PAPR Reduction in OFDM Systems

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Abstract: **Orthogonal Frequency Division Multiplexing (OFDM) is a special form of multicarrier modulation which is particularly suited for transmission over a dispersive channel. It offers several advantages, because of which it is widely used in recent years. However, it has few limitations with the key one is peak-to-average-power ratio (PAPR), which determines the system's power efficiency. High PAPR is one of the major drawbacks in OFDM systems, which causes a significant level of signal distortions, when the modulated signals are amplified through high power amplifiers (HPAs). In particularly, high PAPR increases the complexity of Analogue to Digital (A/D) and Digital to Analogue (D/A) converters and also reduces the efficiency of RF High Power Amplifier (HPA). Therefore, PAPR reduction is very essential. For this purpose, a Zadoff-Chu matrix Transform (ZCT) based precoding and postcoding techniques are quantitatively evaluated in this paper in terms of its parameters *q* and *r*. From the analysis, it is inferred that an optimum selection of *q* and *r* can lead to a significant level of reduction in PAPR.**

Keywords: Zadoff-Chu matrix Transform; OFDM conventional; WHT-OFDM; HPA; Peak to Average Power Ratio and BER.

I. INTRODUCTION

OFDM is a key multicarrier modulation technique [1], which provides high spectral efficiency, low implementation complexity [2], less vulnerability to echoes and non–linear distortion [3]. Due to these advantages of the OFDM system, it is vastly used in various communication systems. However, it has a practical limitation of high peak–to–average power ratio [4]. A large PAPR increases the complexity of the analog–to–digital and digital–to–analog converter and reduces the efficiency of the radio – frequency (RF) power amplifier [5]. There are a number of techniques dealing with the problem of PAPR. The key technologies are: constellation shaping, phase optimization, nonlinear companding transforms [6], tone reservation [7] and tone injection (TI), clipping and filtering [8], partial transmit sequence [9] and precoding based techniques. However, these techniques do PAPR reduction at the expense of increase in transmit signal power, bit error rate [7], data rate loss and computational complexity. Therefore, two novel PAPR reduction techniques, namely, Zadoff-Chu matrix Transform (ZCT) precoding based PAPR reduction technique and ZCT postcoding based PAPR reduction technique for OFDM systems were proposed in this paper. In the proposed schemes, the reshaping of the ZCT is carried out one way for precoding and another way for postcoding. For precoding, the reshaping of the ZCT matrix is row wise and is applied before the IFFT; however, for postcoding, the reshaping of the ZCT matrix is done column wise and it is applied after the IFFT.

II. SYSTEM MODEL

A. OFDM System Model

To design the proposed ZCT precoding and postcoding system for reducing PAPR, consider the block diagram of an OFDM system shown in Figure 1. In Figure 1, an OFDM signal consists of *N* subcarriers that are modulated by *N* complex symbols selected from a particular QAM constellation.

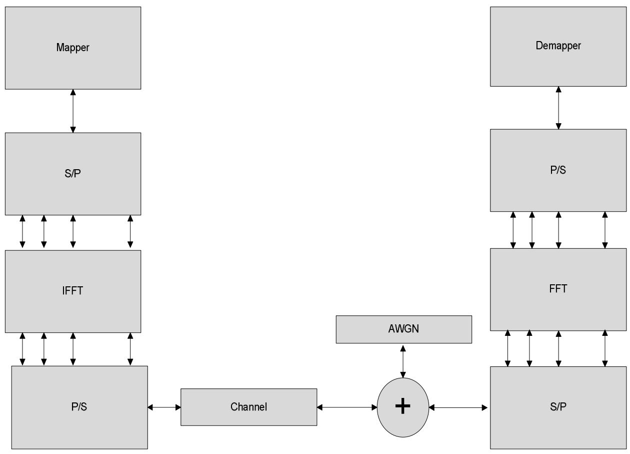

Figure 1: Block diagram of general OFDM system.

