

Optimization of Shaking Force by Discretization of Links Mass of A Planar Four Bar Mechanism

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Abstract

The reduction of the shaking force by redistributing the each link mass of 4-bar mechanism in such a way that the sum of distributed masses are equal to the total mass of each link. The shaking force is minimized by two schemes one is by varying the co-ordinates of discretized mass concentrated points and other is redistributing the discretized mass magnitude keeping the coordinate's constraint by using an optimization technique is Nelder's simplex method.

1. Introduction

The minimizing of shaking force was done by discretizing the each movable link of a 4-bar mechanism in some parts. However, instead of adding the counter weight for minimizing of shaking force, we can modify the shape of links by optimizing the mass concentrated coordinates of discretized mass of linkages. For optimizing co-ordinates an optimization technique "Nelder Simplex Method" is used. The main contribution of the present paper is the proof for planar mechanisms. Counterweight balancing can be reformulated as a convex optimization problem. However, instead of assuming a particular counterweight shape. The counterweight balancing problem is formulated as shaking force optimization problem by discretization.

The Nelder's simplex search method has been known one of the top ten algorithms of the century [1, 2]. The first simplex algorithm has been introduced by [3] as local search method by introducing a gradient activity on a function of problem to reveal the potential solution route [4]. The Nelder simplex method is simple to understand and fast to converge an optimization problem. However, the Nelder algorithm is sensitive to initial value [5]. For example in function optimization problem, different initialization produces different solution. In order to avoid this circumstance, there are two possible ways to initialize these values. First, a very careful initialization selection and second using random generated initialization.

Balancing of shaking force in high speed mechanisms/machines reduces the forces transmitted to the frame, which minimizes the noise and wear and improves the performance of a mechanism. The balancing of shaking force has been studied by various researchers [6–20], and others. A considerable amount of research on balancing of shaking force and shaking moment in planar mechanisms has been carried out in the past [6–20]. In contrast to rapid progress in balancing theory and techniques for planar mechanisms, the understanding of shaking force and shaking

moment balancing of spatial mechanisms is very limited. Kaufman and Sandor [12] presented a complete force balancing of spatial mechanisms like (revolute– spherical– spherical–revolute) RSSR and (revolute–spherical–spherical–prismatic) RSSP. Their approaches are based on the generalization of the planar balancing theory developed by Berkof and Lowen [10], a technique of linearly independent vectors. Using the real vectors and the concept of retaining the stationary centre of total mass, Bagci has obtained the design equations for force balancing of various mechanisms

2. Optimization

2.1 Nelder simplex method:

The basic idea in the simplex method is to compare the values of the objective function at $n+1$ vertices of a general simplex and move this simplex gradually towards the optimum point during the iterative process .the moment of the simplex is achieved by using three operations know as reflection, contraction and expansion.

2.1.1 Reflection

If X_h is the vertex corresponding to the highest value of the objective function among vertices of a simplex, we can expect the point X_r obtained by reflecting the point X_h in the opposite face to have the simple value .if this is the case we can construct a new simplex by rejecting X_h from simplex and including new point X_r :Replection point is given by $X_r = (1 + \alpha)X_o - \alpha X_h$

Where X_h is the vertex corresponding to the maximum function value

$$f(X_h) = \max_{i=1 \text{ to } n+1} f(X_i)$$

X_o is the centroid of all the points X_i except $i=h$ and is given by,

$$X_o = \frac{1}{n} \sum_{\substack{i=1 \\ i \neq h}}^{n+1} X_i$$

And $\alpha > 0$ is the reflection coefficient defined as,

$$\alpha = \frac{\text{distance between } X_r \text{ and } X_o}{\text{distance between } X_h \text{ and } X_o}$$

Thus X_r lie on the line joining X_h and X_o on the far side of X_o

If $f(X_r)$ lies between $f(X_h)$ and $f(X_l)$ where X_l is the vertex corresponding to the minimum function value,

$$f(X_l) = \min_{i=1 \text{ to } n+1} f(X_i)$$

X_h is replaced by X_r and a new simplex is started

2.1.2 Expansion

If a reflection process gives a point X_r for which $f(X_r) < f(X_l)$, if the reflection produces a new minimum ,one can generally expect to decrease the function value further by moving along the direction X_o and X_r .hence we expand X_r to X_e by the relation

$$X_e = \gamma X_r + (1 - \gamma) X_o$$

Where γ is called the expansion coefficient define as,

$$\gamma = \left(\frac{\text{distance between } X_e \text{ and } X_o}{\text{distance between } X_r \text{ and } X_o} \right) > 1$$

if $f(X_e) < f(X_i)$, we replace the point X_h by X_e and restate the process of reflection. on the other hand, if $f(X_r) > f(X_i)$, it means that expansion process is not successful and hence we replace the point X_h by X_r , and start the reflection process again.

