

# PAPR Reduction Using Zadoff-Chu Matrix Transform Based Pre-Coding Technique and Partial Transmit Sequence in OFDM System

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**Abstract:** Orthogonal Frequency Division Multiplexing (OFDM) is an efficient method of data transmission for high Power Ratio (PAPR) of the transmitted signals. OFDM consist of large number of independent subcarriers, as a result of which the amplitude of such a signal can have high peak values. Coding, phase rotation and clipping are among many PAPR reduction schemes that have been proposed to overcome this problem. Here two different PAPR reduction methods e.g. partial transmit sequence (PTS) and Zadoff-Chu Matrix Transform Based Pre-Coding Technique are used to reduce PAPR. Significant reduction in PAPR has been achieved using these techniques. The performances of the two methods are then compared.

**Keywords:** Orthogonal frequency division multiplexing (OFDM), peak-to-average power ratio (PAPR), selected mapping (SLM), partial transmit sequence (PTS), complementary cumulative distribution function (CCDF).

## I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is a multicarrier modulation (MCM) technique which seems to be an attractive candidate for fourth generation (4G) wireless communication systems. OFDM offer high spectral efficiency, immune to the multipath delay, low inter-symbol interference (ISI), immunity to frequency selective fading and high power efficiency. Due to these merits OFDM is chosen as high data rate communication systems such as Digital Video Broadcasting (DVB) and based mobile worldwide interoperability for microwave access (mobile Wi-MAX). However OFDM system suffers from serious problem of high PAPR. In OFDM system output is superposition of multiple sub-carriers. In-off between the capacity of PAPR reduction and transmission power, data rate loss, implementation complexity and Bit-Error-Ratio (BER) performance etc.

In this paper, firstly the distribution of PAPR based on the characteristics of the OFDM signals are investigated then typical PAPR reduction techniques are analyzed.

speed communication systems. However, the main drawback of OFDM system is the high Peak to Average this case some instantaneous power output might increase greatly and become far higher than the mean power of system. To transmit signals with such high PAPR, it requires power amplifiers with very high power scope. These kinds of amplifiers are very expensive and have low efficiency-cost. If the peak power is too high, it could be out of the scope of the linear power amplifier. This gives rise to non-linear distortion which changes the superposition of the signal spectrum resulting in performance degradation. If no measure is taken to reduce the high PAPR, MIMO-OFDM system could face serious restriction for practical applications.

PAPR can be described by its complementary cumulative distribution function (CCDF). In this probabilistic approach certain schemes have been proposed by researchers. These include clipping, coding and signal scrambling techniques. Under the heading of signal scrambling techniques there are two schemes included. Which are Partial transmit sequence (PTS) and Selected Mapping (SLM). Although some techniques of PAPR reduction have been summarized, it is still indeed needed to give a comprehensive review including some motivations of PAPR reductions, such as power saving, and to compare some typical methods of PAPR reduction through theoretical analysis and simulation results directly. An effective PAPR reduction technique should be given the best trade.

## II. OFDM SIGNAL CHARACTERISTICS

An OFDM symbol is made of sub-carriers modulated by constellations mapping. This mapping can be achieved from phase-shift keying (PSK) or quadrature amplitude modulation (QAM). For an OFDM system

with N sub-carriers, the high-speed binary serial input stream is denoted as { a<sub>i</sub> }. After serial to parallel (S/P) conversion and constellation mapping, a new parallel signal sequence {d<sub>0</sub>,d<sub>1</sub>,d<sub>2</sub>,...,d<sub>i</sub>,...,d<sub>N-1</sub>} is obtained, d<sub>i</sub> is a discrete complex-valued signal [6]. Here, d<sub>i</sub> ∈ {±1} when BPSK mapping is adopted. When QPSK mapping is used, d<sub>i</sub> ∈ {±1, ±i}. Each element of parallel signal sequence is supplied to N orthogonal sub-carriers {e<sup>j 2π f<sub>0</sub> t</sup>, e<sup>j 2π f<sub>1</sub> t</sup>, .....e<sup>j 2π f<sub>N-1</sub> t</sup>} for modulation, respectively. Finally, modulated signals are added together to form an OFDM symbol. Use of discrete Fourier transform simplifies the OFDM system structure. The complex envelope of the transmitted OFDM signals can be written as

$$x(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{j2\pi f_k t}, 0 \leq t \leq NT \tag{1}$$

Signals with large N become Gaussian distributed with Probability Density Function (PDF) is given.

$$P_T\{x(t)\} = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{|x(t)|^2}{2\sigma^2}}$$

where σ is the variance of x(t).

