

Optional Inventory Ordering Policies of Multiple Non-Instantaneous Deteriorating Items Under Trade Credit and Price Discount

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Abstract. When ordering multiple items, retailers tend to be used the collaborative procurement, so conducive to the formation of scale in quantities utility. Therefore, this paper established the inventory model of the retailer' in a multiple non-instantaneous deterioration items supply chain, with two kinds of incentives-credits and price discounts. Then we use the theorems to proof the existence of the optimal solution of inventory, and compare the profits under different scenarios. Finally, we use numerical examples to validate the model, we found that the optimal profits of retailer under trade credit is better than that of price discount in the multiple non-instantaneous deterioration items supply chain.

Keywords: Multiple non-instantaneous deteriorating items, trade credit, price discount, supply chain

INTRODUCTION

No-matter how big the company, they all face up two tough problems: the shortage of money and the inventory ordering policies of multiple non-instantaneous deterioration items with uncertainty demand. Especially, the extension of the concept of multiple non-instantaneous deterioration items is involving most of our daily products, with the development of society and technology. One of the most important branches of operation is the inventory model of multiple non-instantaneous deterioration items, and has been systematically summarized. But, in practice, retailers always order multiple non-instantaneous deterioration items, and the single item inventory model is not so precisely for the multiple non-instantaneous deterioration items. Therefore, to tackle the inventory ordering policies of multiple non-instantaneous deterioration items is aroused highly attention in the field of scholars. Bose et al. (1995) built up the EOQ model for deterioration goods with liner demand, allowing shortage and backlogging, and the influence of time value, firstly. And they acquire the inventory policies in difference scenarios. Moon et al. (2005) described a special kind of deterioration items, which the price goes up with time, for example, wine, poultry. And they also build up the inventory model with considering time value; Yang and Chang. (2013) formulated the optimal replenishment model by considering trade credit, time value, two warehouse and allowing shortage and backlogging; Tsao (2010) considered multi-echelon multi-item channels subject to supplier's credit period and retailer's promotional effort. They analyse two trade allowances, the promotion costs sharing and the cash discount, finding that coordination mechanism is better; Wen and Da (2006) discuss single ordering, partly ordering and coordination ordering from the perspective of retailer's, and they got the optimal ordering policies; Wang and Jiang (2007) compare four different circumstance of the combination of decentralized decision and cooperative decision, finding that the cooperative circumstance is better; Mo (2011) considered a model with demand depend on inventory and resisted space. They got the result by line search algorithm; Zhang et al. (2012) provide a completed literature review of the joint inventory policies of multi-deterioration items, and find out the inventory model of multiple deterioration items will be hot in the future.

Above all, they just consider the goods is deteriorating, when the goods arrive at the store, but, in practice, some kinds of goods have a certain life long time. So the existing models is not so precisely to tackle the inventory issues of non-instantaneous deterioration items. Wu (2006) provide the concept of non-instantaneous deterioration items, and build the related models to solve it; Chang et al. (2010) formulated the EOQ model of non-instantaneous deterioration items with considering constant demand, allowing shortage and backlogging, and the backlogging depends on the time of next replenishment time; Geetha et al. (2010) extent the model of Zhang et al.(2012), with considering demand depends on time and price, the room of retailer's shelves house and the maximize of the warehouse; Maihmi et al. (2013a) considered the trade credit and partly backlogging, then Maihmi et al. (2013b) extent their model with demand depend

on time and price; Nita et al. (2013) assume that the demand function is rely on the advertisement and price. What's more, the model is considering a general holding cost function and a general deteriorating rate; Dye et al. (2013) established a maximize the profit model with considering constant demand, the effect of preserve technology, a generalized deterioration rate, and inventory and backlogging depends on time. They got the different factors' influence; Li and Wang (2011) built up the model of minimize the profit of retailer's, and analyse the influence of the time of beginning deterioration, deterioration rate and trade period.

Above all, deterioration items inventory research can be divided into two categories: deteriorating immediately and non-instantaneous deterioration. Even there are some literatures have been studied the multi-deterioration items, but the research about non-instantaneous deterioration items with trade credit is rarely. We build up the inventory ordering model under price discount and trade credit to maximize the retailer's profit, and compare the two strategies, get that the profit under trade credit is better than that of under price discount in multiple non-instantaneous deterioration items supply chain.