2.1.3 Contraction

If the reflection process gives a point X_r for which $f(X_r) > f(X_i)$ for all i excepting $i=h$, and $f(X_r) < f(X_h)$, then we replace the point X_h by X_r . Thus the new X_h will be X_r . In this case, we contract the simplex method as follows:

$$X_c = \beta X_h + (1 - \beta) X_o$$

Where β is called the contraction coefficient ($0 \leq \beta \leq 1$), and is defined as,

$$\beta = \frac{\text{distance between } X_c \text{ and } X_o}{\text{distance between } X_h \text{ and } X_o}$$

if $f(X_r) > f(X_h)$ we still use the x_c with out changing the previous point X_h . if the contraction process produce a point X_c for which $f(X_c) < \min[f(X_h), f(X_r)]$, replace the point X_h in X_1, X_2, \dots, X_{n+1} by X_c and proceed with the reflection process again. on the other hand, if $f(X_c) \geq \min[f(X_h), f(X_r)]$, the contraction process will be a failure and this case, we replace all X_i by $(X_i + X_o)/2$, and restart the reflection process.

3 Problem formulation

3.1 Formulation of the Problem for Planar Mechanism

$$F = \sqrt{(F_x^2 + F_y^2)}$$

$$F_x = -m_1 \ddot{x}_1 - m_2 \ddot{x}_2 - m_3 \ddot{x}_3$$

$$F_y = -m_1 \ddot{y}_1 - m_2 \ddot{y}_2 - m_3 \ddot{y}_3$$

$$\text{Links angles } \phi_3 = 2 \tan^{-1} \left(\frac{A + \sqrt{A^2 + B^2 + C^2}}{B + C} \right)$$

$$A = \sin \phi_1$$

$$B = \cos \phi_1 - \frac{v}{\lambda}$$

$$C = \frac{\lambda^2 + \mu^2 + v^2 - 1}{2\mu\lambda} - \frac{v}{\mu} \cos \phi_1$$

$$\lambda = \frac{a_1}{a_2}$$

$$\mu = \frac{a_3}{a_2}$$

$$v = \frac{a_4}{a_2}$$

$$\phi_2 = \tan^{-1} \left(\frac{\lambda \sin \phi_1 - \mu \sin \phi_3}{\lambda \cos \phi_1 - \mu \cos \phi_3 - v} \right)$$

$$\dot{\phi}_2 = \frac{\lambda}{\tau_3} \sin(\phi_1 - \phi_3) \dot{\phi}_1$$

$$\dot{\phi}_3 = \frac{\tau_1 \lambda}{\tau_3 \mu} \dot{\phi}_1$$

Where

$$\tau_1 = \mu \sin(\phi_1 - \phi_2) + \nu \sin \phi_1$$

$$\tau_3 = \lambda \sin(\phi_1 - \phi_2) + \nu \sin \phi_3$$

Link angular accelerations

$$\ddot{\phi}_2 = \left(\frac{\dot{\phi}_2}{\dot{\phi}_1} \ddot{\phi}_1 \right) + \frac{\nu \lambda}{\tau_3^2} [\cos(\phi_1 - \phi_3) \sin \phi_3 \dot{\phi}_1 - \sin \phi_1 \dot{\phi}_3] \dot{\phi}_1$$

$$\ddot{\phi}_3 = \left(\frac{\dot{\phi}_3}{\dot{\phi}_1} \ddot{\phi}_1 \right) + \frac{\lambda}{\tau_3} \left[\cos(\phi_1 - \phi_3) (\dot{\phi}_1 - \dot{\phi}_3)^2 + \frac{\nu}{\mu \tau_3} (\lambda \cos \phi_1 \dot{\phi}_1^2 - \mu \cos \phi_3 \dot{\phi}_3^2) \right]$$

