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Optimization of natural frequency of vertical storage tower using FEA

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Abstract - *A vertical storage tower is typically used to store liquids or fine powders. To maximize the storage capacity, these columns are usually very tall typically over 27 meters. They are typically susceptible to wind loads, as the bending caused is much greater at these heights. The design problem is that, a vertical storage vessel is to be constructed near an agitator used in the process. The agitator is to generate vibrations at a certain frequency. If the natural frequency of the storage column matches that of the agitator, then it would result in a certain failure of the vessel at resonance. In addition to that, there is an inlet nozzle placed near the top of the column, the nozzle and the piping connected to it might also be susceptible to the vibration. The objective of the project will be to optimize the design and increase the natural frequency of tower in a manner that all failures can be avoided.*

I. INTRODUCTION

Tall cylindrical stacks and towers may be susceptible to wind-induced oscillations as a result of vortex shedding. This phenomenon, often referred to as dynamic instability, has resulted in severe oscillations, excessive deflections, structural damage, and even failure. Once it has been determined that a vessel is dynamically unstable, either the vessel must be redesigned to withstand the effects of wind-induced oscillations or external spoilers must be added to ensure that vortex shedding does not occur.

The deflections resulting from vortex shedding are perpendicular to the direction of wind flow and occur at relatively low wind velocities. When the natural period of vibration of a stack or column coincides with the frequency of vortex shedding, the amplitude of vibration is greatly magnified. The frequency of vortex shedding is related to wind velocity and vessel diameter. The wind velocity at which the frequency of vortex shedding matches the natural period of vibration is called the critical wind velocity. [1]

Wind-induced oscillations occur at steady, moderate wind velocities of 20-25 miles per hour. These oscillations commence as the frequency of vortex shedding approaches the natural period of the stack or column and are perpendicular to the prevailing wind. Larger wind velocities contain high velocity random gusts that reduce the tendency for vortex shedding in a regular periodic manner. A convenient method of relating to the phenomenon of wind excitation is to equate it to fluid flow around a cylinder. In fact this is the exact case of early discoveries related to submarine periscopes vibrating wildly at certain speeds. At low flow rates, the flow around the cylinder is laminar. As the stream velocity increases, two symmetrical eddies are formed on either side of the cylinder. At higher velocities vortices begin to break off from the main stream, resulting in an imbalance in forces exerted from the split stream. The discharging vortex imparts a fluctuating force that can cause movement in the vessel perpendicular to the direction of the stream.
Historically, vessels have tended to have many fewer incidents of wind-induced vibration than stacks. [1]
There is a variety of reasons for this:

1. Relatively thicker walls.
2. Higher first frequency.
3. External attachments, such as ladders, platforms and piping, that disrupt the wind flow around the vessel.
4. Significantly higher damping due to:
   a) Internal attachments, trays, baffles, etc.
   b) External attachments, ladders, platforms, and piping.
   c) Liquid holdup and sloshing.
   d) Soil.
   e) Foundation.
   f) Shell material.
   g) External insulation.

II. DESIGN PROBLEM

The challenge posed is that the agitator and the vertical column are required to be on the same platform. Due to this the Column is susceptible to vibrations from the agitator. The frequency generated by the agitator is directly proportional to the rotations per minute (rpm). If the freq generated by the agitator matches with the natural frequency of the Vertical Column then such a resonance will cause the column to vibrate at max amplitude and may even result in Failure. The issue can be resolved by increasing the natural frequency of the column. Natural freq is a function of the mass (m) and the stiffness (k). If we optimize these parameters using structural modifications we can increase natural freq and then we can operate the agitator at higher rpm reducing the production operate the agitator at higher rpm reducing the production time. [9]

Challenge while increasing the frequency of vertical column is:

Frequency, $f = \frac{1}{2\pi}\sqrt{k/m}$

So if we try to increase the stiffness by increasing stiffeners then weight also increases so no real difference in frequency. [6]