In Figure 1, the baseband modulated symbols are passed through serial to parallel converter which generates complex vector of size N. This complex vector of size N can be expressed as

$$X = [X_0, X_1, X_2, X_3 \ldots X_{N-1}] \quad (1)$$

X is then passed through the IFFT block of size $N \times N$ IFFT matrix. The resulted complex baseband OFDM signal with N subcarriers can then be written as

$$x_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} dX_k e^{\frac{j2\pi kn}{N}} \quad n = 0, 1, \ldots, N-1. \tag{2}$$

After parallel-to-serial conversion, a cyclic prefix (CP) with a length of N_g samples is appended before the IFFT output to form the time-domain OFDM symbol, $s = [s_0, \ldots, s_{N+Ng-1}]$. The useful part of OFDM symbol does not include the N_g prefix samples and has duration of T_u seconds. The samples (s) are then amplified, with the amplifier characteristics is given by function F and the output of amplifier produces a set of samples denoted by y. At the receiver front end, the received signal is applied to a matched filter and then sampled at a rate $T_s = T_u/N$. After dropping the CP samples (N_g), the received sequence z, assuming an additive white Gaussian noise (AWGN) channel, can be expressed as

$$z = F(Wd) + \eta \tag{3}$$

Where, the noise vector η consists of N independent and normally distributed complex random variables with zero mean. Subsequently, the sequence z is fed to the fast Fourier transform (FFT), which produces the frequency-domain sequence r as

$$r = W^H z \tag{4}$$

Where, k_{th} element of r is given by

$$r_k = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} z_n e^{\frac{-j2\pi kn}{N}} \quad k=0, 1, 2, \ldots, N-1 \tag{5}$$

Finally, the estimated symbols vector $\hat{d}$ can be obtained from r. It is to be noted that the demodulation is performed based on the assumption of perfect symbol timing, carrier frequency, and phase synchronization. One of the main disadvantages of OFDM systems is the high PAPR of transmitted signal due to the combination of N modulated subcarriers. The PAPR for a continuous time signal $x(t)$ is defined as:

$$PAPR = \frac{max\{|x(t)|^2\}}{E\{|x(t)|^2\}} \quad 0 \le t \le T_u \tag{6}$$

The PAPR for discrete time signals can be estimated by oversampling the vector d by a factor L and computing NL point IFFT. The PAPR in this case is defined as:

$$PAPR = \frac{max\{|x(n)|^2\}}{E\{|x(n)|^2\}} \quad n = 0,1, \ldots, NL-1 \tag{7}$$

The denominator in above equation represents the average power per OFDM symbol at the amplifier input, which is denoted as P_{in}. PAPR, in quantitative terms, is usually expressed in terms of Complementary Cumulative Distribution function (CCDF) for an OFDM signal, and is mathematically given by

$$P(PAPR > PAPR_0) = 1 - (1 - e^{-PAPR_0})^N \tag{8}$$

Where, $PAPR_0$ is the clipping level. This equation can also be read as the probability that the PAPR of a symbol block exceeds some clip level $PAPR_0$.

B. ZADOFF-CHU SEQUENCES

In general, a Zadoff–Chu sequence is a complex-valued sequence which, when applied to radio signals, gives rise to an electromagnetic signal of constant amplitude, whereby cyclically shifted versions of the sequence imposed on a signal result in zero cross-correlation with one another at the receiver. The sequence exhibits the useful property that cyclically shifted versions of itself are orthogonal to one another, provided, that is, that each cyclic shift, when viewed within the time domain of the signal, is greater than the combined propagation delay and multi-path delay-spread of that signal between the transmitter and receiver. This is expected to reduce PAPR by significant level. Mathematically, Zadoff–Chu sequences can be defined for a sequence of length L as:

$$z(k) = \begin{cases} e^{\frac{j2\pi r}{L}\left(\frac{k^2}{2}+qk\right)} & for\ L\ even \\ e^{\frac{j2\pi r}{L}\left(\frac{k(k+1)}{2}+qk\right)} & for\ L\ odd \end{cases} \tag{9}$$

Where, $k = 0, 1, 2 \ldots L-1$, q is any integer and r is any integer relatively prime to L. From the above equation, q and r comes in the numerator of the exponential power fraction. This means, that their variation can lead to a significant change in the output. This is the key focus of the research reported in this paper. This ZCT sequence can be applied in two forms: pre-coding and post-coding, as discussed below.