1. PROBLEM DESCRIPTION, ASSUMPTIONS AND NOTATIONS

1.1 Problem description

In this paper, we consider the retailer how to make inventory policies in a multiple non-instantaneous deterioration items supply chain. In order opening, the retailer will order all the non-instantaneous deterioration items together, and at the end of ordering period, all products are already completely sold. Items have no effect to others. Meanwhile, suppliers will provide the trade credit or price discount to retailer, to stimulate the order quantity. Modelling the problem based on the process shown in Fig. 1:

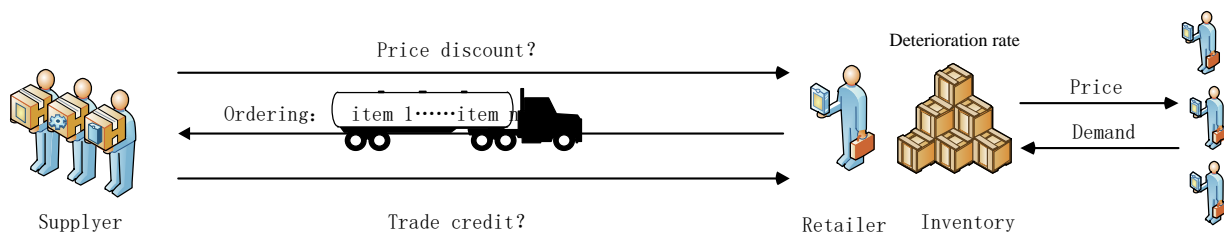


Fig. 1 The structure of modelling

1.2 Assumptions

- (1) Replenishment rate is infant, lead time is zero, and no shortage are allowed.
- (2) The demand of item i depends on time and price, $D_i(p_i, t) = (a_i - b_i p_i) e^{\lambda_i t}$ ($a_i > 0$, $b_i > 0$), in this paper we assume $\lambda_i < 0$.
- (3) Deterioration rate θ_i depends on inventory, and the beginning time of deterioration is smaller than ordering period $\max(t_{di}) < T$.
- (4) Bad items can't be repaired or replaced in the order cycle.
- (5) Capital rate of inventory opportunity cost is greater than the current interest rates, $I_p > I_e$.

1.3 Notations

- S : the ordering cost per cycle, has nothing to do with the kinds of items per order.
- S_i : the additional ordering cost of item i , the particularity of the item, different item in the process of transportation, loading and unloading and additional costs.
- T : ordering period.
- c_i : the purchase cost of per unit item of i .
- h_i : the holding cost of per unit item of i .
- p_i : the price of item i , $p_i > c_i$.
- θ_i : the deterioration rate of item i .
- t_{di} : the beginning time of deterioration of item i .
- Q_i : the order quantity of item i .
- $I_{1i}(t)$: the inventory level of product i during interval 1 at time t .
- $I_{2i}(t)$: the inventory level of product i during interval 2 at time t .
- I_e : the current interest rates.
- I_p : capital rate of inventory opportunity cost.
- r : discount rate.

TP_j : the retailer's total profit of circumstance j , $j=1,2,3$.

2. Model formulation

2.1 The model under price discount

First of all, the supplier provide price discount to retailer, retailers make his inventory policies by maximizing the profit. When , inventory system under the price discount is shown in fig.2:

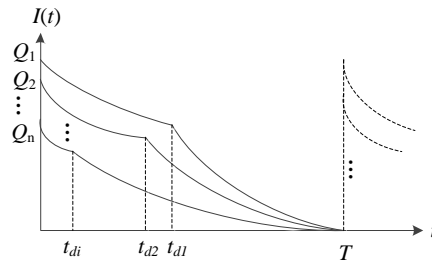


Fig. 2 Retailer's inventory levels under the price discount

The inventory goes like this: during the time interval $[0, t_{di}]$, the inventory level is decreasing only owing to demand. The inventory level is dropping to zero due to demand and deterioration during the time interval $[t_{di}, T]$.

Using the differential equation representing the inventory status is given by:

$$\frac{dI_i(t)}{dt} = \begin{cases} -(a_i - b_i p_i) e^{\lambda_i t} & 0 \leq t \leq t_{di} \\ -(a_i - b_i p_i) e^{\lambda_i t} - \theta_i I_{2i}(t) & t_{di} \leq t \leq T \end{cases} \quad (1)$$

According to the boundary conditions, yields:

$$I_i(t) = \begin{cases} I_{1i}(t) = \frac{(a_i - b_i p_i)}{\lambda_i} [1 - e^{\lambda_i t}] + Q_i & 0 \leq t \leq t_{di} \\ I_{2i}(t) = \frac{(a_i - b_i p_i) e^{-\theta_i t}}{\lambda_i + \theta_i} [e^{(\lambda_i + \theta_i) T} - e^{(\lambda_i + \theta_i) t}] & t_{di} \leq t \leq T \end{cases} \quad (2)$$

