Parameter Estimation of Low Frequency Oscillations using Prony

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***Abstract*—This paper presents Prony method to monitor and to analyse Low Frequency Oscillations (LFO) in large interconnected power system. In power system stability problems are more due the electromechanical oscillations or swinging of generators. The power network increasing rapidly, so the complexity of power system and also the power system stability problems are increases. Oscillations can be low damped or undamped with amplitude constant or increasing. In the recent advancements in power system as wide area measurement, phasor measurement unit to analyse and to take correct control auction on stability problems requires an accurate knowledge of these low frequency oscillations.**

Keywords—Low Frequency Oscillations, Power system, Prony Method, Phasor measurement Unit,Wide area Measurment System (WAMs).

I. INTRODUCTION

Power systems are subjected to wide range of disturbances, which causes the stability problems and it must be able to adjust the changing conditions. Electromechanical oscillations are the result of disturbances either large or small. To maintain the stability of power system has number of monitoring, protection and controlling devices. If the system is unstable which causes the progressively increase in the power angle i.e. generator rotor angle and in same way decrease in the bus voltage or the system frequency deviation.

In power system, Oscillations are classified by the system components that they affect.Electromechanical oscillations are of following types [1].

1. Inter area oscillation mode (02-0.7 Hz) when a group of synchronous generators in an area are linked by the long tie line oscillates against the group of generators in another area.It is observed over a large part of the power system network
2. Intra Plant mode (2-3Hz) in this mode of oscillation, synchronous generators within the plant are oscillating against each other.
3. Local mode (0.8-1.6Hz)when a synchronous generator swings against the large system (as single machine infinite bus system)
4. Control mode oscillation
5. Torsional modes

As the WAMs technology are developing very rapidly and also the Phasor Measurement Units (PMU) places at various locations to measure the correct system data. So the lot of data is coming to the control centre and to take correct control action the control system operator requires the accurate knowledge of parameters like amplitude, frequency, phase angle and main important is damping factor of low frequency oscillation.

II. MODAL ANALYSIS – SMALL SIGNAL STABILITY OF MULTIMACHINE SYSTEM

Monitoring and analysis of transient oscillations in power system is done by different methodological approaches. Each method has its own advantages and applications, provides a different view of systems dynamic behavior. Eigenvalue analysis technique is based on the linearization ofthe nonlinear equations that represent the power system aroundan operating point which is the result of electromechanicalmodal characteristics: frequency, damping and shape.

Analysis of practical power system network involves the simultaneous solution of mathematical equations representing the (i) synchronous machines, and the associated excitation systems and prime movers, (ii) interconnecting transmission network (iii) dynamic (motor) and static loads and (iv) other devices (FACT devices)such, as HVDC converters, static VAR compensators [2]. Electromechanical low frequency oscillations range from less than 1 Hz to 3 Hz other than those with sub-synchronous resonance. In this frequency range the dynamic behavior multi-machine power system is usually expressed as a set of non-linear differential and algebraic equations. The algebraic equations result from the network power balance and generator stator current equations. When the analysis is focused on low frequency electromechanical oscillations then the high frequency network and stator transients are ignored. The initial operating state of the algebraic variables such as bus voltages and angles are obtained through a standard power flow solution. The initial values of the dynamic variables are obtained by solving the differential equations through simple substitution of algebraic variables into the set of differentialequations. The set of differential and algebraic equations isthen linearized around the equilibrium point and a set of lineardifferential and algebraic equations is obtained:

$$\dot{x} = f(x, z, u) \quad (1)$$

$$0 = g(x, z, u) \quad (2)$$

$$y = h(x, z, u) \quad (3)$$

Where f and g are vectors of differential and algebraicequations and h is a vector of output equations. The inputs arenormally reference values such as speed and voltage atindividual units and can be voltage, reactance and power flowasset in FACTS devices. The output can be unit power output,bus frequency, bus voltage, line power or current etc. Bylinearizing the (1) to (3) around the equilibrium point followingequations (4) to (6) are obtained:

$$\Delta\dot{x} = \frac{\partial f}{\partial x}\Delta x + \frac{\partial f}{\partial z}\Delta z + \frac{\partial f}{\partial u}\Delta u \quad (4)$$

$$0 = \frac{\partial g}{\partial x}\Delta x + \frac{\partial g}{\partial z}\Delta z + \frac{\partial g}{\partial u}\Delta u \quad (5)$$

$$\Delta y = \frac{\partial h}{\partial x}\Delta x + \frac{\partial h}{\partial z}\Delta z + \frac{\partial h}{\partial u}\Delta u \quad (6)$$

