Optimization of Rectangular Plate with Central Square Hole Subjected to In-Plane Static Loading for Mitigation of SCF

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ABSTRACT

The study of stress concentration factor (SCF) for the rectangular plate with central square hole under in-plane load is analyzed numerically with the help of Finite Element Method. The material used for the plate is isotropic in nature, from this analysis author found that the variation in the SCF for different S/A ratio in the range of 0.1 to 0.5. In this paper SCF was mitigated by introducing auxiliary circular holes around main square hole. Variation in stresses in different directions with respect to S/A ratio is studied graphically. The results of percentage reduction in SCF for one set of auxiliary holes and two sets of auxiliary holes for considered cases are also shown graphically. The finite element formulation is carried out by using the software ANSYS

Keywords: Square hole, Stress Concentration Factor, Mitigation of Stress Concentration Factor, Finite Element Analysis.

Nomenclature:

A Width of rectangular plate, mm
L Length of rectangular plate, mm
S Diagonal of square hole, mm
k Far field Stress Concentration Factor
Kt Local SCF, Stress concentration factor
R1 Radius of first set of auxiliary holes, mm
R2 Radius of second set of auxiliary holes, mm
L_1 Centre to centre distance between main square hole and first set of auxiliary hole
L_2 Centre to centre distance between main square hole and second set of auxiliary hole
σ Stress applied in the plate lengthwise, N/mm^2
σ_{max} Maximum stress at discontinuity , N/mm^2
σ_{nom} Nominal stress at discontinuity = Applied load / Area at discontinuity

Introduction:

Any discontinuity in structure penetrates the strength of the structure. The flow of stress is altered due to discontinuity. Concentration of stress form near the discontinuity. These stress concentrations are of concern during both the preliminary and detail design of any structure.

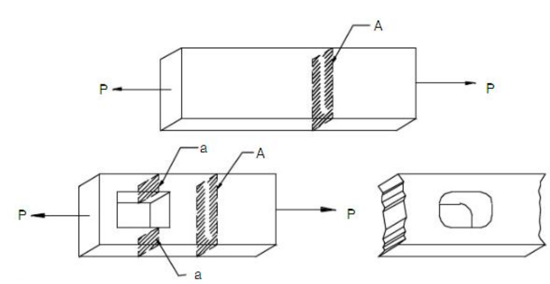

Fig. Plate with uniform thickness subjected to loadP

Through theory and experimentation, it has been shown that some of the stress concentration areas will have a stress value several times that of P/A or P/2a.The ratio of these increased stress values as compared to the P/2a value is termed the stress concentration factor.

$$K = \frac{\text{Local Stress}}{P/2a}$$

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Consider a plate under tensile stress, as in Fig, a K value greater than one signifies that the local stress exceeds the average stress in the smaller cross sectional area, 2a.Value of K less than one but greater than zero indicates that the local stresses are reduced. Value of K is less than zero means that the structure is in compression.

It may be convenient to express the stress concentration factor, as it would relate to the average stress across the entire plate in the absence of openings. This would correspond to the cross sectional area A in Figure 7. This is termed the stress concentration factor k.

$$k = \frac{\text{Local Stress}}{P/A}$$

These two Stress Concentration Factors relate to each other:

$$K = k * 2a/A$$

For a plate with a finite width, 2a/A will always be less than one. Therefore the K value will always be smaller than the k value. To avoid confusion between these two SCF, K will be referred to as the local stress concentration factor and k will be referred to as the basic or far-field stress concentration factor.

Of particular interest to designers are the maximum values of the local and far field stress concentration factors (K and k, respectively). The location and magnitude of this maximum will vary depending on a number of factors. These factors include the size of the discontinuity, the shape of the discontinuity, the number of discontinuity, the location of the discontinuity and the relative size of the discontinuity when compared with the structural member that it is in. In general, these values can be as high as 3 or even higher in the immediate area of the geometrical discontinuity. It is desirable to apply techniques to reduce these factors as much as is practicable.

As previously mentioned, the local and far-field stress concentration factors have been determined theoretically for a number of different opening configurations. There are several ratios that are related to the geometry of an opening in a plate. The stress concentration factors are dependent on these ratios. Figure 8 below shows a typical parameterized opening in a plate.