Let $I_{1i}(t_{di}) = I_{2i}(t_{di})$, we get:

$$Q_i = \frac{(a_i - b_i p_i) e^{-\theta_i t_{di}}}{\lambda_i + \theta_i} [e^{(\lambda_i + \theta_i) T} - e^{(\lambda_i + \theta_i) t_{di}}] - \frac{(a_i - b_i p_i)}{\lambda_i} [1 - e^{\lambda_i t_{di}}] \quad (3)$$

$$= \sum_{i=1}^n (a_i - b_i p_i) (T - \theta_i t_{di} T + \theta_i t_{di}^2)$$

We have the retailer's total profit function:

$$TP^1(p, T, R) = \sum_{i=1}^n [p_i (a_i - b_i p_i) - c_i (1-R) (a_i - b_i p_i) (1 - \theta_i t_{di}) - \frac{h_i (a_i - b_i p_i) (1 - 2(\lambda_i + \theta_i) T) t_{di}}{T (\lambda_i + \theta_i)} - c_i \theta_i (1-R) (a_i - b_i p_i) (T - t_{di}) - \frac{c_i I_p (1-R) (a_i - b_i p_i) (1 - 2(\lambda_i + \theta_i) T) t_{di}}{T (\lambda_i + \theta_i)}] - \frac{1}{T} (S + \sum_{i=1}^n S_i) \quad (4)$$

In the order of Eq.(4) : sales revenue of item i , purchase cost of item i , holding cost of item i , deterioration cost of item i , inventory opportunity cost of item i , ordering cost.

2.2 The model under trade credit

In the multiple non-instantaneous deterioration item inventory system, we build up the inventory model under three scenarios: (1) $M < t_{di} < T$; (2) $t_{di} < M < T$; and (3) $t_{di} < T < M$, with the constrain of $T > t_{di}$. The inventory system under the trade credit is shown in fig.3.

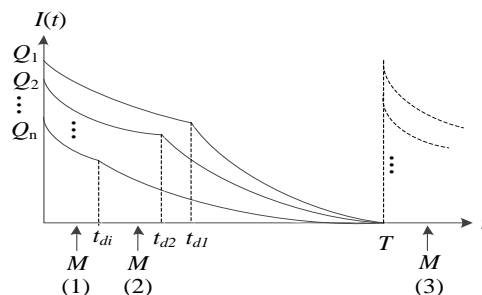


Fig.3 Retailer's inventory levels under the trade credit

According to Fig.3, we can get the retailer's total profit function, when $M < T$.

$$\begin{aligned}
 TP^2(p, T, R) = & \sum_{i=1}^n \left\{ p_i(a_i - b_i p_i) + \frac{p_i I_e(a_i - b_i p_i) M}{T} - c_i(a_i - b_i p_i)(1 - \theta_i t_{di})(1 - rM) - \right. \\
 & \frac{h_i(a_i - b_i p_i)}{T} \frac{(1 - 2(\lambda_i + \theta_i)T)t_{di}}{\lambda_i + \theta_i} - \frac{c_i \theta_i(a_i - b_i p_i)}{T(\lambda_i + \theta_i)} (t_{di} - T)(2 + (\lambda_i + \theta_i)T) - \\
 & Y_i \frac{c_i I_p(a_i - b_i p_i)}{T} [(t_{di} - M)(T - \theta_i t_{di} T + \theta_i t_{di}^2) + T] - \\
 & \left. (1 - Y_i) \frac{c_i I_p(a_i - b_i p_i)(T - M)(1 - \theta_i - r + (\lambda_i + \theta_i)T)}{T(\lambda_i + \theta_i)(\theta_i + r)} \right\} - \frac{1}{T} (S + \sum_{i=1}^n s_i)
 \end{aligned} \tag{5}$$

$$\text{And } Y_i = \begin{cases} 1 & M < t_{di} < T, \text{ for item } i \\ 0 & t_{di} < M < T, \text{ for item } i \end{cases}$$

In the order of Eq.(5) : sales revenue of item i , interest revenue of item i , purchase cost of item i , holding cost of item i , deterioration cost of item i . when $M < t_{di} < T$, the inventory opportunity cost of item i ; when $t_{di} < M < T$, the inventory opportunity cost of item i , and the ordering cost.

When $t_{di} < T < M$, the retailer's profit function is:

$$\begin{aligned}
 TP^3(p, T, R) = & \sum_{i=1}^n [p_i(a_i - b_i p_i) + p_i I_e(a_i - b_i p_i)(1 + T - M) - c_i(a_i - b_i p_i)(1 - \theta_i t_{di})(1 - rM) - \\
 & \frac{h_i(a_i - b_i p_i)}{T} \frac{(1 - 2(\lambda_i + \theta_i)T)t_{di}}{\lambda_i + \theta_i} - \frac{c_i \theta_i(a_i - b_i p_i)}{T(\lambda_i + \theta_i)} (t_{di} - T)(2 + (\lambda_i + \theta_i)T)] - \\
 & \frac{1}{T} (S + \sum_{i=1}^n s_i)
 \end{aligned} \tag{6}$$

In the order of Eq.(6) : sales revenue of item i , interest revenue of item i , purchase cost of item i , holding cost of item i , deterioration cost of item i , and the ordering cost.