The vector algebraic variable Δzis eliminated from (4) and (6), gives:

$$\Delta\dot{x} = A\Delta x + B\Delta u \quad (7)$$

$$\Delta y = C\Delta x + D\Delta u \quad (8)$$

Where A, B, C and D are the matrix of partial derivatives in (4) to(6) evaluated at equilibrium. Normally power system state spacerepresentation is linearized around an operating point which is the result of electromechanical modal characteristics.The symbol A from (7) and (8) is omitted so as to follow thestandard state space making x and u into the incremental values.This is the representation of a linearized differential andalgebraic equations model of a power system on which standardlinear analysis tools.

III. Prony Analysis

Prony analysis isclassical approach to the model identification and oscillation monitoring in the system. It is a signal processing method which extends the Fourier analysis by directly estimating the frequency, damping, amplitude, and relative phase of modal components present in agiven signal. In prony analysis signal sampled at regular interval is expressed as a linear combination of exponential terms. It has a close relationship with the algorithm(least square prediction algorithm) used for AR (Auto Regressive) and ARMA (Auto Regressive Moving Average) parameter estimation.Prony analysis is a method of fitting a signal into a linear combination of complex damped sinusoidal exponential. Each exponential term with different frequency is termed as mode of the signal. Each mode has four element/ parameter as Amplitude, frequency, phase angle and damping factor.

The mathematical formulation of prony method is derived using linear time invariant (LTI) dynamic system is shown below Fig.1.

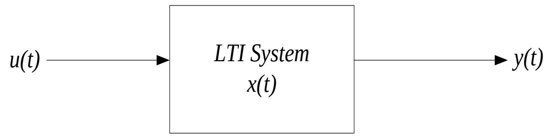

Fig. 1. LTI System

In Fig.1, signal are reffered to as y(t) the system response, u(t) is system input and x(t) is state of the LTI system.the evalution of system state is expressed as

$$\frac{dx(t)}{dt} = A\,x(t) + B\,u(t) \quad (9)$$

Where A and B are constant matrices. If there is no input i.e. u(t)=0 and there are no subsequent input to the system then (9) becomes

$$\frac{dx(t)}{dt} = A\,x(t) \quad (10)$$

Where, A is a n×n matrics and eigenvalues of A are λ_i, right and left eigenvectors are $p_i$and q_i respectively. The system order is reprented by 'n'. The solution to (10) is expressed as sum of n components as (11):

$$x(t) = \sum_{i=1}^{n} (q_i^T x)\, p_i e^{\lambda_i t} \quad (11)$$

Output y(t) for the LTI system is express as

$$y(t) = Cx(t) + Du(t) \quad (12)$$

Where, C and D are constant matrices. If there is no input u(t)=0, then the output of system (12) is given as (13)

$$y(t) = Cx(t) \quad (13)$$

The Prony analysis directly estimates the parameters of the eigen structure described in (11) by fitting a sum of complex damped sinusoids exponentials to evenly spaced samples (in time) values of the output as:

$$\hat{y}(t) = \sum_{i=1}^{L} A_i e^{(\sigma_i t)} \cos(2\pi f_i t + \emptyset_i) \quad (14)$$

In (14) we have used the following notations
A_i: Amplitude of the component i
f_i : Frequency of the component i
$\emptyset_i$: Phaseangle of the component i