C. ZCT PRE-CODING BASED OFDM SYSTEM

The block diagram of ZCT pre-coding based OFDM system is shown in Figure 2. In the ZCT pre-coding based OFDM system, the baseband modulated data is passed through S/P convertor which generates a complex vector of size N that can be written as $X = [X_0, X_1, X_2 \ldots X_{N-1}]^T$. ZCT pre-coding is then applied to this complex vector which transforms this complex vector into new vector of length N that can be written as $Y=[Y_0, Y_1, Y_2 \ldots Y_{N-1}]^T$; where, R is a ZCT based row-wise precoding matrix of size $L=N*N$. By reordering,

$$k = mN + l \tag{10}$$

And the matrix R with row wise reshaping can be written as

$$R = \begin{bmatrix} r_{00} & \cdots & r_{0(N-1)} \\ \vdots & \ddots & \vdots \\ r_{(N-1)0} & \cdots & r_{(N-1)(N-1)} \end{bmatrix} \tag{11}$$

Where, R is a $N*N$, ZCT complex orthogonal matrix with length $L^2 = N*N$. By letting, q=1 and r=1, the ZCT for even L can be written as $r_k = exp[(j*pi*k^2) / L^2]$. Accordingly, pre-coding X gives rise to Y as follows:

$$Y = RX \tag{12}$$

Or,

$$Y_m = \sum_{l=0}^{N-1} r_{m,l} X_l \quad m = 0,1,2, \ldots \ldots, N-1 \tag{13}$$

Where, $r_{m,l}$ means the m^{th} row and l^{th} column of pre-coder matrix. Therefore, the complex baseband OFDM signal with N subcarriers with ZCT pre-coding is given by

$$\hat{x}_n = \frac{1}{\sqrt{N}} \sum_{m=0}^{N-1} \left\{ e^{\left(\frac{j2\pi mn}{N}\right)} \left[e^{j\pi m^2} \sum_{l=0}^{L-1} \left(Y_l . e^{\frac{j\pi l^2}{L^2}} \right) e^{\frac{j2\pi ml}{L}} \right] \right\} \tag{14}$$

The above equation can be expressed as $\hat{x}_n$ the IFFT of constellation data X_l pre-multiplied with ZCT matrix. The PAPR of this signal is same as before.

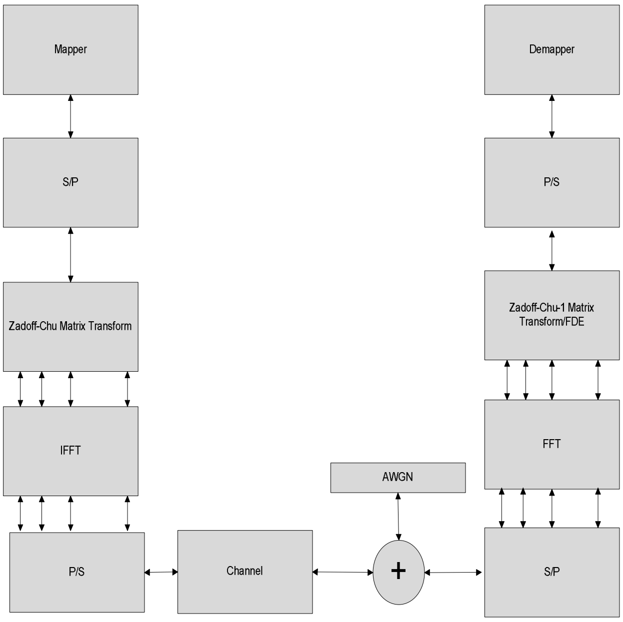

Figure 2: Block diagram of an OFDM system with ZCT pre-coding approach.

D. ZCT POST-CODING BASED OFDM SYSTEM

The block diagram of ZCT post-coding based OFDM system is shown in Figure 3. In the ZCT post-coding based OFDM system, the baseband modulated data is passed through S/P convertor which generates a complex vector of size N that can be written as $X = [X_0, X_1, X_2 \ldots X_{N-1}]^T$. Then, IFFT is performed to this complex vector which transforms this complex vector into new vector of length N that can be written as $y = \text{IFFT}\{X\} = [y_0, y_1, y_2 \ldots y_{N-1}]^T$. Similarly, re-ordering as in pre-coding to have:

$$k = m + lN \qquad (15)$$

And, ZCT matrix C with column wise reshaping is written as

$$C = \begin{bmatrix} c_{00} & \cdots & c_{0(N-1)} \\ \vdots & \ddots & \vdots \\ c_{(N-1)0} & \cdots & c_{(N-1)(N-1)} \end{bmatrix} \qquad (16)$$