Above all, the function (3)-(6) can be described as:

$$TP = \begin{cases} TP^1(p, T, R) & \text{price discount } R \\ TP^2(p, T, M) & t_{di} < T \ \& \ M < T \\ TP^3(p, T, M) & t_{di} < T < M \end{cases} \tag{7}$$

3. Theoretical results

3.1 Results with price discount

We discuss the optimal ordering quantity and ordering period by maximizing the profit of retailer. As p_i has the same identity and no effect to each other. We use p to alternative p_i for simple. First, we discuss the (p, T) not only exist and the only, for any R . Then, we discuss the R not only exist and the only, for any p and T .

Taking the partial derivative of TP^1 with respect to p and T , we have:

$$\frac{\partial TP^1(p, T, R)}{\partial p} = \sum_{i=1}^n [a_i - 2b_i p_i + b_i c_i (1 - R)(1 - \theta_i t_{di}) + \frac{b_i t_{di} (1 - 2(\lambda_i + \theta_i)T)}{T(\lambda_i + \theta_i)} (h_i + c_i I_p(1 - R)) + b_i c_i \theta_i (1 - R)(T - t_{di})] \tag{8}$$

$$\frac{\partial TP^1(p, T, R)}{\partial T} = \frac{1}{T^2} (S + \sum_{i=1}^n s_i) + \sum_{i=1}^n \left[\frac{(a_i - b_i p_i) t_{di}}{T^2 (\lambda_i + \theta_i)} (h_i + c_i I_p(1 - R)) - c_i \theta_i (1 - R)(a_i - b_i p_i) \right] \tag{9}$$

Taking the second partial derivative of TP^1 with respect to p and T , and we have the Hessian matrix:

$$\begin{aligned}
 \Delta(H^1) = & \left(\frac{\partial^2 TP^1}{\partial p \partial T} \right)^2 - \frac{\partial^2 TP^1}{\partial p^2} \times \frac{\partial^2 TP^1}{\partial T^2} \\
 = & \left[\sum_{i=1}^n \left[\frac{-b_i t_{di}}{T^2} (h_i + c_i I_p(1 - R)) + b_i c_i \theta_i (1 - R) \right] \right]^2 - 4 \sum_{i=1}^n b_i \times \left[\frac{1}{T^3} (S + \sum_{i=1}^n s_i) + \sum_{i=1}^n \frac{(a_i - b_i p_i) t_{di}}{T^3 (\lambda_i + \theta_i)} (h_i + c_i I_p(1 - R)) \right]
 \end{aligned} \tag{10}$$

Theorem 1 If $1 - R \in [0, 1]$, there are optimal values of p and T for maximum value of the total profit TP^1 .

Theorem 2 if $n + \sum_{i=1}^n \theta_i T + \frac{I_p t_{di}}{T(\lambda_i + \theta_i)} > \sum_{i=1}^n 2(I_p - \theta_i) t_{di}$, retailer's profit is proportional to the price discount rate; if

$n + \sum_{i=1}^n \theta_i T + \frac{I_p t_{di}}{T(\lambda_i + \theta_i)} \leq \sum_{i=1}^n 2(I_p - \theta_i) t_{di}$, retailer's profit is inversely proportional to the price discount rate.

Let Eq.(8) and Eq.(9) to be zero, we obtain the optimal value of p_i^* and T in price discount:

$$p_i^* = \frac{1}{2b_i} [a_i + b_i c_i (1-R)(1-\theta_i t_{di}) + \frac{b_i t_{di} (1-2(\lambda_i + \theta_i) T)}{T(\lambda_i + \theta_i)} (h_i + c_i I_p (1-R)) + b_i c_i \theta_i (1-R)(T - t_{di})]$$

$$T^{1*} = \sqrt{\frac{S + \sum_{i=1}^n s_i + \sum_{i=1}^n t_{di} (a_i - b_i p_i) (h_i + c_i I_p (1-R)) / (\lambda_i + \theta_i)}{\sum_{i=1}^n c_i \theta_i (1-R) (a_i - b_i p_i)}}$$

According to Theorem1, we get:

$$R^* = \begin{cases} \bar{R} & n + \sum_{i=1}^n \theta_i T + \frac{I_p t_{di}}{T(\lambda_i + \theta_i)} > \sum_{i=1}^n 2(I_p - \theta_i) t_{di} \\ 0 & n + \sum_{i=1}^n \theta_i T + \frac{I_p t_{di}}{T(\lambda_i + \theta_i)} < \sum_{i=1}^n 2(I_p - \theta_i) t_{di} \end{cases}$$

3.2 Results under trade credit

First of all, we should to determine the (p_i, T) of item i is not only exist but the only for any M . For any p_i and T , the M is not only exist but the only.