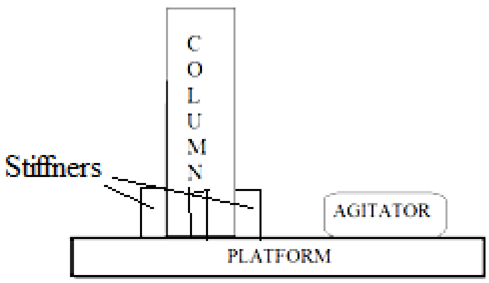

Figure 1: Block diagram of column, agitator and Stiffeners modifications

III. MODAL ANALYSIS

A modal analysis determines the vibration characteristics (natural frequencies and corresponding mode shapes) of a structure or a machine component. It can serve as a starting point for other types of analyses by detecting unconstrained bodies in a contact analysis or by indicating the necessary time-step size for a transient analysis, for example. In addition, the modal-analysis results may be used in a downstream dynamic simulation employing mode-superposition methods, such as a harmonic response analysis, a random vibration analysis, or a spectrum analysis. The natural frequencies and mode shapes are important parameters in the design of a structure for dynamic loading conditions. [6]

Modes are inherent properties of a structure, and are determined by the material properties (mass, damping, and stiffness), and boundary conditions of the structure. Each mode is defined by a natural (modal or resonant) frequency, modal damping, and a mode shape (i.e. the so-called "modal parameters"). If either the material properties or the boundary conditions of a structure change, its modes will change. For instance, if mass is added to a structure, it will vibrate differently. To understand this, we will make use of the concept of single and multiple-degree-of-freedom system. [2]

We find the equation of motion using modal analysis, v_1 and v_2 be the eigenvectors of the matrix K. These vectors are orthogonal (unless they correspond to the same eigenvalue, in which case they should be made orthogonal). If they have also been normalized, then they form an orthonormal set. Now let's define the matrix of eigenvectors P to consist of these orthonormal eigenvectors. In an equation this is

$$P = [\, v_1 \;\; v_2 \,]$$

This matrix is an orthogonal matrix (as its columns are orthonormal). Such matrices have the convenient property that $P^T P = I$. Matrix mode shapes S is defined as

$$S = M^{-1/2} P$$

$$x(t) = M^{-1/2} q(t) = M^{-1/2} Pr(t) = Sr(t)$$

Using all the equations we can rewrite the system of differential equations to $\ddot{r}(t) + \Lambda r(t) = 0$ Where the matrix Λ is given by $\Lambda = P^T \acute{K} P = \begin{bmatrix} \omega_1^2 & 0 \\ 0 & \omega_2^2 \end{bmatrix}$

So we remain with differential equations

$$\ddot{r}_1 + \omega_1^2 r = 0$$
$$\ddot{r}_2 + \omega_2^2 r = 0$$

The deferential equations have been decoupled! They don't depend on each other, and therefore can be solved using simple methods. The two decoupled equations above are called the modal equations. Also the coordinate system r (t) is called the modal coordinate system.

IV. NATURAL FREQUENCY AND MODE SHAPES (EIGENVALUES AND EIGENVECTORS)

If the shell is of constant diameter and thickness for its full length, the period of vibration maybe easily found from the work of C E FREESE. The graph is given by Author from the general formula for the period of the first mode of vibration of a cantilever beam [1]:

Let's take a multi-degree of freedom system. For generalizing the method for determining its natural frequencies and mode shape the differential equations of motion for the system is, [3]

$$\left.\begin{aligned} &[m_1\ddot{x}_1 + (k_1 + k_2)\, x_1] - k_2 x_2 = 0 \\ &-k_2 x_1 + [m_2\ddot{x}_2 + (k_2 + k_3)\, x_2] - k_3 x_3 = 0 \end{aligned}\right\}$$

$-k_3 x_2 + [m_3 \ddot{x}_3 + (k_3 + k_4)\, x_3] - k_4 x_4 = 0$ ------ [A]