In other words, the N^2 point long Zadoff-Chu sequence fills the pre-coding matrix column-wise. The matrix C is $N*N$, ZCT complex orthogonal matrix with length $L^2 = N*N$. By letting, q=1 and r=1, the ZCT for even L can be written as $c_k = exp[j*pi*k^2/L^2]$. Accordingly, postcoding y gives rise to w as follows:

$$w = C.y \qquad (17)$$

Where, C is a ZCT column wise post-coding matrix of size $L^2 = N*N$. For q = 1 and r =1, the complex baseband ZCT postcoding based OFDM signal with N subcarriers can be written as:

$$w_m = e^{\frac{j\pi m^2}{L^2}} \sum_{l=0}^{L-1} \left[e^{j\pi l^2}.x_l\right].e^{\frac{j2\pi ml}{L}} \; ; \quad m = 0,1, \ldots \ldots, N-1 \qquad (18)$$

Where, w_m is the IFFT of constellation data X_l premultiplied with quadratic phase and IFFT postcoded, and then alternated with ±1. The PAPR of this ZCT postcoding based OFDM signal is same as before.

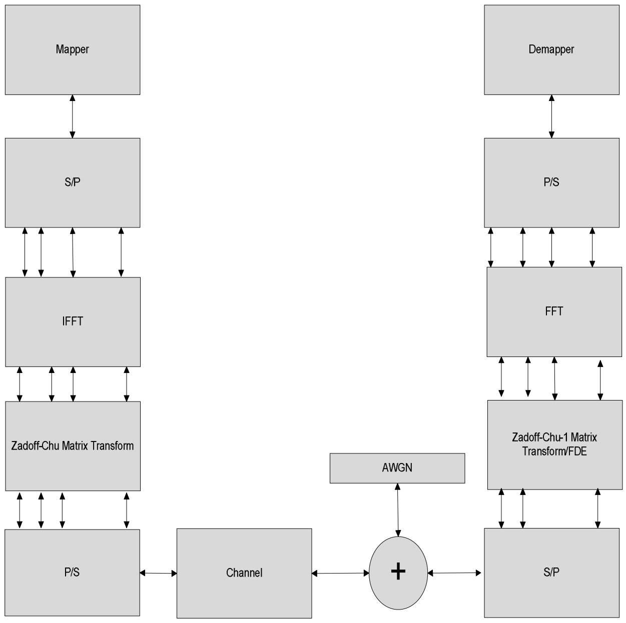

Figure 3: Block diagram of an OFDM system with ZCT post-coding approach.

III. IMPLEMENTATION

To do the PAPR analysis and to evaluate performance of ZCT precoding and postcoding based OFDM systems, the following key steps are undertaken.

1. To do the PAPR analysis of ZCT precoding and postcoding based OFDM, firstly binary data is generated randomly.
2. The generated binary data is then converted into symbols and is then modulated by M-QAM (where M=4, 16, 64, 256). The ZCT matrix is calculated for the length of the data.
3. The serial data is then converted into parallel data and IFFT is performed. The ZCT matrix (Transform) is applied before and after the IFFT, respectively for precoding and postcoding. The PAPR is then calculated and compared.

4. The parallel signal is then converted to serial data and is passed through a multipath channel with AWGN noise added.
5. The received signal is again converted from serial to parallel data. ZCT inverse matrix is applied before and after the FFT. The final demodulated signal is parallel data, which is then converted into serial data. The serial data is then demodulated to retrieve back the signal.

IV. RESULT & DISCUSSION

To evaluate the performance of ZCT precoding and postcoding based OFDM systems, extensive simulations have been performed in Matlab. The simulations are performed for different values of *q* and *r*. To show the PAPR analysis of ZCT precoding and postcoding based OFDM systems, data is generated randomly and then modulated by M-QAM (where M = 4, 16, 64, 256). The order of FFT and IFFT is taken as 64. The results obtained are shown in Figures 4 to 11, for various combinations of *q* and *r*; whereas, *q*= 1 & 7 and *r* =2, 13, 31, 53, 73, 101. The key conclusions drawn from the results are discussed below:

1. From Figures 4 to 11, it can be interpreted that the results obtained for ZCT precoding and postcoding are different. The PAPR is more reduced in case of ZCT precoding for the same value of *q* and *r*.
2. From Figures 4 to 11, for ZCT precoding, the PAPR is more reduced when value of *r* is increased from 2 to 101, for any value of *q*. Zadoff-Chu sequences is directly proportional to the exponential raised to power *r* and product of rk^2 for even *L*. Therefore, the increase in value of *r* will yield greater output and so the PAPR will increase as observed from Figures 4 to 11.
3. As the constellation size increases, i.e., from M = 4, 16, 64, 256; there is significant reduction in PAPR for the same values of *q* and *r*. This is clearly observed in Figures 4 and 11.
4. It can also be seen that ZCT postcoding has quick and sharp roll off than ZCT pre-coding. Also, there is less irregularity in ZCT postcoding compared to ZCT precoding.
5. It can also be seen that ZCT precoding curves overlap for small time and some of these are intersecting each other.

From the above discussion, the value of *q* and *r* significantly changes the PAPR. Also, for same values of *q* and *r*, the variation in constellation order also significantly changes the PAPR. It is also found that ZCT precoding is more effective that ZCT postcoding. Therefore, the selection of value of *q*, *r*, M and ZCT precoding/postcoding depends upon the level of PAPR reduction required. Here in this paper, it is preferred to choose a small value of *q* and large value of *r* for ZCT post coding and both small values of *q* and *r* for ZCT pre-coding. It is preferred to choose ZCT post coding because of its sharp roll-off; however, at the expense of less reduction in PAPR or requirement of large values of *r* if results comparable to ZCT pre-coding needs to be achieved.

CASE I: Constellation size = 4

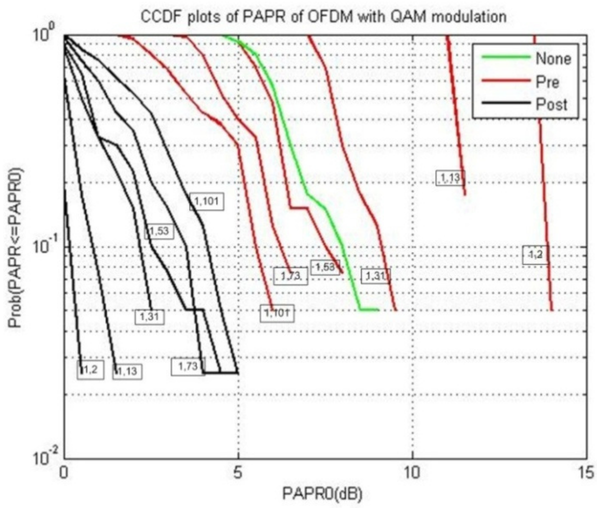

Figure 4: CCDF plots of PAPR of OFDM with QAM modulation for constellation size 4 and with ZCT precoding & postcoding. The values are: *q*=1 and *r*=2, 13, 31, 53, 73, 101.

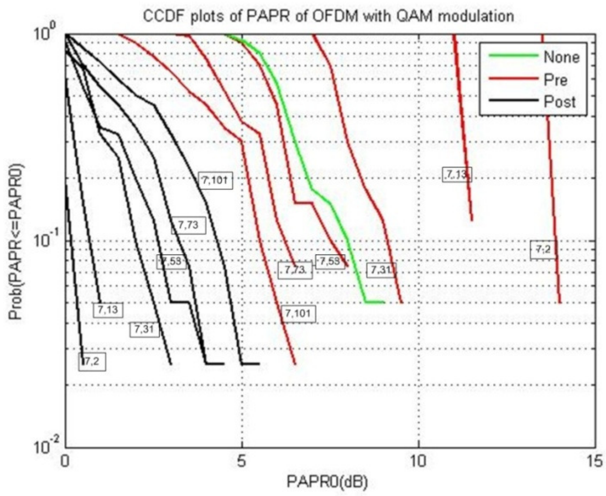

Figure 5: CCDF plots of PAPR of OFDM with QAM modulation for constellation size 4 and with ZCT precoding & postcoding. The values are: *q*=7 and *r*=2, 13, 31, 53, 73, 101.

