{\Omega}_+)-\frac{k^2\delta_2^2|\xi|^2}{(\Delta_1^2-2i\Gamma_e\Omega_1)}\right]^{-1}. \qquad (17b)$$

The above equations reveal that $(\chi^{(2)})_r$ and $(\chi^{(2)})_i$ characterize the steady state optical response of the medium and govern the nonlinear wave propagation through the medium in presence of transverse magnetostatic field. Moreover, $(\chi^{(2)})_r$ is responsible for parametric dispersion while $(\chi^{(2)})_i$ give rise to parametric amplification/absorption and oscillation. The present paper deals with the study of parametric amplification/absorption of optically excited coherent collective mode in a semiconductor magneto-plasma only. The study of parametric dispersion and parametric oscillation will be the research work of future publication.

To incite parametric interactions in the medium, the pump amplitude should exceed certain threshold value $\zeta_{0,th}$ necessary to supply minimum required energy to the medium. This may be obtained from eq. (15) by setting $P_{nl}(\Omega_1) = 0$ as

$$\xi_{0,th}=\left|\frac{m}{ek}\frac{(\Omega_0^2-\Omega_c^2)}{\Omega_0^2}\frac{\Delta_1\Delta_2}{\delta_2}\right|. \qquad (18)$$

The above equation reveals that $\zeta_{0,th}$ is strongly influenced by external parameters: wave vector (k), magnetostatic field B_0 (via Ω_c), and carrier concentration n_0 (via Ω_p). It is evident that threshold pump field is inversely proportional to wave vector k, therefore, lower threshold could be achieved for higher values of wave vector.

In order to investigate the amplification characteristics in a doped semiconductor, we employ the relation

$$g_{para}=\frac{k_0}{2\varepsilon_1}(\chi^{(2)})_i\,|\xi_0|, \qquad (19)$$

Substituting the value of $(\chi^{(2)})_i$ from equation (17b), the expression of parametric gain coefficient becomes

$$g_{para}=\frac{2ik_0kq^3N\Omega_0\Omega_p^2\delta_1\Gamma_{pl}\bar{\Omega}_+|\xi_0|}{2\varepsilon_0\varepsilon_1M_{pl}^2\Omega_1(\Omega_0^2-\Omega_c^2)[(\bar{\Omega}_+^2+\Omega_{cc}^2)^2+4\Gamma_{pl}^2\bar{\Omega}_+^2]}$$

$$\times\left[(\Delta_2^2+2i\Gamma_e\bar{\Omega}_+)-\frac{k^2\delta_2^2|\xi|^2}{(\Delta_1^2-2i\Gamma_e\Omega_1)}\right]^{-1}. \qquad (20)$$

The above equation reveals that g_{para} is strongly influenced by external parameters: magnetostatic field B_0 (via Ω_c), carrier concentration n_0 (via Ω_p) as well as pump field amplitude ζ_0.

III. RESULTS AND DISCUSSION

In order to establish the validity of present model, we have chosen a weakly-polar narrow band-gap semiconductor crystal viz. n-InSb) at 77 K as the medium; it is assumed to be irradiated by a 10.6 μm ($\Omega_0 = 1.78\times10^{14}$ s^{-1}) pulsed CO_2 laser. Around this temperature, absorption coefficient of the sample is low around 10 μm and one may safely neglect contribution due to band-to-band transition mechanism. The physical parameters of n-InSb at 77 K are given Ref. [36].

Utilizing the material parameters of n-InSb-CO_2 laser system, the numerical estimations depicting threshold and amplification characteristics are plotted in Figs. 1 - 5.

Fig. 1 shows the variation of threshold pump amplitude $\zeta_{0,th}$ with magnetostatic field B_0 for two different doping concentrations ($n_0 = 1.5\times10^{23}$ m^{-3} and $n_0 = 3.1\times10^{23}$ m^{-3}) at $k = 2\times10^8$ m^{-1}. It can be observed from this figure that in both the cases, higher magnetostatic field reduces the pump amplitude required to incite parametric processes. Moreover, higher values of n_0 lowers the $\zeta_{0,th}$ required for the onset of OPA. Thus, one may reduce the threshold pump field by significant amount by increasing the externally applied magnetostatic field even at different doping concentrations in the semiconductor.