**PEAK POWER AVERAGE RATIO:**

In general, the PAPR of OFDM signals x(t) is defined as the ratio between the maximum instantaneous power and its average power

$$PAPR[x(t)] = \frac{P_{PEAK}}{P_{AVERAGE}} = 10 \log_{10} \frac{\max[|X(n)|^2]}{E[|x_n|^2]}$$

where P<sub>PEAK</sub> represents peak output power, P<sub>AVERAGE</sub> means average output power. E[·] denotes the expected value, x<sub>n</sub> represents the transmitted OFDM signals which are obtained by taking IFFT operation on modulated input symbols X<sub>k</sub>. x<sub>n</sub> is expressed as:

$$x_n = \frac{1}{\sqrt{N}} \sum_{K=0}^{N-1} X_k W_N^{nk}$$

The instantaneous output of an OFDM system often has large fluctuations compared to traditional single-carrier systems. This requires that system devices, such as power amplifiers, A/D converters and D/A converters, must have large linear dynamic ranges. If this is not satisfied, a series of undesirable interference is encountered when the peak signal goes into the non-linear region of devices at the transmitter, such as high out of band radiation and inter-modulation distortion. PAPR reduction techniques are therefore of great importance for OFDM systems. Also due to the large fluctuations in power output the HPA (high power amplifier) should have large dynamic

range. This results in poor power efficiency.

**III. PROPOSED TECHNIQUES AND SYSTEM MODEL**

**(i) ZCT METHOD**

**A. Zadoff-Chu Sequences**

Zadoff-Chu sequences are class of poly phase sequences having optimum correlation properties. Zadoff-Chu sequences have an ideal periodic autocorrelation and constant magnitude. The Zadoff-Chu sequences of length L can be defined as:

$$z(k) = \begin{cases} e^{\frac{j2\pi r}{L}(\frac{k^2}{2}+qk)} & \text{for } L \text{ Even} \\ e^{\frac{j2\pi r}{L}(\frac{k(k+1)}{2}+qk)} & \text{for } L \text{ Odd} \end{cases}$$

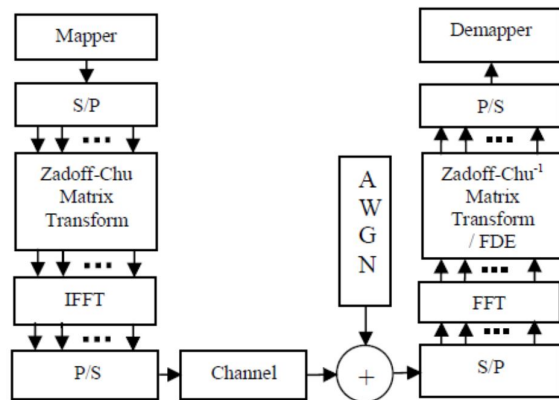


Figure 1. Block diagram of ZCT precoding based OFDM system

Then ZCT precoding is applied to this complex vector which transforms this complex vector into new vector of length N that can be written as **Y=PX= [Y<sub>0</sub>, Y<sub>1</sub>, Y<sub>2</sub>... Y<sub>N-1</sub>]<sup>T</sup>**, Where **R** is a ZCT based row-wise precoding matrix of size L = N × N With the use of reordering as given in equation (2)

$$k = mN + 1 \tag{2}$$

Matrix **R** with row wise reshaping can be written as

$$R = \begin{bmatrix} r_{00} & r_{01} & \dots & r_{0(N-1)} \\ r_{10} & r_{11} & \dots & r_{1(N-1)} \\ \vdots & \vdots & \ddots & \vdots \\ r_{(N-1)0} & r_{(N-1)1} & \dots & r_{(N-1)(N-1)} \end{bmatrix} \quad (3)$$

In other words, the point long Zadoff-Chu sequence fills the precoding matrix row-wise. R is  $N \times N$ , ZCT complex orthogonal matrix with length  $L^2 = N \times N$ . By letting,  $q = 1$  and  $r = 1$ , the ZCT for Even L can be written as  $r_k = \exp [(j*\pi*k^2) / L^2]$ . Accordingly, precoding X gives rise to Y as follows:

$$Y=RX \quad (4)$$

$$Y_m = \sum_{l=0}^{N-1} r_{m,l} X_l \quad m = 0, 1, \dots, N - 1 \quad (5)$$

means  $m^{th}$  row  $n^{th}$  column of precoder matrix. The complex baseband OFDM signal with N subcarriers without precoding is given by

$$x_n = \frac{1}{\sqrt{N}} \sum_{m=0}^{N-1} X_m \cdot e^{j2\pi \frac{nm}{N}}, \quad n=0, 1, 2, \dots, N-1 \quad (6)$$