CASE II: Constellation size = 16

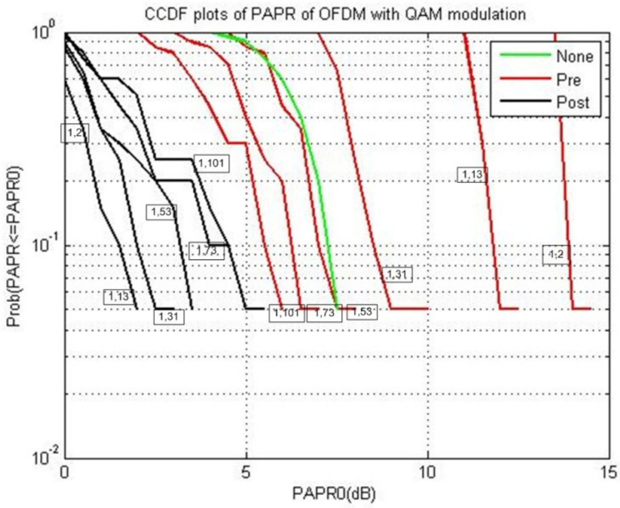

Figure 6: CCDF plots of PAPR of OFDM with QAM modulation for constellation size 16 and with ZCT precoding & postcoding. The values are: *q*=1 and *r*=2, 13, 31, 53, 73, 101.

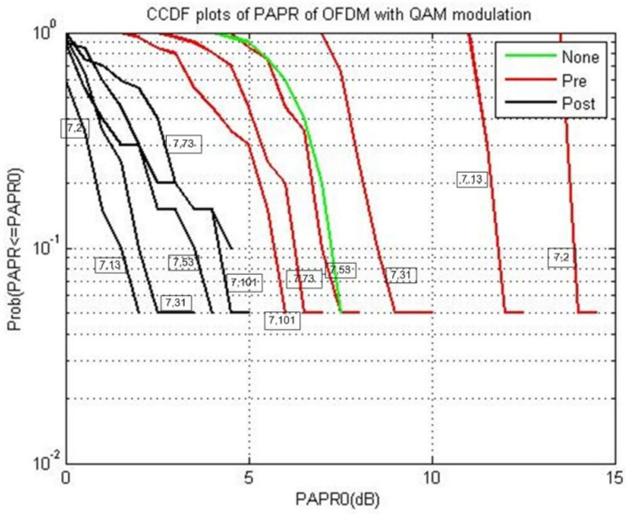

Figure 7: CCDF plots of PAPR of OFDM with QAM modulation for constellation size 16 and with ZCT precoding & postcoding. The values are: *q*=7 and *r*=2, 13, 31, 53, 73, 101.

CASE III: Constellation size = 64

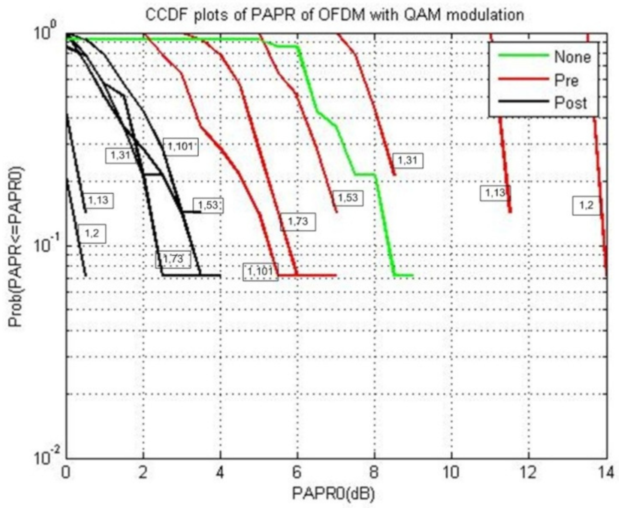

Figure 8: CCDF plots of PAPR of OFDM with QAM modulation for constellation size 64 and with ZCT precoding & postcoding. The values are: *q*=1 and *r*=2, 13, 31, 53, 73, 101.