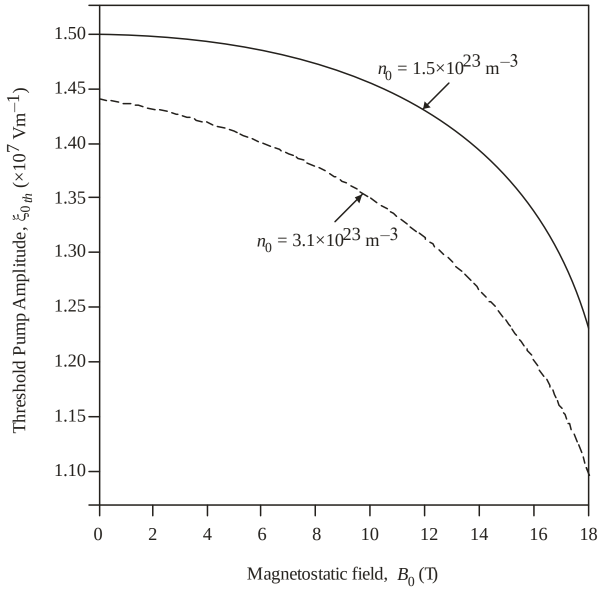

Fig. 1: Variation of $\zeta_{0,th}$ with B_0 at $k = 2\times10^8$ m^{-1}.

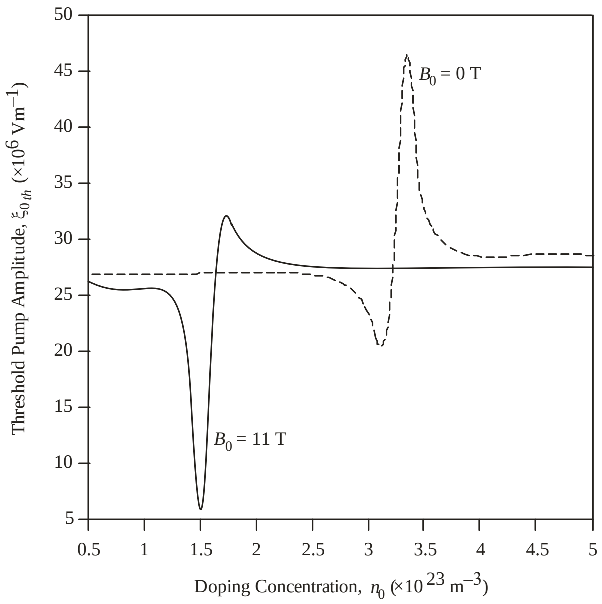

Fig. 2: Variation of $\zeta_{0,th}$ with n_0 at $k = 2\times10^8$ m^{-1}.

Fig. 2 shows the variation of threshold pump amplitude $\zeta_{0,th}$ with doping concentration n_0 in the absence (B_0 = 0T) and presence of magnetostatic field (B_0 = 11T) at $k = 2\times10^8$ m^{-1}. It can be inferred that in both the cases threshold pump amplitude decreases on increasing carrier concentration up to $n_0 = 1.5\times10^{23}$ m^{-3} (for B_0 = 11T) and $n_0 = 3.1\times10^{23}$ m^{-3} (for B_0 = 0T) and attains minima due to resonance between plasma frequency and modified normal and normal mode frequencies respectively. Beyond this value, plasma frequency becomes greater than modified normal mode frequency, responsible for enhancement in the value of threshold pump amplitude. Further increment in carrier concentration reduces threshold pump field. Presence of magnetostatic field reduces the magnitude of pump field required to incite parametric interactions.

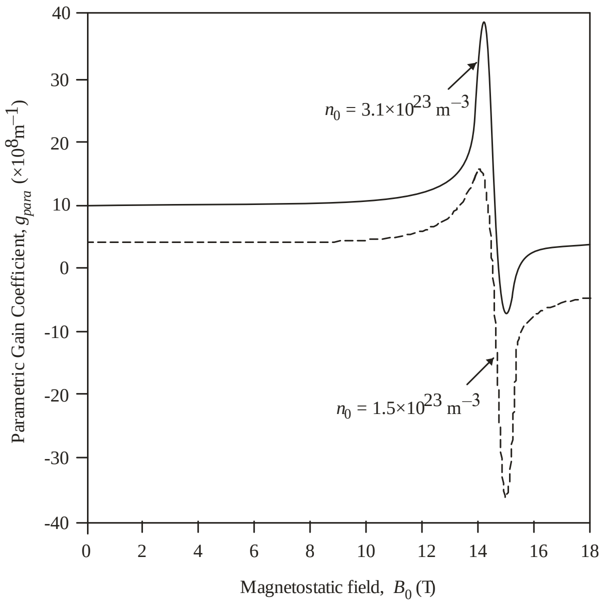

Fig. 3: Variation of g_{para} with B_0 at $k = 2\times10^8$ m^{-1} and $\zeta_0 = 1.3\times10^7$ Vm^{-1}.