However, expanding (6) while using  $q = 1$  and  $r = 1$  in (1), gives complex baseband ZCT precoding based OFDM signal with N subcarriers as

$$\hat{x}_n = \frac{1}{\sqrt{N}} \sum_{m=0}^{N-1} \left\{ e^{j \frac{2\pi mn}{N}} \left[ e^{j\pi m^2} \sum_{l=0}^{L-1} \left( Y_l \cdot e^{j \frac{\pi l^2}{L^2}} \right) e^{j \frac{2\pi ml}{L}} \right] \right\} \quad (7)$$

The expression in (7) suggests that  $x_n$  are IFFT of constellation data  $X_l$ , premultiplied with quadratic phase and IFFT precoded, and then alternated with  $\pm 1$ . The PAPR of ZCT-OFDM signal in (7) can be written as

$$PAPR = \frac{\max\{|\hat{x}_n|^2\}}{E\{|\hat{x}_n|^2\}} \quad (8)$$

where  $E[\cdot]$  denotes expectation and the CCDF for an ZCT based OFDM signal can be written as

$$P(PAPR > PAPR_0) = 1 - (1 - e^{-PAPR_0})^N \quad (9)$$

Where  $PAPR_0$  is the clipping level.

(ii) Partial Transmit Sequence (PTS)

Partial Transmit Sequence (PTS) algorithm is a technique for improving the statistics of a multicarrier signal. The basic idea of partial transmit sequences algorithm is to divide the original OFDM sequence into several sub-sequences and for each sub-sequences multiplied by different weights until an optimum value is chosen.

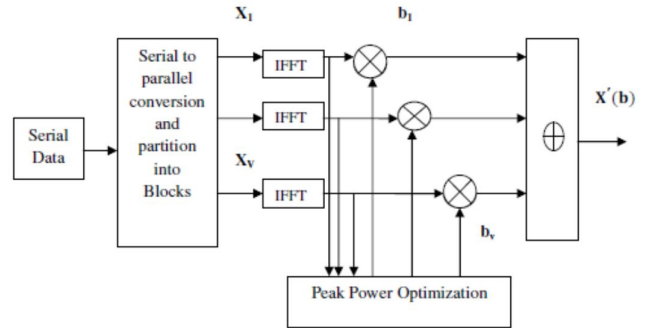


Figure 2. The Block diagram of PTS Technique

Figure 2 is the block diagram of PTS technique. From the left side of diagram, the data information in frequency domain X is separated into V non-overlapping sub-blocks and each sub block vectors has the same size N. So for each and every sub-block it contains N/V nonzero elements and set the rest part to zero. Assume that these sub-blocks have the same size and no gap between each other. The sub-block vector is given by

$$X = \sum_{v=1}^V b_v X_v$$

where  $b_v = e^{j\phi_v}$  ( $\phi_v \in [0, 2\pi]$ ) ( $v = 1, 2, \dots, X_v$ )

is a weighting factor been used for phase rotation. The signal in time domain is obtained by applying IFFT operation on, that is

$$\hat{x} = IFFT(X) = \sum_{v=1}^V b_v IFFT(X_v) = \sum_{v=1}^V b_v X_v$$

For the optimum result one of the suitable factor from combination

$b = [b_1, b_2, \dots, b_v]$  is selected and the combination is given by

$$b = [b_1, b_2, \dots, b_v] = \arg \min_{(b_1, b_2, \dots, b_v)} (\max_{1 \leq n \leq N} |\sum_{v=1}^V b_v X_v|^2)$$

where  $\arg \min [(\cdot)]$  is the condition that minimize the output value of function.