σ_i: Damping factor of the component i
L: total number of damped exponential component
$\hat{y}(t)$: estimated data for y(t) having N samples y(tk)=y(k), k=0,1,2,…..(N-1) that are evenly sampled.
Using Euler's theoremand letting t=kT, T is the sampling period less than Nyquistperiod, then the samples of $\hat{y}(t)$ in (14) are

$$y(k) = \sum_{i=1}^{L} C_i \mu_i^k \qquad (15)$$

$$C_i = \frac{A_i}{2} e^{j\phi_i} \qquad (16)$$

$$\mu_i = e^{(\sigma_i + j2\pi f_i)T} \qquad (17)$$

C_i is the output residue for the poles λ_i ($\lambda_i = \sigma_i + j2\pi f_i$). The objective is to find the poles, residue and L of the system that force the system to fit y(t). The prony analysis computes residue C_i and μ_i in three basic steps. First, construct the linear prediction model (LPM) (17) using observed data set y(t) and then compute the coefficient of Linear Prediction Model.

$$y(k) = a_1 y(k-1) + a_2 y(k-2) + \cdots + a_L y(k-L) \qquad (18)$$

In (18), y(k) is computed for various values of k=L, L+1,L+2…., N-1. And we can write the y(k) in matrix form for various values of k as

$$\begin{bmatrix} y(L) \\ y(L+1) \\ \vdots \\ y(N-1) \end{bmatrix} = \begin{bmatrix} y(L-1) & y(L-2) & \cdots & y(0) \\ y(L) & y(L-1) & \cdots & y(1) \\ \vdots & \vdots & \vdots & \vdots \\ y(N-2) & y(N-3) & \cdots & y(N-L-1) \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_L \end{bmatrix} \qquad (19)$$

Co-efficient vector a of linear prediction are calculated by solving the over determined least square problem assuming N>2L.

In second step, the roots μ_n of characteristics polynomial (20) form by the coefficients of LPM (18) are derived.

$$\mu^L - a_1\mu^{L-1} - \ldots - a_{L-1}\mu - a_L = (\mu-\mu_1)(\mu-\mu_2)\ldots(\mu-\mu_L) \qquad (20)$$

Using the roots derived from (19) is used to calculate the damping factor σ_n and frequency f_n according to (17).

In last step, the magnitudes A_n and phase angle ϕ_n are calculated by least square sense. Using the roots μ_n of the polynomial (20),(21) is built according to (15).

$$Y = UC \qquad (21)$$

Where,

$$Y = \begin{bmatrix} y(0) \\ y(1) \\ y(2) \\ \vdots \\ y(k-1) \end{bmatrix}, \quad C = \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ \vdots \\ C_L \end{bmatrix} \text{ and}$$

$$U = \begin{bmatrix} 1 & 1 & \cdots & \cdots & 1 \\ \mu_1 & \mu_1 & \cdots & \cdots & \mu_1 \\ \mu_1^2 & \mu_2^2 & \cdots & \cdots & \mu_L^2 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \mu_1^{k-1} & \mu_2^{k-1} & \cdots & \cdots & \mu_L^{k-1} \end{bmatrix}$$

As C and μ are now known, Amplitude, frequency, phase angle and important thing damping factor are calculated by using (16) and (17).

IV. Simulation and Results

When a disturbance occurs in a power system, it creates animbalance between the electrical power beingsupplied to the power system and the mechanical power being supplied to a generator by its turbine. This imbalance is translated intoa change in the kinetic energy of the rotor. In other words thegenerators begin to speed up or slow down. Normally variousdamping phenomena within the power system will act so thatthe system will attain a new steady state operating point.

To identify the low frequency oscillations, a two area four machine interconnected power systems shown in Fig.2 is considered. All generators present in this two-area system are equipped with a fast static exciter with a gain of 200.Each area is equipped with two identical round rotor generators rated 20 kV/900 MVA. The load is represented as constant impedances and split between the areas in such a way that area 1 is exporting 413 MW to area 2. Since the surge impedance loading of a single line is about 140 MW [4], the system is somewhat stressed, even in steady-state. In thispaper for analyzing the modes present in the system, key variable of the machine-1 of area-1 i.e., accelerating power is used, which is also root cause for occurrence of LFOs in power system. Hence Prony method uses the accelerating power of machine-1 for identification of modes and estimating the parameters of LFO.