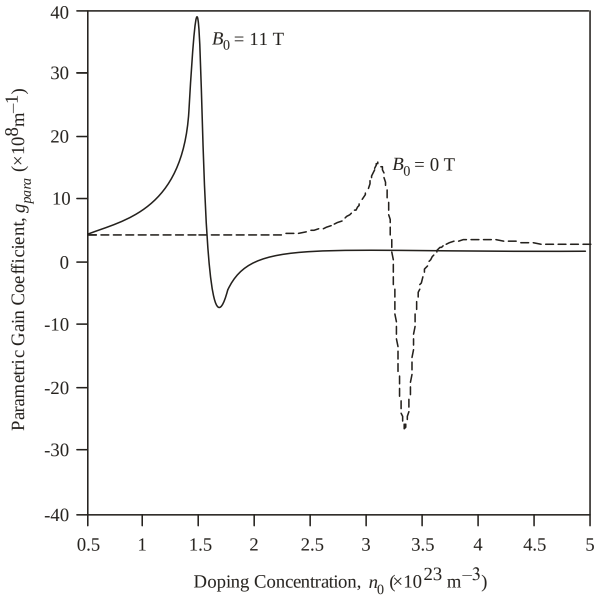

Fig. 4: Variation of g_{para} with n_0 at $k = 2\times10^8$ m^{-1}, B_0 = 11T and $\zeta_0 = 1.3\times10^7$ Vm^{-1}.

Fig. 3 shows the variation of parametric gain coefficient g_{para} with magnetostatic field B_0 for two different doping concentrations ($n_0 = 1.5\times10^{23}$ m^{-3} and $n_0 = 3.1\times10^{23}$ m^{-3}) at $k = 2\times10^8$ m^{-1} and $\zeta_0 = 1.3\times10^7$ Vm^{-1}. It can be observed that in both the cases parametric gain coefficient is positive (which indicates amplification) and remains almost constant with the increase in magnetostatic field. However, as B_0 ~ 14T, g_{para} shoots up to its maximum value (3.8×10^9 m^{-1} for $n_0 = 3.1\times10^{23}$ m^{-3} and 1.6×10^9 m^{-1} for $n_0 = 1.5\times10^{23}$ m^{-3}) followed by a very sharp fall making g_{para} = 0 at B_0 = 14.2T. Beyond this point g_{para} becomes negative (which indicates absorption) fall rapidly acquiring minimum value (-3.7×10^9 m^{-1} for $n_0 = 1.5\times10^{23}$ m^{-3} and -0.8×10^9 m^{-1} for $n_0 = 3.1\times10^{23}$ m^{-3}). A rise in gain constant is again witnessed if magnetostatic field is further increased which may be explained as follows: For B_0 < 14.2 T, $\Omega_c < \Omega_0$ and the term ($\Omega_0^2 - \Omega_c^2$) in the denominator of Eq. (20) is positive and give rise positive values of g_{para}. As B_0 ~ 14.2T, the term $\Omega_0^2 - \Omega_c^2$ becomes smaller thereby increasing g_{para}. Further, as B_0 > 14.2T, $\Omega_c > \Omega_0$ and the term $\Omega_0^2 - \Omega_c^2$ in the denominator of Eq. (20) becomes negative and give rise negative values of g_{para}. A comparison between the two curves infer that at low doping concentration, parametric absorption is higher while at high doping concentration, parametric gain is higher. Thus we conclude from this figure that by suitably choosing the external magnetostatic field around resonance, one may control the amplification/attenuation characteristics of the medium for the optically excited coherent collective mode in semiconductor plasmas. This feature can be utilized for the design of optical switches.

Fig. 4 shows the variation of parametric gain coefficient g_{para} with doping concentration n_0 in the absence (B_0 = 0T) and presence of magnetostatic field (B_0 = 11T) at $k = 2\times10^8$ m^{-1} and $\zeta_0 = 1.3\times10^7$ Vm^{-1}. It has been found that the nature of variation is almost similar in both the cases but presence of magnetostatic field enhances magnitude of gain by a factor of 10. Initially g_{para} is positive exhibiting parametric gain, which increases with increasing doping concentration till a particular carrier density (n_0 ~ 1.5×10^{23} m^{-3}) at which maximum gain is achieved. On elimination of magnetostatic field higher doping concentration n_0 ~ 3.1×10^{23} m^{-3} is needed for the same. The maxima are due to resonance between plasma frequency and coupled plasmon–phonon mode frequency. Beyond this doping level a sharp increment shows absorption due to sign reversal of g_{para}. So we realize that presence of magnetostatic field leads higher gain at lower doping level. First cross over point signifies a particular doping concentration $n_0 = 1.5\times10^{23}$ m^{-3}, up to which the presence of magnetostatic field leads to gain whereas in the absence of magnetostatic field gain begins at this doping concentration. Therefore $n_0 = 1.5\times10^{23}$ m^{-3} is found to be favourable for the efficient coupling between phonon-plasmon modes in the absence of magneto-plasma excitations. On the other hand coupling between plasma frequency and modified normal mode frequency resulting into amplification initiates at quite lower doping and this continues till $n_0 = 1.5\times10^{23}$ m^{-3}. Favourable doping concentrations reported in this figure are the same which are already interpreted in Fig. 1 for both the cases. Beyond this concentration an abrupt decrease of gain is observed. Such behaviour can also be utilized in the construction of optical switches.