IV. PERFORMANCE EVALUATION AND SIMULATION RESULTS

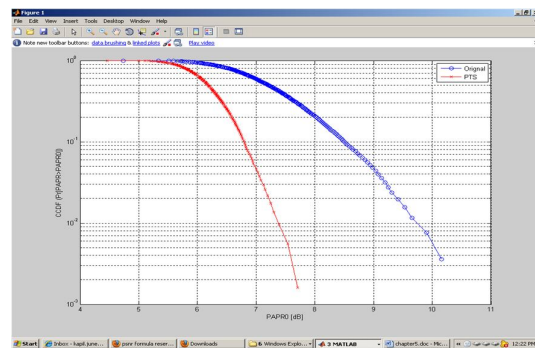


Figure 3 PAPR (Standard OFDM Vs. PTS) (N=4)

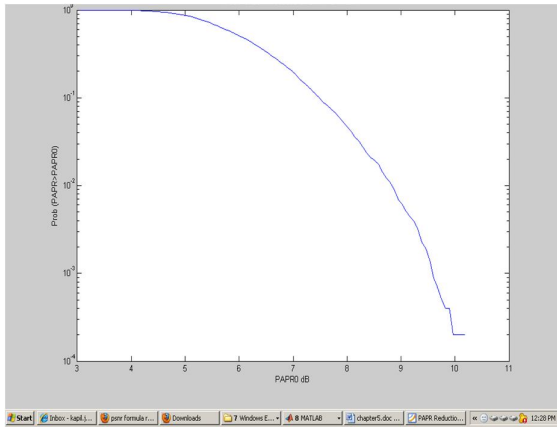


Figure 4 PAPR (ZadOff-Chu Matrix Transform Based Technique) (N=4)

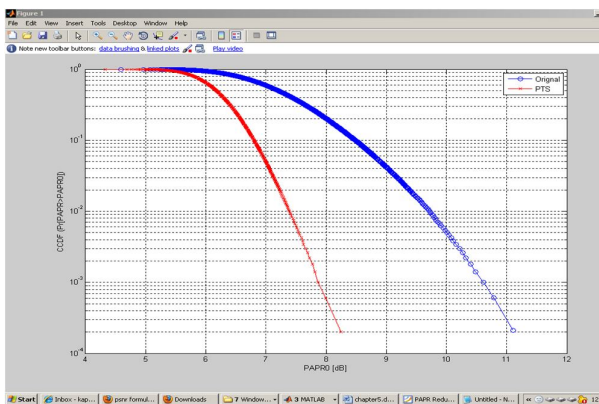


Figure 5 PAPR (Standard OFDM Vs. PTS) (N=16)

Here figure 3 is showing the result analysis of PAPR reduction. Here we have compared standard OFDM and PTS. As we can see the reduction rate of PTS is much more than standard OFDM. The modulation Type is QPSK and its taken for N=4. It is easy to observe from Figure, that  $10^{-2}$  of CCDF the PAPR values in case of standard OFDM filter is 9.5dB and in case of PTS it is 7.4. After PTS implementation it is reduced by 2.1 db.

Here figure 4 is showing the result analysis of PAPR reduction. Here we have compared ZadOff-Chu Matrix Transform Based Technique. As we can see the reduction rate of ZadOff-Chu Matrix Transform is much more than Standard OFDM. The modulation type is QPSK and its taken for N=4. Here we can observe that  $10^{-2}$  of CCDF the PAPR values in case of standard OFDM filter is 9.5 dB where as it was about 8.5. After ZadOff- Chu Matrix Transform approach the implementation it is reduced by 1 db.

Here figure 5 is showing the result analysis of PAPR reduction. Here we have compared standard OFDM and PTS. As we can see the reduction rate of PTS is much more than standard OFDM. The modulation Type is

16QAM. It is easy to observe from Figure.3, that  $10^{-4}$  of CCDF the PAPR values in case of standard OFDM filter is 11.5B and in case of PTS it is 8.5. After PTS implementation it is reduced by 3 db.

## V CONCLUSIONS

OFDM is a very attractive technique for wireless communications due to its spectrum efficiency and channel robustness. A OFDM channel is having number of problems in it including synchronization, PAPR ratio, phase noise etc. PAPR is one of the major problem in OFDM that occur when multiple carrier collectively define a larger peak value then the average peak value of a signal. To increase the linearity in the signal and to reduce the error rate. It is required to reduce the PAPR from the signal. In this present paper we have implemented a Zadoff-Chu Matrix Transform Based approach to reduce the PAPR from the signal. Other than a comparative analysis is being performed using PTS and the standard OFDM.

Partial transmit sequence (PTS) achieve more PAPR reduction than Zadoff-Chu Matrix Transform Based approach at the expense of data rate loss, computational complexity increase. Various parameters like loss in data rate, transmit signal power increase, BER increase, computational complexity increase should be taken into consideration before choosing the appropriate PAPR technique.

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