Peterson[1] has developed good theory and charts on the basis of mathematical analysis and presented excellent methodology in graphical form for evaluation of SCF in isotropic plates with different types of discontinuities.

The literature contains extensive work detailing the formulas and magnitudes of the SCF around different shapes of notches and holes, Roark and Young [2].

Ukadgaonkar [3] has given a new approach of stress analysis of an infinite plate with elliptical hole or crack with tensile stress. The closed form equations for SCF are given for anisotropic plate.

T. Hasan [4] has reported stress analysis of steel plate having holes of various shapes, sizes and orientations using finite element method. Finite element results are carried out by using the commercial software COMSOL 3.3. Results are presented in the form of non-dimensional graphs. Effects of hole shape are critically analyzed from the results of the finite element method and analytical methods. Comparisons between the results by the present finite element method and the analytical solution technique yield good agreement.

Nagpal, Jain, Sanyal [5] has compiled the work of many researchers in analysis of stress concentration and its mitigation. They have given the various methods used by different researchers for stress analysis of plate with different singularities. Comparison of stress concentration and its mitigation has also been done. From these results we have considered the area reduction method for mitigation of SCF around different discontinuities.

Problem Description:

The geometry of the problem is shown in Fig 1. Rectangular plate of 400mm*100mm and of 1mm thickness is considered for analysis. The isotropic material of the plate is high strength alloy steel; Poisson's ratio ν=0.3, Young's modulus E=39 GPa. Uniform tensile load (σ) is applied to the plate's lengthwise direction. For S/W 0.2, the model has been analyzed for different element size and element type. It has been concluded that an 8 nodded structural plane 82 element type with element size of 1mm with free meshing gives the same results as reported in literature and the same has been selected for further analysis on the bases of convergence test .The figure shows the element . Each node has two degrees of freedom, making a total 16 degrees of freedom per element. Due to the symmetry of the problem, one quarter of the plate for each case is discretized and analyzed.

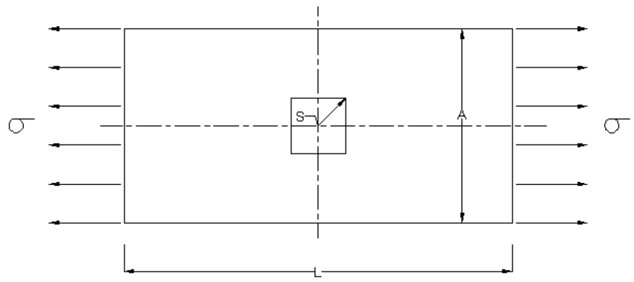

Fig.1. Plate with central square hole

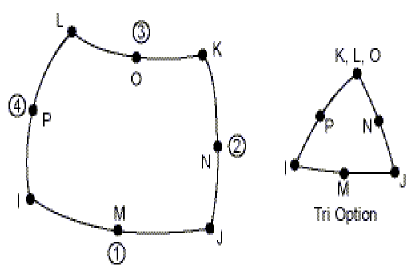

Fig.2. Noded Plane 82 Element Geometry

Analysis of Rectangular Plate with Central Square Hole

ANSYS analysis

Finite element analysis software ANSYS is a capable way to analyze a wide range of different problems. ANSYS analysis has the following steps for problem solving:

Modelling : Includes the system geometry definition and material property selection. In this step user can draw either 2D or 3D representation of the problem.

Meshing: This step involves discritizing the model according to predefined geometric element.

Table1. Stress Variation with S/A

S/A	σx	σy	σxy	σvon
0.1	2.612	1.014	0.2714	3.1848
0.2	3.4262	1.5595	0.37164	4.2245
0.3	4.104	1.873	0.44733	5.0644
0.4	4.868	2.032	0.51835	5.873
0.5	6.003	1.962	0.548	6.9318

Solution: In this step, we apply boundary conditions and loads to the system than solves the problem.

Post Processing: This step involves plotting/listing nodal solutions, which may be displacements in different directions, stresses in different directions, reactive forces etc.

The plate has been analyzed for different dimensions of square hole in rectangular plate. The results are given in Table. We have studied the stresses in x direction, y direction, xy stresses and von-mises stresses. All the stresses are increases with the size of hole. The magnitude of stress in x direction and von-mises stress is significant. The magnitude of stresses in y direction and xy are not significant as compared to other.