Fig. 5 shows the variation of parametric gain coefficient g_{para} with pump field ζ_0 at $k = 2\times10^8$ m^{-1}, B_0 = 11T and $n_0 = 1.5\times10^{23}$ m^{-3}. It can be seen that the gain coefficients increases quadrically with increasing pump amplitude. Hence, we conclude from this figure that high pump field amplitude yield larger gain coefficients.

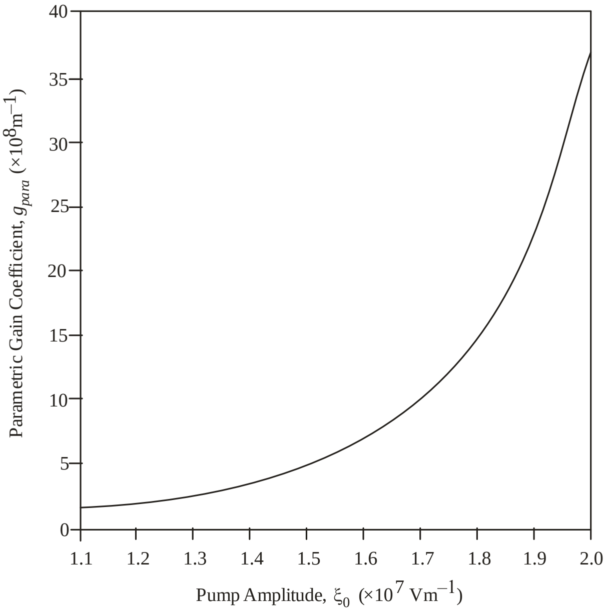

Fig. 5: Variation of g_{para} with ζ_0 at $k = 2\times10^8$ m^{-1}, B_0 = 11T and $n_0 = 1.5\times10^{23}$ m^{-3}.

Using Mathematica software, we calculate the threshold pump amplitude and parametric gain coefficient (for $\zeta_0 = 1.3\times10^7$ Vm^{-1}) for the onset of OPA in n-InSb crystal. The calculated values are given in Table 1.

TABLE I.

$\zeta_{0,th}$ (Vm^{-1})	g_{para} (m^{-1})	B_0 (T)	n_0 (m^{-3})
2.10×10^7	1.60×10^9	0	3.10×10^{23}
2.05×10^7	1.65×10^9	1	2.95×10^{23}
2.00×10^7	1.70×10^9	2	2.81×10^{23}
1.95×10^7	1.75×10^9	3	2.66×10^{23}
1.89×10^7	1.81×10^9	4	2.52×10^{23}
1.82×10^7	1.92×10^9	5	2.37×10^{23}
1.73×10^7	2.12×10^9	6	2.23×10^{23}
1.62×10^7	2.33×10^9	7	2.08×10^{23}
1.50×10^7	2.58×10^9	8	1.94×10^{23}
1.30×10^7	2.85×10^9	9	1.79×10^{23}
1.00×10^7	3.26×10^9	10	1.64×10^{23}
0.60×10^7	3.80×10^9	11	1.50×10^{23}

IV. CONCLUSIONS

The present paper deals with the analytical investigations of parametric amplification of an intense electromagnetic wave due to coherent collective mode in weakly-polar narrow band-gap semiconductors. The role of cyclotron excitations and coupled plasmon-phonon modes has been studied at length. Numerical estimations made for n-InSb-CO_2 laser system enables one to draw the following conclusions:

(i) It has been found that the correction to the parametric amplification/absorption coefficient due to electron-phonon interaction and magneto-plasma excitations depends strongly on doping concentration.

(ii) The presence of external magnetostatic field modifies the carrier dynamics effectively. Higher magnetostatic field is favourable for the parametric interactions and also for significant enhancement of amplification coefficient.

(iii) The coupled collective magneto-plasma excitations tend to enhance the parametric amplification/ absorption coefficient and lower threshold pump amplitude. At higher values of magnetostatic fields, these favourable parameters shift towards lower values of doping concentration.

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