Accelerations of center of mass

$$\ddot{x}_1 = -\dot{\phi}_1^2 (p_1 \cos \phi_1 - q_1 \sin \phi_1) - \ddot{\phi}_1 (p_1 \sin \phi_1 + q_1 \cos \phi_1)$$

$$\ddot{y}_1 = -\dot{\phi}_1^2 (p_1 \sin \phi_1 - q_1 \cos \phi_1) - \ddot{\phi}_1 (p_1 \cos \phi_1 + q_1 \sin \phi_1)$$

$$\ddot{x}_2 = -a_1 \sin \phi_1 \ddot{\phi}_1 - a_1 \dot{\phi}_1^2 \cos \phi_1 - \ddot{\phi}_2 (p_2 \sin \phi_2 + q_2 \cos \phi_2) - \dot{\phi}_2^2 (p_2 \cos \phi_2 - q_2 \sin \phi_2)$$

$$\ddot{y}_2 = a_1 \cos \phi_1 \ddot{\phi}_1 - a_1 \dot{\phi}_1^2 \sin \phi_1 + \ddot{\phi}_2 (p_2 \cos \phi_2 - q_2 \sin \phi_2) - \dot{\phi}_2^2 (p_2 \sin \phi_2 + q_2 \cos \phi_2)$$

$$\ddot{x}_3 = -\dot{\phi}_3^2 (p_3 \sin \phi_3 + q_3 \cos \phi_3) - \ddot{\phi}_3 (p_3 \cos \phi_3 - q_3 \sin \phi_3)$$

$$\ddot{y}_3 = \dot{\phi}_3^2 (p_3 \cos \phi_3 - q_3 \sin \phi_3) - \ddot{\phi}_3 (p_3 \sin \phi_3 + q_3 \cos \phi_3)$$

3.2 Formulation of the shaking force reducing Problem for Planar Mechanisms after discretization

When the links mass is discretized the expression for the shaking force is given by

$$F = \sqrt{(F_x^2 + F_y^2)}$$

$$F_x = -\sum_{i=1}^n \delta m_{1i} \ddot{x}_{1i} - \sum_{i=1}^n \delta m_{2i} \ddot{x}_{2i} - \sum_{i=1}^n \delta m_{3i} \ddot{x}_{3i}$$

$$F_y = -\sum_{i=1}^n \delta m_{1i} \ddot{y}_{1i} - \sum_{i=1}^n \delta m_{2i} \ddot{y}_{2i} - \sum_{i=1}^n \delta m_{3i} \ddot{y}_{3i}$$

$$\sum_{i=1}^n \delta m_{1i} = m_1$$

$$\sum_{i=1}^n \delta m_{2i} = m_2$$

$$\sum_{i=1}^n \delta m_{3i} = m_3$$

$$\sum_{i=1}^n \ddot{X}_{1i} = \ddot{X}_1$$

$$\sum_{i=1}^n \ddot{X}_{1i} = \ddot{X}_2$$

$$\sum_{i=1}^n \ddot{X}_{1i} = \ddot{X}_3$$

$$\sum_{i=1}^n \ddot{Y}_{1i} = \ddot{Y}_1$$

$$\sum_{i=1}^n \ddot{Y}_{1i} = \ddot{Y}_2$$

$$\sum_{i=1}^n \ddot{Y}_{1i} = \ddot{Y}_3$$

3.3 Optimum variables

Each moving link of the mechanism, i.e. links $i = (1,2,3)$ is discretized. The general choice of mass parameters for a planar mechanism are its mass m_i , its center of gravity (COG) position (p,q) with respect to the local coordinate system of link i to mass concentrating point.

$$\text{Mass constrain } \sum_{i=1}^n \delta m_{ii} = m_1$$

3.4 Objective function

Instead of minimizing a weighted combination of the three balancing effect indices, the balancing trade-off is controlled based on the following approach.

Minimize F

Subjected to $\sigma \leq \sigma_y$

The advantage of this approach is that the shaking force is minimized while the designer directly controls, through the designer-specified upper bounds the maximum allowed increase (> 1), or the minimum wanted reduction (< 1) of the shaking force

4 Numerical results

The origin and use of the force will now be explained by means of an example, using a mechanism with the following dimensions