The deflection in all directions are also analyzed, the magnitude of deflections is of the range of 10^{-08}. These deflections are not considered for further analysis as it not very significant.

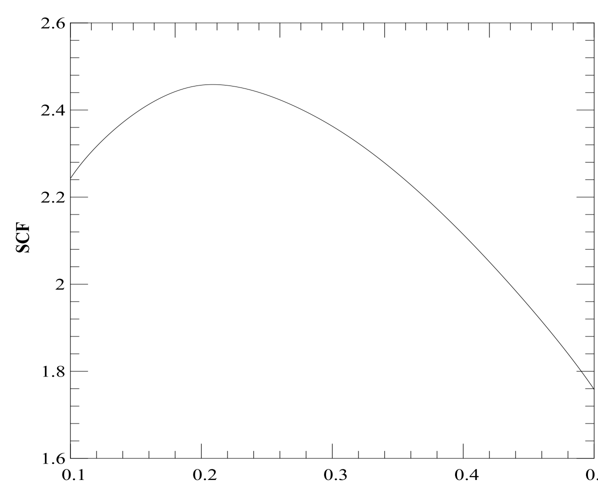

Fig. 3(a) variation in SCF with S/A

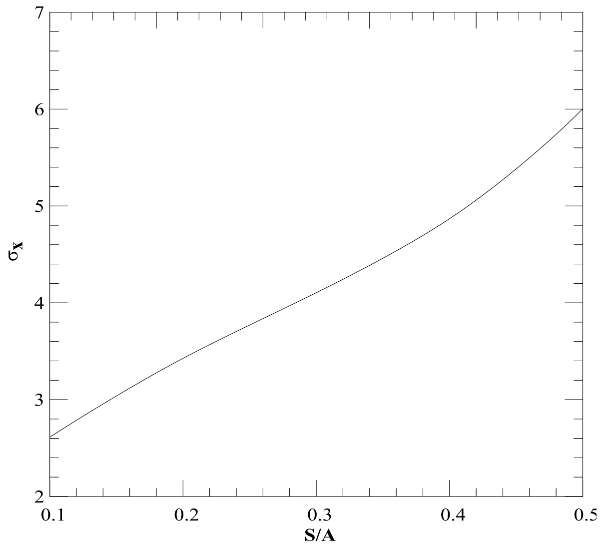

Fig3.(b) Variation in σx with S/A

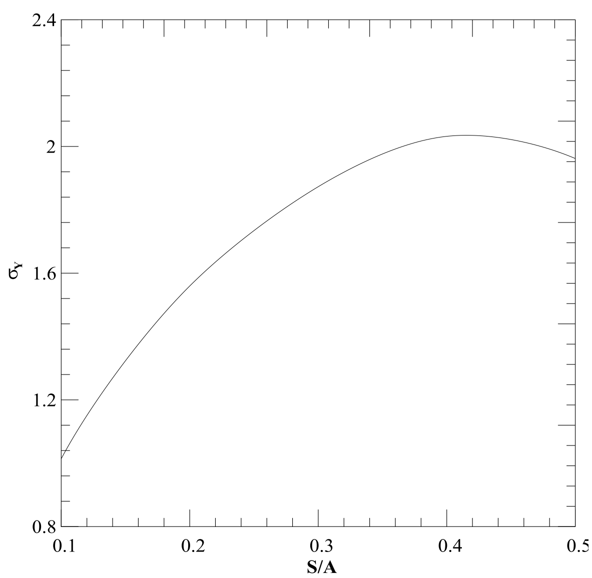

Fig.3(c) Variation in σy with S/A

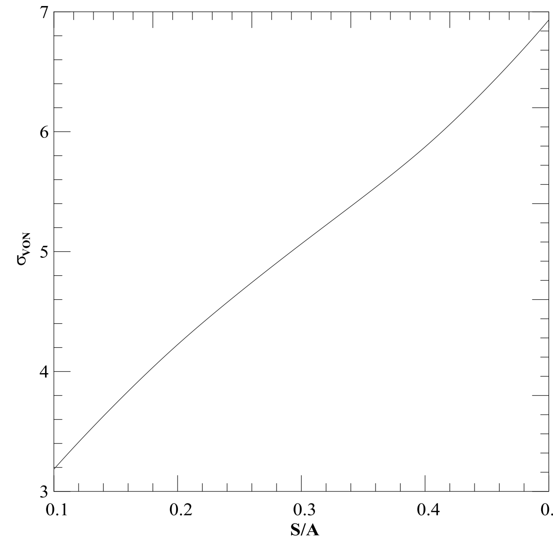

Fig3. (d) Variation in σvon with S/A

The Fig3 (a) shows the variation in SCF around main square hole with respect to S/A ratio. As size of hole increases the magnitude of SCF reduces. The Fig3 (b),(c) and (d) shows the variation in stresses in different directions with respect to size of square hole. From curves we can predict that all the stresses are increasing with increased size of hole. The magnitude of stress is significant for stress in x direction and von-mises stress.

Mitigation of Stress Concentration Factor

The methods of minimizing SCF in different elements have a long history dating back to eighteenth century. Reducing stress concentration or controlling the stress failure criteria is essential in mechanical engineering design. In literature many methods are proposed for stress analysis around any discontinuity. Many methods are given for mitigation of SCF around discontinuity.

Five basic models of finite rectangular plate with square hole are generated for S/A ratio from 0.1 to 0.5 in ANSYS. It has been already mentioned in literature that by introducing auxiliary holes, introducing fillets at square corners and by reinforcement on discontinuity, we get mitigation of SCF. We have studied the effect of several ratios on SCF that are related to the geometry of discontinuity in the plate. In present analysis all the models are also analyzed by introducing auxiliary holes of different sizes and at different distance from the main square hole.