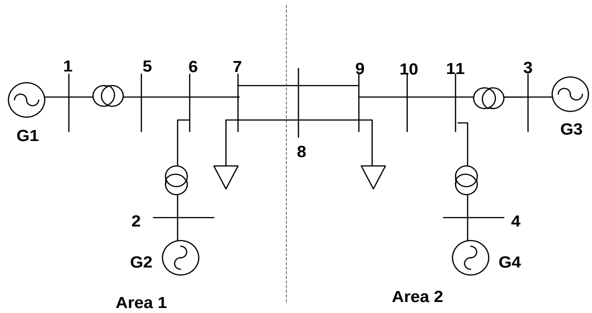

Fig. 2. Two area system

The area 1 and area 2 are interconnected by a weak tie line. Simulation of two area sysem is done in MATLAB SIMULINK. The data for prony analysis is taken by creating some disturbance in system is used. It consists of 1200 samples of accelerating power of machine1 in a time period of 20 sec to focus on the low frequency modes.

Below Fig. 3 shows the measured signal of accelerating power of machine-1 in area1. Reconstruction of measured signal using prony method is shown in Fig.4

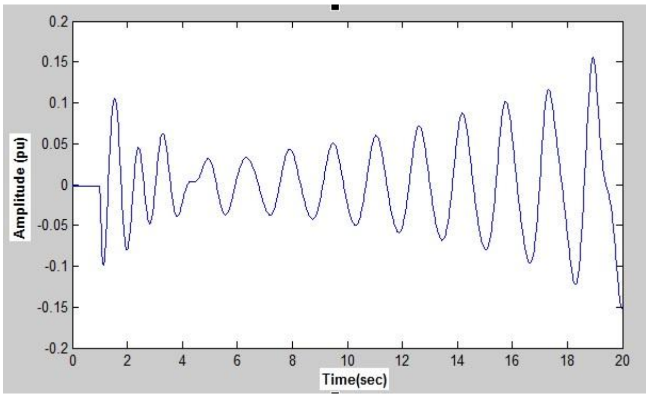

Fig. 3. Measured Signal

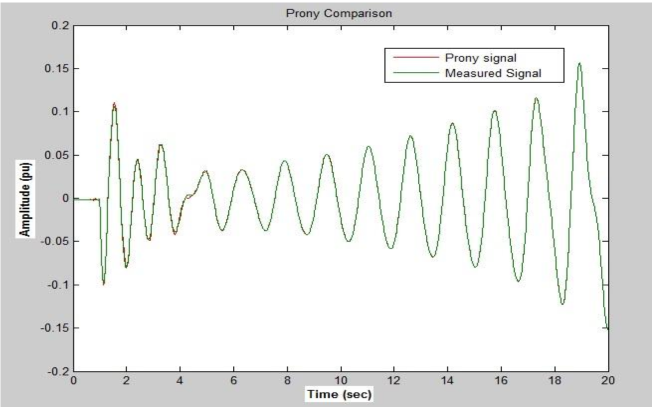

Fig. 4. Reconstructed Signal using prony method

A modal analysis of accelerating power of machine-1 in two-area system shows the following dominant modes having parameters:

1) An Inter-area mode of frequency = 0.64 Hz, damping ratio = -0.026 involving the whole area 1 against area 2.

2)Local mode of area 1 having frequency = 1.12 Hz, and damping ration = 0.08 involving this area's machines against each other.

TABLE I. PARAMETERS OF LOW FREQUENCY OSCILLATIONS MODES

Sr. No.	Frequency f (Hz)	Damping Factor (σ)	Phase Angle (rad)	Amplitude (pu)
1	0.64	-0.026	0.45881	0.00893
2	1.12	0.08	1.82819	0.17006

V. CONCLUSION

The low frequency oscillation has been a universal andserious problem in modern large-scale power systems.Monitoring and studying of large scale power system is a bigchallenge. Thus using prony method we can estimate accurately the parameters of the low frequency oscillations in power system. The parameters as damping ratio of LFO are known so we can predict the nature of these dominant low frequency oscillations and take corrective control action to damp out these oscillations.

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