Scenario 1 if $M < T$, there is an optimal solution of the profits.

Taking the partial derivative of TP^2 with respect to p and T , we have:

$$\frac{\partial TP^2}{\partial p} = \sum_{i=1}^n \left\{ (a_i - 2b_i p_i) - \frac{b_i p_i I_e M}{T} + b_i c_i (1 - \theta_i t_{di}) (1 - rM) + \frac{b_i h_i t_{di}}{\lambda_i + \theta_i} \left(\frac{1}{T} - 2(\lambda_i + \theta_i) \right) + \frac{b_i c_i \theta_i}{\lambda_i + \theta_i} \left[\frac{2t_{di}}{T} - (\lambda_i + \theta_i) T + (\lambda_i + \theta_i) t_{di} \right] + Y_i b_i c_i I_p [(t_{di} - M)(1 - \theta_i t_{di} + \frac{\theta_i t_{di}^2}{T}) + 1] + (1 - Y_i) \frac{b_i c_i I_p}{(\lambda_i + \theta_i)(\theta_i + r)} [(\lambda_i + \theta_i) T - \frac{(1 - \theta_i - r)M}{T} + 1 - \theta_i - r - M(\lambda_i + \theta_i)] \right\} \quad (11)$$

$$\frac{\partial TP^2}{\partial T} = \sum_{i=1}^n \left[\frac{-p_i I_e (a_i - b_i p_i) M}{T^2} + \frac{(h_i + 2c_i \theta_i)(a_i - b_i p_i) t_{di}}{T^2 (\lambda_i + \theta_i)} + Y_i c_i \theta_i (a_i - b_i p_i) + \frac{Y_i c_i \theta_i I_p (a_i - b_i p_i) t_{di}^2}{T^2} (t_{di} - M) - (1 - Y_i) \frac{c_i I_p (a_i - b_i p_i)}{\theta_i + r} - (1 - Y_i) \frac{c_i I_p (a_i - b_i p_i) (1 - \theta_i - r) M}{T^2 (\lambda_i + \theta_i) (\theta_i + r)} \right] + \frac{1}{T^2} (S + \sum_{i=1}^n s_i) \quad (12)$$

Taking the second partial derivative of TP^2 with respect to p and T , and we have the Hessian matrix:

$$\Delta(H^2) = \left\{ \sum_{i=1}^n \left[\frac{b_i p_i I_e M}{T^2} - \frac{b_i t_{di} (h_i + 2c_i \theta_i)}{T^2 (\lambda_i + \theta_i)} - Y_i b_i c_i \theta_i - \frac{Y_i b_i c_i \theta_i I_p t_{di}^2}{T^2} (t_{di} - M) + (1 - Y_i) \frac{b_i c_i I_p}{\theta_i + r} + (1 - Y_i) \frac{b_i c_i I_p (1 - \theta_i - r) M}{T^2 (\lambda_i + \theta_i) (\theta_i + r)} \right]^2 - 4 \sum_{i=1}^n b_i \times \left\{ \sum_{i=1}^n \left[\frac{-p_i I_e (a_i - b_i p_i) M}{T^3} + \frac{(h_i + 2c_i \theta_i)(a_i - b_i p_i) t_{di}}{T^3} + \frac{Y_i \theta_i I_p (a_i - b_i p_i) t_{di}^2}{T^3} (t_{di} - M) - (1 - Y_i) \frac{c_i I_p (a_i - b_i p_i) (1 - \theta_i - r) M}{T^3 (\lambda_i + \theta_i) (\theta_i + r)} \right] + \frac{1}{T^3} (S + \sum_{i=1}^n s_i) \right\} \right\} \quad (13)$$

Theorem 3 if $M < T$, retailer always get the optimum values for any M .

Scenario 2 if $M > T$, there is an optimal solution of the profits.

Taking the partial derivative of TP^3 with respect to p and T , we have:

$$\frac{\partial TP^3}{\partial p} = \sum_{i=1}^n a_i - 2b_i p_i - b_i p_i I_e (T - M + 1) + b_i c_i (1 - rM)(1 - \theta_i t_{di}) + \frac{b_i h_i t_{di}}{\lambda_i + \theta_i} \left(\frac{1}{T} - 2(\lambda_i + \theta_i) \right) + \frac{b_i c_i \theta_i}{\lambda_i + \theta_i} \left[\frac{2t_{di}}{T} - (\lambda_i + \theta_i) T + (\lambda_i + \theta_i) t_{di} - 2 \right] \quad (14)$$

$$\frac{\partial TP^3}{\partial T} = \sum_{i=1}^n \left[p_i I_e (a_i - b_i p_i) + \frac{(h_i + 2c_i \theta_i)(a_i - b_i p_i) t_{di}}{T^2 (\lambda_i + \theta_i)} + c_i \theta_i (a_i - b_i p_i) \right] + \frac{1}{T^2} (S + \sum_{i=1}^n s_i) \quad (15)$$