Effect of corner radius on SCF

From Fig4 we can see that the maximum stress concentration occur as the radius becomes very small relative to the width of the opening. Also, as the radius approaches ½ the width of the opening such that one side of the opening is a half circle, the stress concentration factors approach that of a circular opening. The minimum seems to occur where r/b is equal to 5/16. For mitigation of SCF it requires that for r/b value of a least 1/8 be used and a value of 1/4 is typically recommended. Fig. is applicable to minor and medium sized discontinuity. Sensitivity to the relative size of the radius with respect to the size of discontinuity decreases, as the size of square hole becomes significant the curves would tend to shift left as well as change shape. By significant it is to be meant that the w/b ratio becomes small.

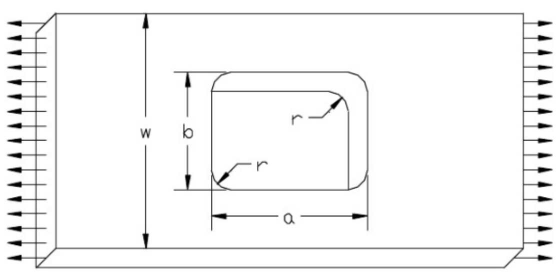

Fig4. Rectangular Plate with parameterized opening

This effect is shown through the b/a ratio (hole width normal to loading/hole length parallel to loading). It should be apparent from Fig. that if one were to take an opening with a b/a ratio of 3:1 and rotate it 90 degrees such that its b/a ratio becomes 1:3 a significant advantage is gained. The maximum SCF will drop to roughly half its original value. This illustrates the importance of orientation of openings. Another case where orientation is important occurs for a square opening with rounded corners. It can be shown that the opening should be oriented with one of its dimensions parallel to the direction of loading, rather than orienting it such that one of its diagonals is parallel to the direction of loading. The magnitude of this difference is dependent on the radius of the corners of the opening. For small radii (r/b=1:32), the stress concentration factor will double for the latter case. For larger radii (3:8), the stress concentration factor is roughly 20% greater for the latter case.