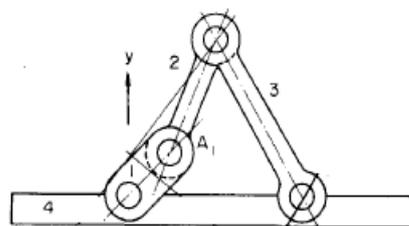


Figure 1 slanted 4-bar configuration

Link lengths is given as

$$a_1 = 50.8 \text{ mm}$$

$$a_2 = 101.6 \text{ mm}$$

$$a_3 = 152.4 \text{ mm}$$

$$a_4 = 152.4 \text{ mm}$$

Angular acceleration is given as $\dot{\phi}_1 = 100 \text{ rad/sec}$

4.1 Results For Scheme-I

In scheme-I the magnitude of the discrete mass is fixed and varying the mass concentrated points i.e. coordinates of the discrete mass. Finding the set of coordinates which gives optimal forces

| Φ (degree) | Force (N) |
|-----------------|-----------|
| 0 | 108.4387 |
| 71.74577 | 108.1865 |
| 107.9968 | 109.8092 |
| 144.0359 | 108.8155 |
| 179.9947 | 106.7311 |
| 215.9478 | 100.2193 |
| 251.9926 | 105.5625 |
| 287.9686 | 62.0558 |
| 323.9503 | 133.3234 |
| 359.9894 | 108.4418 |

Table 1 Shaking Forces For Scheme- I

| p11 | p12 | p13 | q11 | q12 | q13 |
|---------|----------|----------|---------|----------|---------|
| 0.19558 | 11.59256 | 1.15824 | 0.03048 | -0.58928 | 0.10668 |
| 0.57658 | 1.15824 | 2.31394 | 0.30988 | 0.39116 | 0.03048 |
| 0.96266 | 1.9304 | 3.08356 | 0.1651 | -2.81178 | 0.10922 |
| 1.34874 | 2.70002 | 4.24434 | 0.69342 | 1.46558 | 0.30988 |
| 1.73736 | 3.47218 | 5.01396 | 0.10668 | -0.52832 | 0.1651 |
| 2.1209 | 4.24434 | 6.16966 | 0.14224 | -0.35814 | 0.17526 |
| 2.50444 | 5.01396 | 6.94182 | 0.03048 | -0.04826 | 0.1651 |
| 2.89052 | 5.78104 | 8.09752 | 0.17526 | 0.3556 | 0.69342 |
| 3.27914 | 6.55574 | 8.87222 | 0.1651 | -1.00584 | 0.03048 |
| 3.47218 | 6.94182 | 10.02792 | 0.10922 | -0.44958 | 0.14224 |

Table 2 New Co-Ordinates of Mass Concentration for optimum force

4.2 Results for Scheme –II

In scheme-II mass concentrated points i.e. coordinates of the discrete mass is fixed and varying the magnitude of the discrete mass. Finding the set of magnitude of mass which gives optimal forces.

| input angle \emptyset (degree) | Shaking force F (N) |
|-------------------------------------|------------------------|
| 0 | 86.6824 |
| 71.74577 | 91.7267 |
| 107.9968 | 90.2019 |
| 144.0359 | 86.7403 |
| 179.9947 | 79.8967 |
| 215.9478 | 71.8965 |
| 251.9926 | 63.5197 |
| 287.9686 | 57.0764 |
| 323.9503 | 113.1934 |
| 359.9894 | 90.9896 |

Table 3 Shaking Forces for Scheme-II

| Masses for link 1 (Kgs) | Masses for link 2 (Kgs) | Masses for link 3 (Kgs) |
|----------------------------|----------------------------|----------------------------|
| 1.21e-4 | 6.17e-5 | 1.08e-5 |
| 2.34e-4 | 1.80e-4 | 3.29e-5 |
| 1.87e-4 | 1.80e-4 | 1.41e-5 |
| 1.24e-5 | 9.57e-4 | 1.74e-5 |
| 4.41e-5 | 2.628e-4 | 3.87e-4 |
| 7.44e-5 | 2.21e-4 | 8.21e-5 |
| 2.16e-4 | 2.88e-4 | 8.21e-5 |
| 4.36e-5 | 5.27e-4 | 2.39e-5 |
| 1.97e-4 | 3.88e-5 | 5.49e-5 |
| 6.51e-4 | 1.08e-4 | 2.85e-5 |

Table 4 New Set of Magnitude of Masses for optimal force

Conclusion

In this work a procedure to minimize the shaking force in a four bar mechanism is presented. The shaking force is minimized by the redistribution of the mass in all the three links except the first link that constitute the mechanism the redistribution is carried out by using two schemes. One scheme involves variation of the locations of the distributed masses that constitute each link. The other schemes carries out the redistribution by varying the magnitude of the discretized masses, in keeping their locations fixed. For determining the redistributed magnitude or the locations of discretized masses for minimum shaking forces, a non linear optimal method, Nelder Mead simplex is adopted.

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