Optimum Design of a Variable Speed Wind Turbine with a Permanent Magnet Synchronous Generator

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Abstract:

The main motive of this work is to analyze a typical configuration of a Wind Turbine Generator System [WTGS] equipped with variable speed generator. Now-a-days doubly fed Induction generators (DfIGs) are being widely used in WTGS. It is based on an Induction generator with multi phase wound rotor & multi phase slip ring assembled with brushes for access to the rotor windings. So, facing drawbacks such as decrease in efficiency, cost and size. If electromagnetic synchronous generator is used which has rotor current, losses increases there by efficiency decreases. So, In order to overcome this drawback, we adopt permanent magnet synchronous generator in which the rotor current is zero which is very beneficial. And in this work, “Direct drive” technique is adopted where gearbox is not present which in turn reduces the weight of nacelle and reduction of cost. Apart from the generator, the analyzed WTGS consists of another three parts: wind speed, wind turbine and drive train. These elements have been designed and equations that explain their behavior have presented in and the total system is implemented in MATLAB/SIMULINK interface.

Key words: PMSG, WTGS, Gear-box, Direct-drive

I. INTRODUCTION

The wind turbine first came into being as a horizontal axis windmill for mechanical power generation, used since 1000 AD in Persia, Tibet and China. Transfer of mechanical windmill technology from the Middle East to Europe took place between AD 1100s and 1300s, followed by further development of the technology from the Middle East to Europe between AD 1300 and 1500s, and finally in the 16th century. The advent of DC electric power in 1882, and the advent of AC electric power in 1882, provided a technological basis for constructing wind turbines that generated electricity. The Danish physicist and engineer Poulsen in 1882, and the Danish mathematician and engineer Poulsen in 1882, provided a technological basis for constructing wind turbines that generated electricity. The Danish scientist and engineer Poul La Cour is the most widely known pioneer of electricity generation using wind power. In 1891 in Askov, Denmark he introduced a four shuttle sail rotor design generating approximately 10kW of DC electric power. He also applied the DC current for water electrolysis, and utilized the hydrogen gas for gas lamps to light up the local school grounds. La Cour’s efforts started research, development and commercialization of wind electricity in Europe and thus Europe gained its leadership role in wind energy electricity generation.

In order to raise the voltage, a transformer is placed between the inverter and the Point of Common Connection (PCC) by avoiding losses in the transport of the current. The layout of the electrical part is depicted in Fig. 1.
The present work considers a constant wind speed equal of the wind speed in Simulink implies the consideration of the base wind speed component, as shown in Fig. 3.

Fig. 3. Wind Speed model with simulink

It must be noted that this study is dedicated to analyze and implement the model from the wind turbine to the PMSG. For this reason, transformer, grid, rectifier and inverter models (and their controls) will not be considered.

III. Sub system models

A. Wind Speed Model

In order to study and simulate the spatial effect of wind behavior, including gusting, rapid (ramp) changes, and background noise, a model is required. The wind speed is modeled as the sum of the four components listed above.

\[ v_{\text{tot}}(t) = v_{\text{base}}(t) + v_{\text{ramp}}(t) + v_{\text{gust}}(t) + v_{\text{noise}}(t) \]  

where \( v_{\text{base}} \) = base (constant) wind component in \( \text{m/s} \),

\( v_{\text{ramp}} \) = ramp wind component in \( \text{m/s} \),

\( v_{\text{gust}} \) = gust wind component in \( \text{m/s} \),

\( v_{\text{noise}} \) = base noise wind component in \( \text{m/s} \).

Fig. 2. shows the waveforms of the non-constant wind speed components: (a) ramp (b) gust and (c) noise components.

(a) Ramp  
(b) Gust  
(c) Random noise

Fig. 2. Non-constant wind speed components.

The present work considers a constant wind speed equal to 20 m/s. Consequently, the model implementation of the wind speed in Simulink implies the consideration of the base wind speed component, as shown in Fig. 3.