Table2. SCF around discontinuity for different geometries Fig4

Discontinuity	b/a	w/b	r/b	SCF
Rectangular hole with major dimension normal to load applied on the plate	3:1	3.24	0.5	3.25
			0.45	3.00
			0.4	2.9
			0.35	3.0
			0.33	3.1
Rectangular hole with major dimension normal to load applied on the plate	2:1	2.92	0.5	2.6
			0.45	2.3
			0.4	2.25
			0.35	2.36
			0.33	2.5
Square hole	1:1	3.24	0.5	2.24
			0.45	2.2
			0.4	2.1
			0.35	2.1
			0.33	2.1
Rectangular hole with major dimension parallel to load applied on the plate	1:2	3.24	0.5	1.6
			0.45	1.4
			0.4	1.35
			0.35	1.4
			0.33	1.51

In Table2 the effect of b/a ratio on Stress Concentration Factor is shown. From these results we can say that the shape of discontinuity which is most desirable in structures for minimum SCF is rectangular with larger dimension parallel to the direction of loading. From these data we can also said that the orientation of the discontinuity is also very important.

Mitigation of SCF by Area Reduction Method

To mitigate the SCF by improving the stress flow lines around the main hole, set of auxiliary circular holes are proposed. In this method the designer utilize the superposition of stress fields of main square hole and reduced stress areas by introducing auxiliary circular holes.

Optimization of Radius of Auxiliary Holes

For same main square hole, the plate has been analyzed by considering different size of auxiliary holes at

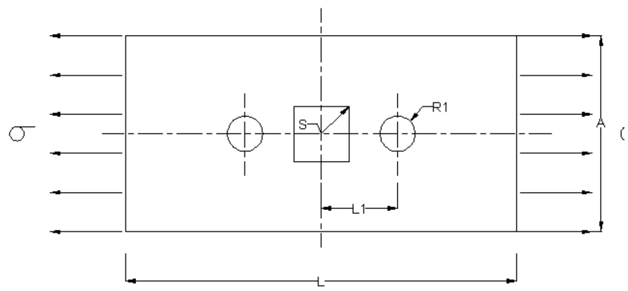

Fig.5. Plate with central square hole and one set of auxiliary circular hole

different positions. Table3 shows the reduction in maximum stress by introducing the one set of auxiliary holes. The minimum distance between the centre of main hole and auxiliary hole should be more than the sum of radius of auxiliary hole and half of side of square. It has been observed that the maximum reduction is reported for radius of 4.5mm (=0.9*S/2). Table shows the variation in maximum stress with one set of auxiliary hole with optimized radius by varying the distance. It has been observed that the maximum reduction is obtained at 1.5S+R1.

Optimization of Peripheral Distance

To mitigate the SCF by improving the stress flow lines around the main hole, set of auxiliary circular holes are proposed at different distances from main square hole. As the distances between the holes changes the stress also changes. As the distance increases the area of reduced stress increases. For closely spaced holes the area of reduced stress is more than that of area of unreduced stress by introduction of auxiliary holes. Table4 shows the variation in SCF with peripheral distance.

The SCF at considered area of main square hole can be determined by the following formula;

$$SCF = \sigma_{max} * (1-1.4142*S/A)$$

Table3. Maximum stress with one set of auxiliary hole

S/A	S	R1	L1	σmax without ah	σmax 1ah
0.1	10	0.5	10.5	2.612	2.593
0.1	10	1	11	2.612	2.596
0.1	10	1.5	11.5	2.612	2.598
0.1	10	2	12	2.612	2.576
0.1	10	2.5	12.5	2.612	2.563
0.1	10	3	13	2.612	2.516
0.1	10	3.5	13.5	2.612	2.446
0.1	10	4	14	2.612	2.361
0.1	**10**	**4.5**	**14.5**	**2.612**	**2.28**
0.1	10	5	15	2.612	2.284

Table4. Maximum stress for varying distance

S/A	S	R1	L1	σmax without AH	σmax
0.1	10	4.5	12	2.612	2.536
0.1	10	4.5	14.5	2.612	2.323
0.1	10	4.5	17	2.612	2.306
0.1	**10**	**4.5**	**19.5**	2.612	**2.281**
0.1	10	4.5	22	2.612	2.306
0.1	10	4.5	24.5	2.612	2.407
0.1	10	4.5	27	2.612	2.494

Variation in maximum stress has been studied by introducing second set of auxiliary hole along with the optimized one set of auxiliary holes as shown in Fig6. It has been observed that the maximum reduction is reported at R2=R1 at L2=36 mm.