Taking the second partial derivative of TP^3 with respect to p and T , and we have the Hessian matrix:

$$\Delta(H^3) = \left[\sum_{i=1}^n (b_i p_i I_e - \frac{b_i t_{di} (h_i + 2c_i \theta_i)}{T^2 (\lambda_i + \theta_i)} - b_i c_i \theta_i) \right]^2 - 2 \sum_{i=1}^n b_i \times \left[\sum_{i=1}^n \left[\frac{(2h_i + 4c_i \theta_i)(a_i - b_i p_i) t_{di}}{T^3 (\lambda_i + \theta_i)} \right] + \frac{2}{T^3} (S + \sum_{i=1}^n s_i) \right] \tag{16}$$

$$= \sum_{i=1}^n b_i \times \left\{ b_i \sum_{i=1}^n (p_i I_e - c_i \theta_i - \frac{b_i t_{di} (h_i + 2c_i \theta_i)}{T^2 (\lambda_i + \theta_i)}) \right\}^2 - 4 \sum_{i=1}^n \left[\frac{(h_i + 2c_i \theta_i)(a_i - b_i p_i) t_{di}}{T^3 (\lambda_i + \theta_i)} \right] - \frac{4}{T^3} (S + \sum_{i=1}^n s_i) \left\} \right.$$

Theorem 4 if $T^3 b_i \sum_{i=1}^n (p_i I_e - c_i \theta_i)^2 < 8(S + \sum_{i=1}^n s_i)$, or any M , the retailer can obtain the optimal value.

When $M < T$, let Eq.(11) and Eq.(12) equal to zero, we have:

$$p_i^{2*} = \frac{1}{2b_i} \left\{ a_i - \frac{b_i p_i I_e M}{T} + b_i c_i (1 - \theta_i t_{di})(1 - rM) + \frac{b_i h_i t_{di}}{\lambda_i + \theta_i} \left(\frac{1}{T} - 2(\lambda_i + \theta_i) \right) + \frac{b_i c_i \theta_i}{\lambda_i + \theta_i} \left[\frac{2t_{di}}{T} - (\lambda_i + \theta_i)T + (\lambda_i + \theta_i)t_{di} \right] + \right.$$

$$\left. Y_i b_i c_i I_p \left[(t_{di} - M)(1 - \theta_i t_{di} + \frac{\theta_i t_{di}^2}{T}) + 1 \right] + (1 - Y_i) \frac{b_i c_i I_p}{(\lambda_i + \theta_i)(\theta_i + r)} \left[(\lambda_i + \theta_i)T - \frac{(1 - \theta_i - r)M}{T} + 1 - \theta_i - r - M(\lambda_i + \theta_i) \right] \right\}$$

$$T^{2*} = \sqrt{\frac{S + \sum_{i=1}^n s_i + \sum_{i=1}^n (a_i - b_i p_i) \times \left[\frac{(h_i + 2c_i \theta_i) t_{di}}{\lambda_i + \theta_i} - p_i I_e M + Y_i c_i \theta_i I_p t_{di}^2 (t_{di} - M) - (1 - Y_i) \frac{c_i I_p (1 - \theta_i - r) M}{(\lambda_i + \theta_i)(\theta_i + r)} \right]}{\sum_{i=1}^n c_i I_p (1 - Y_i) (a_i - b_i p_i) / (\theta_i + r) - Y_i c_i \theta_i (a_i - b_i p_i)}}$$

According to dTP^2 / dM and dTP^3 / dM , we have:

$$M^{2*} = \begin{cases} T & \sum_{i=1}^n rc_i (1 - \theta_i t_{di}) < \sum_{i=1}^n p_i I_e \\ \bar{M} & \sum_{i=1}^n rc_i (1 - \theta_i t_{di}) > \sum_{i=1}^n p_i I_e \end{cases} \quad \text{and} \quad M^{3*} = \begin{cases} T & \sum_{i=1}^n rc_i (1 - \theta_i t_{di}) < \sum_{i=1}^n p_i I_e \\ \bar{M} & \sum_{i=1}^n rc_i (1 - \theta_i t_{di}) > \sum_{i=1}^n p_i I_e \end{cases}$$