-- = 0

$- k_n x_{n-1} + (m_n \ddot{x}_n + k_n x_n) = 0$

For the principal mode of vibration, let us assume the solution as,

$$\left.\begin{aligned} x_1 &= X_1 \sin \omega t \\ x_2 &= X_2 \sin \omega t \\ x_3 &= X_3 \sin \omega t \\ &\text{---------------} \\ x_n &= X_n \sin \omega t \end{aligned}\right\} \text{---------------------------------- [B]}$$

Substituting equations (B) in equations (A) and canceling out the common term sin ωt

$[(k_1 + k_2) - m_1\, \omega_2]\, X_1 - k_2 X_2 = 0$

$-k_2 X_1 + [(k_2 + k_3) - m_2 \omega_2]\, X_2 - k_3 X_3 = 0$

$-k_3 X_3 + [(k_3 + k_4) - m_3 \omega_3]\, X_3 - k_4 X_4 = 0$

-- = 0

$-k_n X_{n-1} + (k_n - m_n\, \omega^2)\, X_n = 0$

For the above equations, the solution, other than X1 =X2 =X3 = ….Xn = 0 is possible only when the determinant composed of the coefficients of X's vanishes, or

$$\begin{bmatrix} [(k_1 \quad k_2) \quad m_1\omega^2] & k_2 & \cdots & 0 & 0 \\ k_2 & [(k_2 \quad k_3) \quad m_2\omega^2 & \cdots & 0 & 0 \\ 0 & k_3 & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & k & (k \quad m\,\omega^2) \end{bmatrix} = 0$$

V. COMBINED RECTILINEAR AND ANGULAR MODES

The body having mass M and moment of inertia J, supported as shown in the fig 2(a) and capable of oscillating in the directions x and θ. Let, at any instant, the body be displaced through rectilinear distance x and angular distance θ as shown in fig. 2(b) at this instant taking θ be small, springs k_1, and k_2 are compressed through $(x\text{-}l_1\theta)$ and $(x\text{+}l_2\theta)$ respectively, beyond their equilibrium position. The differential equations of the system are written for motions in the x and θ directions by taking the forces and the moments in the respective directions acting on the system.[5]

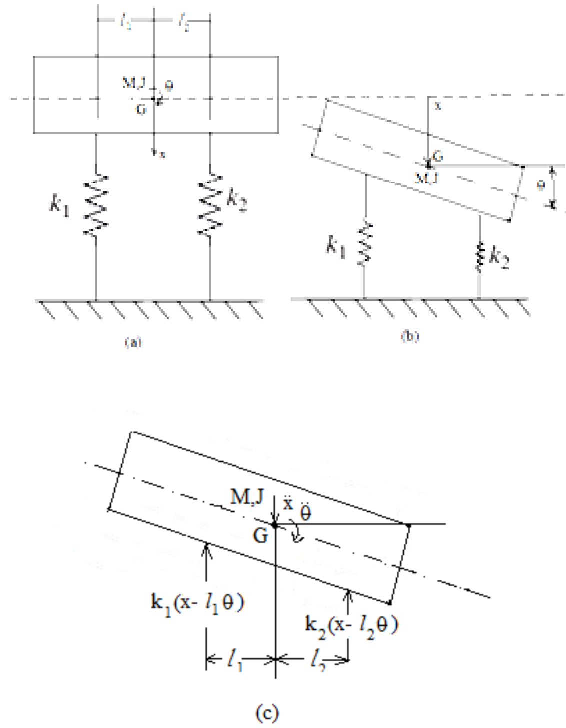

Figure 2: Analysis of a combined rectilinear and angular mode system

$$\left.\begin{aligned} M\ddot{x} &= - k_1\, (x - l_1\theta) - k_2\, (x + l_2\theta) \\ J\ddot{\theta} &= k_1\, (x - l_1\theta)\, l_1 - k_2\, (x + l_2\theta)\, l_2 \end{aligned}\right\} \text{------------- [C]}$$