Fig. 3. Wind Speed model with simulink

B. Wind Turbine Model

The rotor aerodynamic statistics are presented by the well-known static relations

\[ P_{\text{ww}} = \frac{1}{2} \rho A v_{\text{ww}}^3 \]  

Where \( P_{\text{ww}} \) = power extracted from the wind [W],

\( \rho \) = air density, which is equal to 1.225 kg/m\(^3\) at sea level at temperature

\( T=288 \text{ K} \),

\( C_p \) = Power coefficient,

\( v_{\text{ww}} \) = the wind speed upstream of the rotor [m/s]

\( A \) = area swept by the rotor [m\(^2\)] \( A=\pi r^2 \), being \( R \) the radius of the blade [m].

The amount of aerodynamic torque \( (\tau_{\text{ww}}) \) in N·m is given by the ratio between the power extracted from the wind \( (P_{\text{ww}}) \), in W, and the turbine rotor speed \( (\omega_{\text{w}}) \), in rad/s, as

\[ \tau_{\text{ww}} = \frac{P_{\text{ww}}}{\omega_{\text{w}}} \]  

The key point noted here is the mechanical torque transmitted to the generator \( (\tau_{\text{w-wg}}) \) is same as that of aerodynamic torque, since there is no gearbox i.e., gearbox ratio is \( n_b=1 \). Therefore \( \tau_{\text{ww}} = \tau_{\text{w-wg}} \).

The power coefficient \( c_p \) reaches a maximum value equal to \( c_p=0.593 \) which means that the power extracted from the wind is always less than 59.3% which is known as Betz’s limit, because various aerodynamic losses depend on the rotor construction (number and shape of blades, weight, stiffness, etc.). This is the well-known low efficiency to produce electricity from the wind.

The power coefficient can be utilized in the form of look-up tables or in form of a function. In the present work, the second approach has taken, where the general function defining the power coefficient as a function of the tip-speed ratio and the blade pitch angle is defined as

\[ c_p(\lambda, \beta) = c_1 (c_2 \frac{1}{\beta} - c_3 - c_4 \beta - c_5 \beta^2) e^{-c_6 x} \]  

As the above function depends on the type of wind turbine rotor, the coefficients \( c_1 \) to \( c_6 \) and \( x \) can be different for various turbines. The proposed coefficients are as follows: \( c_1=0.4 \), \( c_2=115 \), \( c_3=0.4 \), \( c_4=0 \), \( c_5=5 \), \( c_6=20 \). Since \( c_4=0 \), \( x \) is not used here. Additionally, the parameter \( \beta \) is also defined in different ways. For example, the parameter \( 1/\beta \) is defined as

\[ \frac{1}{\beta} = \frac{1}{4+0.899 x} - \frac{0.035}{19 x} \]  

where \( \beta \) is the pitch angle which is the angle between the ["\], plane of rotation and the blade cross-section chord (Fig. 4), and the tip-speed ratio \( \lambda \) is defined as

\[ \lambda = \frac{\omega_{\text{w}} R}{v_{\text{ww}}} \]  

where \( \omega_{\text{w}} \) is the angular velocity of rotor [rad/s], \( R \) is the rotor radius [m] and, \( v_{\text{ww}} \) is the wind speed upstream of the rotor [m/s].
The $C_p = C_p(\lambda, \theta)$ characteristics computed taking into account (12) and (13), the above parameters $c_1$-$c_6$ and the wind turbine parameters (table I), for various blade pitch angles, are presented in Fig 5.

![Fig. 4. Blade pitch angle $\theta$.](image)

The model of the wind turbine implemented in Simulink is shown in Fig. 6.

![Fig. 6. Wind Turbine modeled with Simulink.](image)

Table I shows the parameters of the analyzed wind turbine.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air density</td>
<td>$\rho$</td>
<td>1.205kg/m$^3$</td>
</tr>
<tr>
<td>Rotor radius</td>
<td>$R$</td>
<td>38 m</td>
</tr>
<tr>
<td>Rated wind speed</td>
<td>$V_w$ rated</td>
<td>11.8 m/s</td>
</tr>
<tr>
<td>Maximum $C_p$</td>
<td>$C_{p_{max}}$</td>
<td>0.4412</td>
</tr>
</tbody>
</table>