And when $M \geq T$, let Eq.(14) and Eq.(15) equal to zero, we have:

$$p_i^{3*} = \frac{1}{2b_i} \left\{ a_i - b_i p_i I_e (T - M + 1) + b_i c_i (1 - rM)(1 - \theta_i t_{di}) + \frac{b_i h_i t_{di}}{\lambda_i + \theta_i} \left(\frac{1}{T} - 2(\lambda_i + \theta_i) \right) + \frac{b_i c_i \theta_i}{\lambda_i + \theta_i} \left[\frac{2t_{di}}{T} - (\lambda_i + \theta_i)T + (\lambda_i + \theta_i)t_{di} - 2 \right] \right\}$$

$$T^{3*} = \sqrt{\frac{-S - \sum_{i=1}^n s_i - \sum_{i=1}^n (h_i + 2c_i \theta_i)(a_i - b_i p_i) t_{di} / (\lambda_i + \theta_i)}{\sum_{i=1}^n (p_i I_e + c_i \theta_i)(a_i - b_i p_i)}}$$

Above all, the retailer's optimal value can be summarized as:

$$TPM^* = \text{Max} \begin{cases} TP^{2*}(p_i^*, T^{2*}, M^{2*}) & M < t_{di} < T \\ TP^{3*}(p_i^*, T^{3*}, M^{3*}) & t_{di} < T < M \end{cases} \tag{17}$$

If $TPM^* < TP^{1*}$, retailer should be ordering in the form of price discount;

If $TPM^* > TP^{1*}$, retailer should be ordering in the form of trade credit;

If $TPM^* = TP^{1*}$, if the type of retailer is limited funds, small and medium-sized enterprises, he should take the way of the trade credit to ensure that the flounce of internal cash flow.

4. Numerical examples

We solve the model using the Matlab2010. We chose two items, and the parameters as following:

Item 1: $S = 300$, $s_1 = 0$, $c = 20$, $h = 10$, $R = \sum_{i=1}^n e^{-Q} \rightarrow 0.58$, $D(p,t) = (600 - 2p)e^{-0.98t}$, $\theta = 0.08$, $r = 0.10$, $I_e = 0.12$, $I_p = 0.15$, $t_d = 1/12$.

Item 2: $s_2 = 10$, $c = 30$, $h = 12$, $R = \sum_{i=1}^n e^{-Q} \rightarrow 0.58$, $D(p,t) = (500 - p)e^{-0.98t}$, $\theta = 0.05$, $r = 0.10$, $I_e = 0.12$, $I_p = 0.15$, $t_d = 1/10$.

5.1 Numerical example analysing for price discount

According to parameters value, we can acquire the result of inventory ordering policies under price discount, as describe in table1.

Table1 Computational result under price discount

R	P_1^*	P_2^*	T^*	Q_1^*	Q_2^*	TP^{1*}
0	158.5759	262.5736	0.7474	210.14	176.677	93839.3
0.1	157.6166	261.1643	0.7724	218.656	183.683	95147
0.2	156.6585	259.7567	0.8027	228.732	191.988	96467.3
0.3	155.7017	258.3511	0.8399	240.932	202.063	97801.1
0.4	154.7466	256.9482	0.8871	256.143	214.647	99149.7
0.45	154.2699	256.248	0.9158	265.308	222.239	99830.1
0.5	153.7937	255.5488	0.9492	275.861	230.988	100515
0.55	153.3184	254.8507	0.9884	288.188	241.215	101204.8
0.6	152.8438	254.154	1.0353	302.842	253.381	101899.8

According to table 1, the price of the two non-instantaneous deterioration items decrease with the price discount rate; the ordering quantity of the two non-instantaneous deterioration items increase with the price discount rate. Under the coordination order cycle, due to different deterioration rate and deterioration time will lead to different retailer's strategy of different items order quantity and price. It means if the deterioration time is shorter, the retailer will order more items and the price will be lower. The profit and order period are increasing with the discount rate.

5.2 Numerical example analysing for trade credit

Scenario 1: when the trade credit in the interval of $[0, T]$, the result can be seen in Fig.4. According to theorem 4, if $M < T$, the optimal ordering period is $T^{2*}=0.373$. According to Fig.4, the profit is increasing with trade credit, and the optimal profit is $TP^{2*}=107066.7$, when $M^{2*}=T$. So the two kinds items' inventory ordering parameters are $p_1^*=155.15$, $p_2^*=260.39$, $Q_1^*=107.48$, $Q_2^*=89.04$.

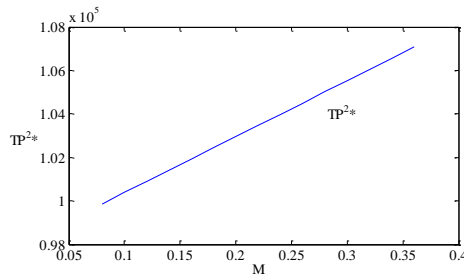


Fig.4 when $M < T$, the trend of profit

Scenario 2: when the trade credit in the interval of $[T, +\infty]$, the result can be seen in table 2 and Fig.5.