Letting,

$$\left.\begin{aligned} \frac{k1+k2}{M} &= a \\ \frac{k_1 l_1 - k_2 l_2}{M} &= b \\ \frac{k_1 l_1^2 + k_2 l_2^2}{J} &= c \end{aligned}\right\} \text{-------------------------------- [D]}$$

and remembering that $J = mr^2$, the equations [C] reduces as below

$\ddot{x} + ax = b\theta$

$\ddot{\theta} + c\theta = (b/r^2)\, x$

Here b = coupling coefficient,

If b = 0, then two equations are independent of each other and therefore the two motions, rectilinear and angular, can exist independently of each other with their respective natural frequencies $\sqrt{a}$ and $\sqrt{c}$. Thus for the case of uncoupled system when b = 0, i.e. $k_1 l_1 = k_2 l_2$, the natural frequencies in the rectilinear and angular modes respectively, are [3], [4]

$$\omega_{n1} = \sqrt{\frac{k_1 + k_2}{M}}$$

$$\omega_{n2} = \sqrt{\frac{k_1 l_1^2 + k_2 l_2^2}{I}}$$

VI. FINITE ELEMENT ANALYSIS

The finite element mode shapes of various frequencies helps in understanding the cylinder's vibration behavior and also assists in choosing the frequency range and mode shapes of interest, selecting suitable accelerometer and impact hammer, selecting locations for accelerometer placement, etc. The placement of the accelerometer is very critical in obtaining valid data. The data obtained won't be useful if the accelerometer is placed at a node point of the structure's mode shape for any frequency within the frequency range of interest. The thin uniform circular cylindrical shell used for the Experimental Modal Analysis is modeled and analyzes for its frequencies using normal mode analysis in MSC.NASTRAN byAlzahabi, B.; Natarajan, L. K [2,3].

The finite element method (FEM), sometimes referred to as finite element analysis (FEA), is a computational technique used to obtain approximate solutions of boundary value problems in engineering. Simply stated, a boundary value problem is a mathematical problem in which one or more dependent variables must satisfy a differential equation everywhere within a known domain of independent variables and satisfy specific conditions on the boundary of the domain. [9] Boundary value problems are also sometimes called field problems. The field is the domain of interest and most often represents a physical structure. The field variables are the dependent variables of interest governed by the differential equation. The boundary conditions are the specified values of the field variables on the boundaries of the field. Depending on the type of physical problem being analyzed, the field variables may include physical displacement; temperature, heat flux, and fluid velocity to name only a few Vertical columns have a few characteristics which make it difficult to design. Firstly the model has no natural roll-off at high frequencies and it is modally very rich. It is characterized by resonant peaks which dominate the dynamics besides the vertical column themselves; complete systems contain many other components such as stiffeners, supports, vibration isolators etc. [6]

Stiffeners are attached to column to strengthen the panel against pressure loads, to reduce vibrations of the stiffen plate against buckling. Stiffeners must be spaced at some convenient spacing so that plate stress is less than allowable stress. Stiffener may also function as column supports as well as plate reinforcement. Stiffeners are generally welded to the column although there are other methods to join.

i. MODELING OF VERTICAL TOWER

Vertical tower and supports are modeled in ANSYS workbench. There are 20 stiffeners attached to column. All these stiffeners are welded to stiffeners.

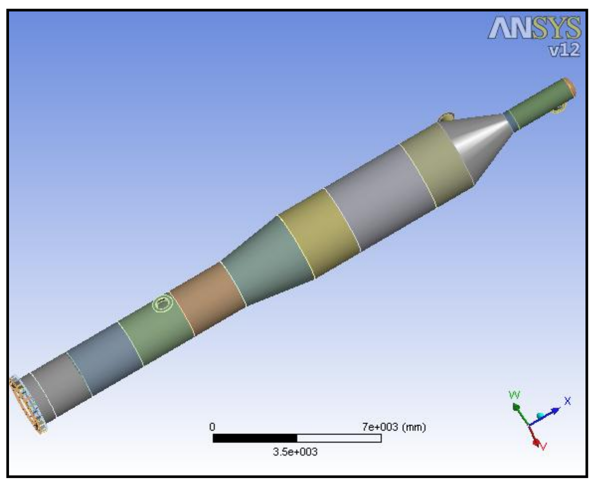

Figure 3: FE model of vertical column

ii. MESHING OF THE MODEL

Meshing is the method of dividing the model into the number of element to obtain the good accuracy in the analysis. As the number of elements increase the accuracy of analysis increases.