C. Drive Train Model

The drive train of a wind turbine generator system consists of a blade-pitching mechanism with a spinner, a hub with blades, a rotor shaft and a gearbox with breaker and generator. It must be noted that gearbox is not considered because the analyzed system consists of a wind turbine equipped with a multi-pole PMSG. The acceptable (and common) way to model the drive train is to treat the system as a number of discrete masses connected together by springs defined by damping and stiffness coefficients as shown in Fig 7. Therefore, the equation of $i$th mass motion can be described as follows:

$$
\frac{d^2 \theta_i}{dt^2} = \frac{v_i c_i}{J_i} \frac{d\theta_i}{dt} + \frac{v_i^2 c_i}{J_i} \theta_i + \frac{v_i^2 c_{i+1}}{J_i} \frac{d\theta_{i+1}}{dt} + \frac{v_i^2 k_i}{J_i} \theta_i + \frac{v_i^2 k_{i+1}}{J_i} \frac{d\theta_{i+1}}{dt} + \frac{\tau_i}{J_i} - D_i \frac{d\theta_i}{dt}
$$

where $v_i$ is the transmission rate between $i$ and $i$-masses, $c_i$ is the shaft viscosity [kg/(ms)], $k_i$ is the shaft elastic constant [N/m], $J_i$ is the moment of inertia of the $i$th mass [kgm$^2$], $\tau_i$ is the external torque [Nm] applied to the $i$th mass and $D_i$ is the damping coefficient [Nm/s], which represents various damping effects. For the purposes of the present research, neither viscosity nor damping effects have been considered.

![Fig. 7. Transmission model of $N$ masses connected together.](image)

When the complexity of the study varies, the complexity of the drive train differs. For example, when the problems such as torsional fatigue are studied, dynamics from all parts have to be considered. For these purposes, two-lumped mass or more sophisticated models are required. However, when the study focuses on the interaction between wind farms and AC grids, the drive train can be treated as one-lumped mass model for the sake of time efficiency and acceptable precision.

The last approximation has been considered in the present study and it is defined by the following equation...
\[
\frac{d\omega_g}{dt} = \frac{\tau_e - \tau_{w,g}}{J_{eq}} - B_m \cdot \omega_g
\]  

where the sub-index \(g\) represents the parameters of the generator side, \(\omega_g\) is the mechanical angular speed [rad/s] of the generator, \(B_m\) is the damping coefficient [N-m/s], \(\tau_e\) is the electromechanical torque [N-m], \(\tau_{w,g}\) is the aerodynamic torque that has been transferred to the generator side (3), which is equal to the torque produced in the rotor side because there is no gearbox, and \(J_{eq}\) is the equivalent rotational inertia of the generator [kg.m\(^2\)], which is derived from

\[
J_{eq} = J_g + \frac{J_w}{n_g^2}
\]

where \(J_g\) and \(J_w\) are the generator and the rotor rotational inertias [kg.m\(^2\)] respectively, \(n_g\) is the gear ratio, which is equal to 1, because no gearbox is utilized.

The model of the one mass drive train implemented in Simulink is depicted in Fig. 8.

![Fig. 8. Drive Train modeled with Simulink.](image)

Table II shows the parameters of the drive train that has been considered.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value and Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gear ratio</td>
<td>(n_g)</td>
<td>1</td>
</tr>
<tr>
<td>Rotational damping coefficient</td>
<td>(B_m)</td>
<td>0</td>
</tr>
<tr>
<td>Equivalent inertia (turbo-generator)</td>
<td>(J_{eq})</td>
<td>0.3 kg.m(^2)</td>
</tr>
</tbody>
</table>

**D. Generator Model**

The PMSG has been considered as a system which makes possible to produce electricity from the mechanical energy obtained from the wind.

The dynamic model of the PMSG is derived from the two-phase synchronous reference frame, which the q-axis is 90° ahead of the d-axis with respect to the direction of rotation. The synchronization between the d-q rotating reference frame and the abc-three phase frame is maintained by utilizing a phase locked loop (PLL) [10]. Fig. 9 shows the d-q reference frame used in a salient-pole synchronous machine (which is the same reference as the one used in a PMSG), where \(\theta\) is the mechanical angle, which is the angle between the rotor d-axis and the respective axis of the machine.