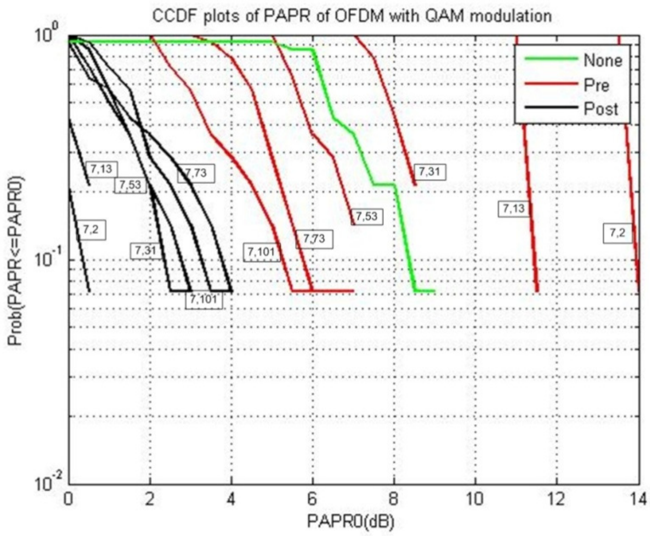

Figure 9: CCDF plots of PAPR of OFDM with QAM modulation for constellation size 64 and with ZCT precoding & postcoding. The values are: *q*=7 and *r*=2, 13, 31, 53, 73, 101.

CASE IV: Constellation size =256

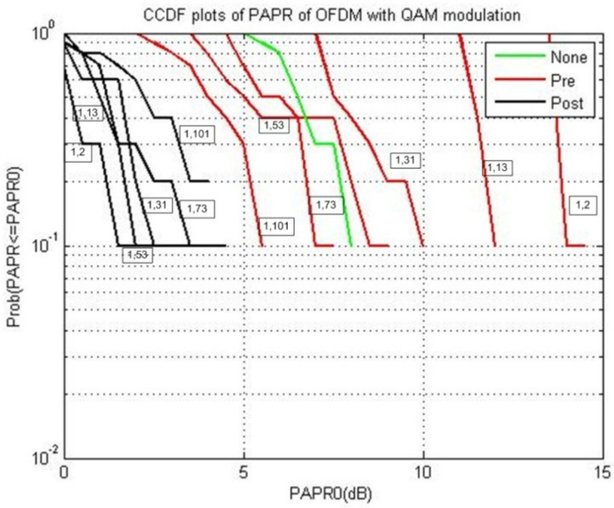

Figure 10: CCDF plots of PAPR of OFDM with QAM modulation for constellation size 256 and with ZCT precoding & postcoding. The values are: q=1 and r=2, 13, 31, 53, 73, 101.

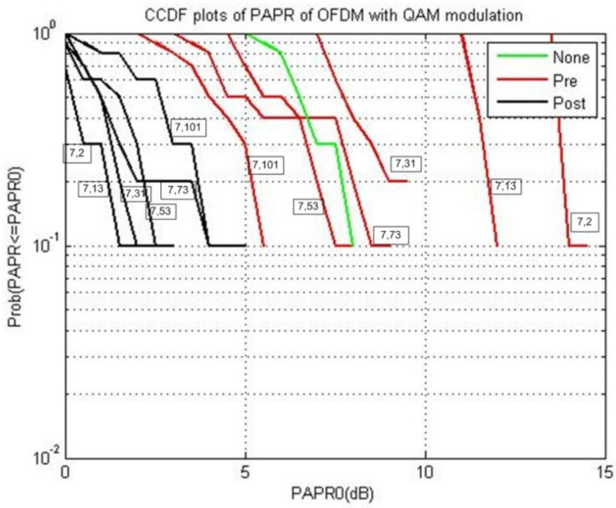

Figure 11: CCDF plots of PAPR of OFDM with QAM modulation for constellation size 256 and with ZCT precoding & postcoding. The values are: q=7 and r=2, 13, 31, 53, 73, 101.

V. CONCLUSION

This paper presented the results obtained on applying a ZCT pre-coding and post-coding for the purpose to reduce PAPR in OFDM systems with M-QAM modulation, where as M=4, 16, 64, 256. It is observed from the graphs that the response of both ZCT pre-coding and post-coding is different for r>1 and q>1. The PAPR significantly varies with the values of r and q. For any particular value of r and q, the PAPR reduction is more for ZCT pre-coding than for ZCT pre-coding. From the results, it is preferred to choose a small value of q and large value of r for ZCT post coding and both small values of q and r for ZCT pre-coding. It is preferred ZCT post coding because of its sharp roll-off; however, at the expense of less reduction in PAPR or requirement of large values of r if results comparable to ZCT pre-coding needs to be achieved.

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