The meshing is done as second order meshing method using shell 93.

iii. SIZE OF STIFFENERS

The size of stiffener is R/2 of largest diameter of the tower; width wise and R/4 length wise. The stiffeners equally separated at 15 inch distance along the three sides.

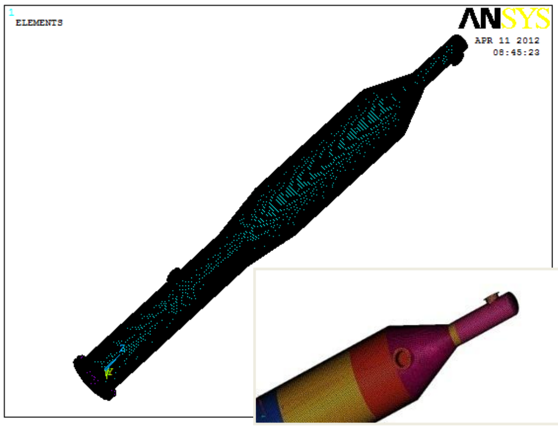

Figure 4: FE mesh of the vertical tower model

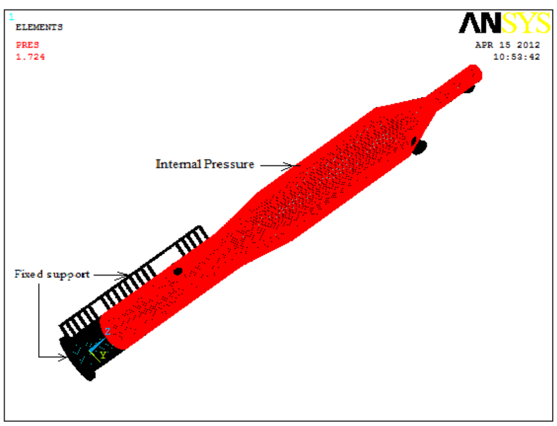

Figure 5: Boundary conditions

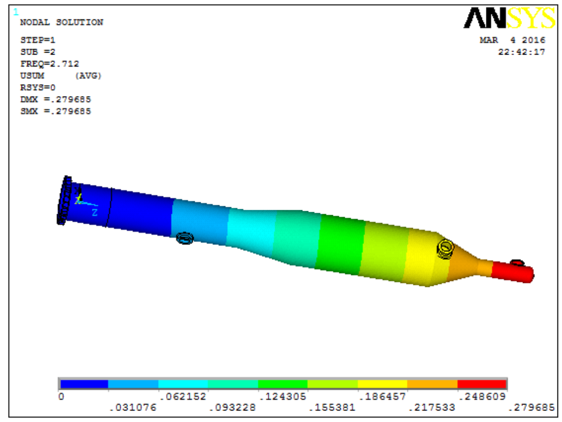

Figure 6: Natural Frequency of vertical tower when no stiffeners are added0

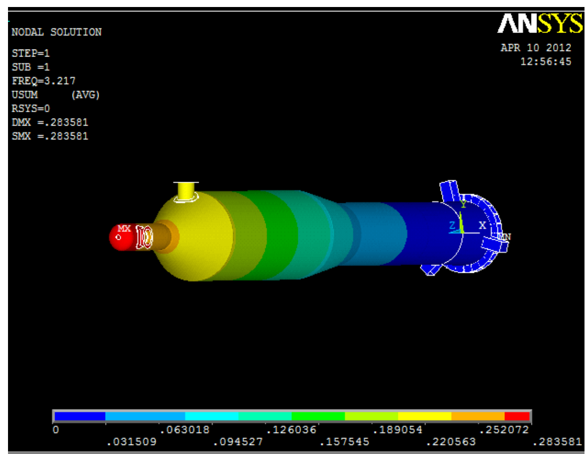

Figure 7: 5 stiffeners added

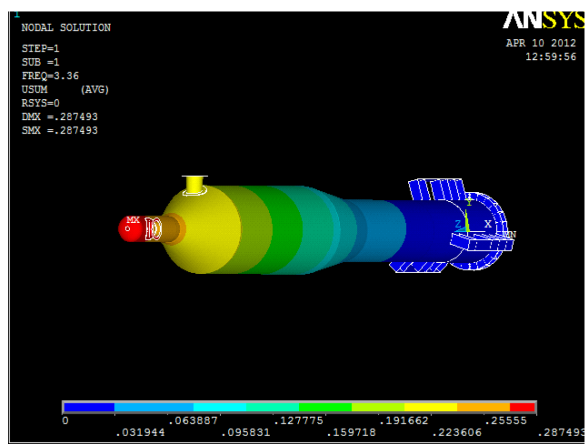

Figure 8: 12 stiffeners added

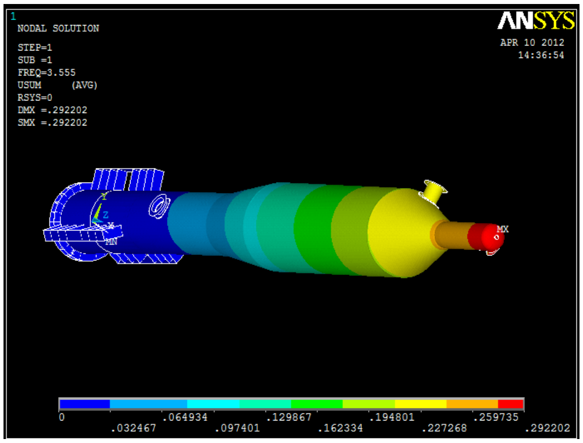

Figure 9: 15 stiffeners added

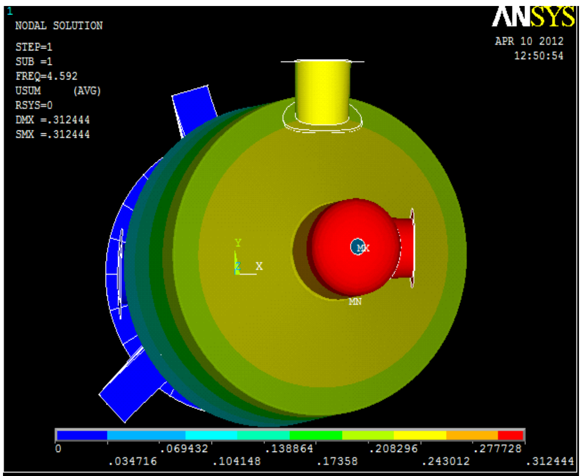