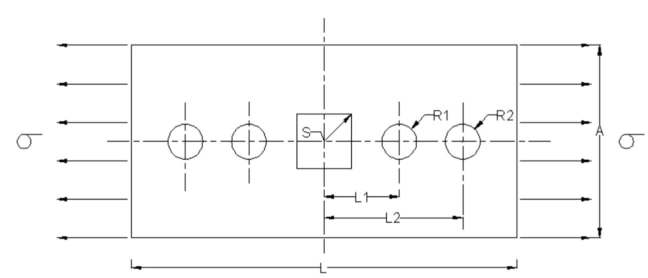

Fig.6. Plate with central square hole and two set of auxiliary circular hole

The Fig7. shows the percentage reduction in SCF obtained by introducing auxiliary holes. The reduction in SCF around square hole is significant. As S/A ratio increases, the reduction obtained decreases. For S/A=0.2, maximum reduction has been reported. This reduction gradually decreases as the size of hole increases.

For optimum distance and radius of auxiliary holes, it has been observed that the mitigation in SCF reported for two sets of auxiliary holes is less as compared to one set of auxiliary holes.

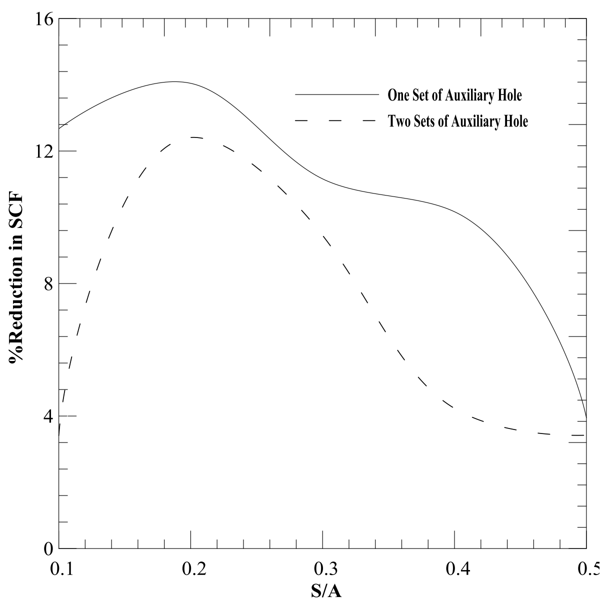

Fig7. S/A Vs % Reduction in SCF by introducing auxiliary holes

Results & Discussion

Numerical results obtained from 2D finite element analysis for different case studies of uni-axial loaded rectangular isotropic plate with square hole are presented in curves.

SCF is maximum for S/A 0.2, its value decreases with increase in value of S/A . Maximum stress in x direction and Von mises stress increases with increase in S/A ratio. The same trend has been reported in both curves. The stress in y direction also increases with S/A ratio , the trend of curve is non-linear.

SCF decreases by introduction of auxiliary holes. The percentage of reduction of SCF reported is upto 14-15%. The percentage reduction in SCF is maximum for one set of auxiliary holes.

The mitigation in SCF is reported by introducing the fillet at sharp corners of discontinuity and by area reduction method. These are called compensation methods. The SCF can be further mitigated by increasing the thickness of the plate.

Reinforcement is the other method of further reducing local and far-field stress concentration factors present around openings in primary structure. Reinforcement is typically applied to minor or medium sized openings. This technique generally involves modification to an opening in the primary structure after the opening has been cut. In other words, no modification is made to the primary structural scantlings (strakes) that will be penetrated with the exception of making the opening. Reinforcement is then applied locally around the opening boundaries.

Conclusion

Stresses are higher at the edge of square hole which can be reduced by providing fillet ends instead of sharp ends, area reduction method and reinforcement.

We can safely conclude from these results that with proper knowledge of stress variation at different S/A ratio, we can suggest exact size and position of material removal area. These results enable us to find the optimum distance between centre of main hole and centre of auxiliary hole for maximum stress reduction.

Another very useful guideline is that rectangular openings oriented such that their long dimensions are parallel to the direction of loads result in lower stress concentration factors than if they were oriented normal to the direction of loading.

When there is choice of shape of discontinuity then it is highly recommended to choose discontinuity with of rectangular shape with larger dimension parallel to the direction of loading, larger dimension is two times the smaller dimension.

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