Table2 when $M > T$, the result of retailer under trade credit

M	P_1^*	P_2^*	T^*	Q_1^*	Q_2^*	TP^{3*}
0.25	149.1053	246.8186	0.2495	74.972	62.9878	107244.2
0.252	149.1203	246.8438	0.2495	74.9578	62.9759	107222.1
0.254	149.1353	246.869	0.2495	74.9436	62.964	107200.1
0.256	149.1504	246.8942	0.2495	74.9295	62.9521	107178.1
0.258	149.1654	246.9194	0.2494	74.9153	62.9401	107156
0.26	149.1804	246.9446	0.2494	74.9011	62.9282	107133.9
0.261	149.1879	246.9572	0.2494	74.894	62.9223	107122.9
0.262	149.1954	246.9698	0.2494	74.8869	62.9163	107112

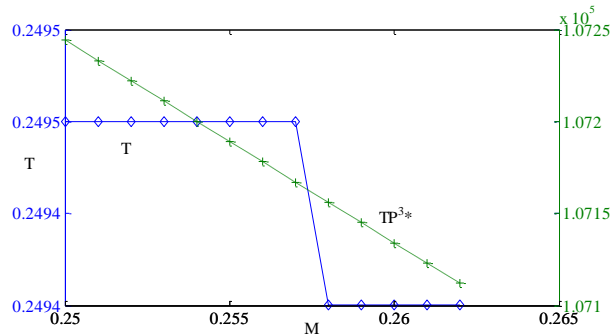


Fig.5 when $M > T$, the trend of profit and ordering period

According to table 2, if $M > T$, the ordering quantity and price is stable of the two kinds non-instantaneous deterioration items. When $M \leq 0.256$, we get $T^* = 0.295$; when $M \geq 0.258$, we get $T^* = 0.294$. From Fig.5, the profit is decreasing with trade credit. So retailer will obtain the optimal value at the point of $M = T$. And the profit value is $TP^{3*} = 107244.2$.

Table3 cooperation of different circumstance

	R^*	M^*	p_1^*	p_2^*	Q_1^*	Q_2^*	T^*	TP^1	TPM^*	
									TP^{2*}	TP^{3*}
Price discount	0.6	-	152.84	254.15	302.84	253.3	1.035	101899.8	-	-
Trade credit	-	0.25	149.11	246.82	74.97	62.99	0.25	-	107066.7	107244.2

From table 3, it is easy to find that $TP^1 < TPM$. So when $t_d < T$, the profit of retailer under trade credit is better than that of price discount. And in the trade credit, the profit under $M < T$ is better than under $M > T$. This is because of the former has bigger ordering quantity. Also, when $0.25 < T < 0.36$, it is better for retailer to order less. Fig.6 describes the change of profit, and shows that the optimal value are $M^* = 0.25$, $p_1^* = 149.11$, $p_2^* = 246.82$, $Q_1^* = 74.97$, $Q_2^* = 62.99$, $T^* = 0.25$, $TP = TPM = 107244.2$.

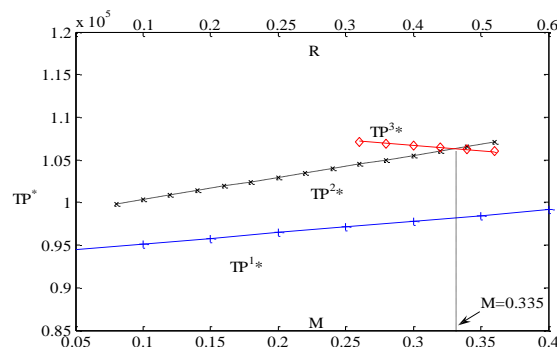


Fig.6 the trend of profit under different circumstance

Conclusions

In this paper, we build up an inventory ordering model for non-instantaneously deteriorating items based on trade credit and price discount. First, established inventory ordering model under different situations by maximize retailer total revenue; then through the numerical calculation to prove the existence of the optimal solution, and obtain the optimal solution; lastly, through numerical calculate out: the retailer's profit increases with the price discount rate or the trade credit ($M < T$); the retailer's profit decreases with the trade credit ($M > T$); and the retailer's profit under trade credit is better than price discount, when multiple non-instantaneously deteriorating items are ordered coordination. The retailer will be obtain his optimal profit, when $M = 0.25$. What's more, this work can be extended in several ways: for instance, discussing the circumstances of $t_d > T$, uncertain demands could be considered by allowing the demand function to be driven by probability, since uncertain issues are very important on policy or decision making (Sun et al., 2018). In addition, a constant or random ordering lead time could be considered.

ACKNOWLEDGEMENTS

This work was supported by the National Social Science Foundation of China [17CGL019] and the Humanities and Social Sciences Program of Shandong Universities [J17RB094].

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