Figure 10: 20 stiffeners added

VII. RESULTS:

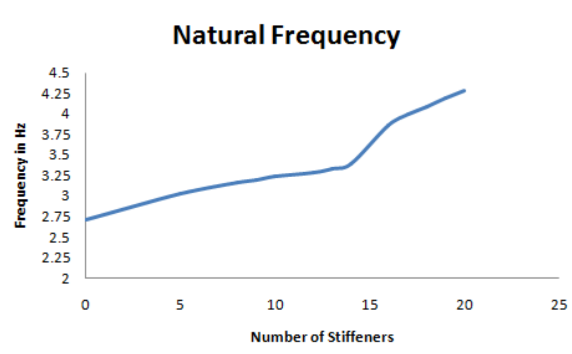

Figure 11: Graph showing increase in natural frequency

CONCLUSION:

- The additional support structure is having an effect on the natural frequency of the Tower.
- As the number of stiffeners increases the frequency is observing a corresponding increase
- At around 14 stiffeners which is at a height of 210 inch, there seems to be a Threshold point, beyond which there is rapid rise in the frequency
- After around 17 stiffeners, the trend seems to be again linear, although further study is needed to determine if there is another Threshold point.
- Based on the study, having around 15 Stiffeners is ideal from both cost and frequency considerations.

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