![Fig. 9. d-q and \(\alpha\)-\(\beta\) axis of a typical salient-pole synchronous machine.](image)

The mathematical model of the PMSG for power system and converter system analysis is usually based on the following assumptions: the stator windings are positioned sinusoidal along the air-gap as far as the mutual effect with the rotor is concerned; the stator slots cause no appreciable variations of the rotor inductances with rotor position; magnetic hysteresis and saturation effects are negligible; the stator winding is symmetrical; damping windings are not considered; the capacitance of all the windings can be neglected and the resistances are constant (this means that power losses are considered constant).

The mathematical model of the PMSG in the synchronous reference frame (in the state equation form) is given by

\[
\frac{di_d}{dt} = \frac{1}{L_{ds} + L_{hs}}(-R_s i_d + \omega_e (L_{qs} + L_{qs}) i_q + u_d)
\]

\[
\frac{di_q}{dt} = \frac{1}{L_{qs} + L_{hs}}(-R_s i_q - \omega_e (L_{ds} + L_{qs}) i_d + \psi_f) + u_q
\]

Where subscripts \(d\) and \(q\) refer to the physical quantities that have been transformed into the d-q synchronous rotating reference frame, \(R_s\) is the stator resistance [Ω], \(L_{ds}\) and \(L_{qs}\) are the inductances [H] of the generator on the d and q axis, \(L_{dsd}\) and \(L_{dqs}\) are the leakage inductances [H] of the generator on the d and q axis, respectively, \(\psi_f\) is the permanent magnetic flux [Wb] and \(\omega_e\) is the electrical rotating speed [rad/s] of the generator, defined by

\[
\omega_e = p \cdot \omega_g
\]

Where \(p\) is the number of pole pairs of the generator.

In order to complete the mathematical model of the PMSG the mechanical equation is needed, and it is described by the following electromagnetic torque equation [10]

\[
\tau_e = 1.5 p ((L_{ds} - L_{ds}) i_d i_q + i_q \psi_f)
\]
Fig. 10 shows the equivalent circuit of the PMSG in the d-q synchronous rotating reference frame.

![Equivalent circuit of the PMSG](image)

The model of the PMSG implemented in simulink is depicted in Fig. 11.

![PMSG modeled with simulink](image)

By analyzing the power produced by the wind turbine at various wind and rotor speeds, as depicted in Fig. 12, it can be appreciated that an optimum power coefficient constant $k_{p \text{ opt}}$ exists. This coefficient shows the generated power associated with the corresponding optimum rotor speed. $k_{p \text{ opt}}$ is calculated from individual wind turbine characteristics. By measuring generated power, the corresponding optimum rotor speed can be calculated and set as the reference speed according to

$$\omega_{r \text{ opt}} = \sqrt{\frac{P_{\text{gen}}}{k_{p \text{ opt}}}} \tag{13}$$

where $\omega_{r \text{ opt}}$ is the optimum rotor speed [rad/s] and $P_{\text{gen}}$ is the measured generated power [W].

This is the base of the well-known Maximum Power Point Tracking (MPPT) from the prior treatment of the wind turbine model. It can be appreciated that in order to extract the maximum amount of power from the incident wind, $C_p$ should be maintained at a maximum. In order to achieve this objective, it can be appreciated from Fig. 5 that the speed of the generator rotor must be optimized according to instantaneous wind speed (this optimization is achieved by using (13)).

![Power vs. speed curves](image)

The model of a variable speed wind turbine with a permanent magnet synchronous generator has been treated. The model has been implemented in MATLAB/Simulink in order to validate it. $C_p$ curves and power-speed characteristics have been obtained.

The generator has been modeled in the d-q synchronous rotating reference frame, taking into account different simplifications. Moreover, the concept of the maximum power point tracking has been presented in terms of the adjustment of the generator rotor speed according to instantaneous wind speed.

### IV. CONCLUSION

The modeling of a variable speed wind turbine with a permanent magnet synchronous generator has been treated. The model has been implemented in MATLAB/Simulink in order to validate it. $C_p$ curves and power-speed characteristics have been obtained.

The generator has been modeled in the d-q synchronous rotating reference frame, taking into account different simplifications. Moreover, the concept of the maximum power point tracking has been presented in terms of the adjustment of the generator rotor speed according to instantaneous wind speed.
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