Optimization of IRM - Parallel-Series Redundant System

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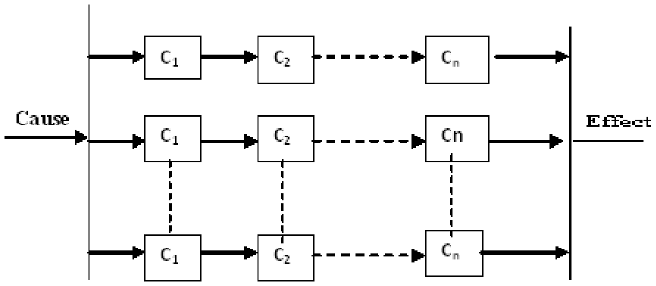
ABSTRACT

An Integrated Reliability Model (IRM) for the parallel series redundant system considers both the unknowns i.e. the Component Reliabilities and the Number of Components in each stage for given constraints to maximize the System Reliability. Integrated Reliability Model for a Redundant System by treating Weight and Volume as additional constraints apart from the conventional Cost constraint to optimize the System Reliability, to negotiate the hidden impact of the additional constraints Weight and Volume. Integrated Reliability Model for a Parallel – Series Redundant System is proposed by applying the Lagrangean Multiplier Method to find out the number of Components, Component Reliabilities, Stage Reliabilities to optimize System Reliability and as these values are found to be in real, to derive an integer solution the Dynamic Programming Method is applied to obtain an integer solution.

1. INTRODUCTION

The reliability of a system can be increased by keeping redundant units, or by using components of greater reliability, or by employing both methods simultaneously [3,4]. Either of them consumes additional resources. Optimizing of System Reliability, subject to resource availability such as Cost, Weight, Volume is considered. Generally, Reliability is treated as a function of Cost; but when applied to real-life problems, the hidden impact of other constraints like Weight, Volume, etc, will have a definite impact on optimizing the Reliability. The novel application of a Redundant Reliability model with multiple constraints is considered to optimize the proposed system.



PARALLEL – SERIES CONFIGURATION

The problem considers the unknowns – that is, the Number of Components (x_j), the Component Reliabilities (r_j), and the Stage Reliability (R_j) at each stage for a given multiple of constraints to maximize the System Reliability which is termed as an Integrated Reliability Model (IRM). In the literature Integrated Reliability Models are

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optimized using Cost constraints where there is an established relation between Cost and Reliability. The novelty aspect of the proposed work is consideration of Weight and Volume as additional constraints along with Cost to design and optimize the Redundant Reliability System for a Parallel-Series System configuration.

2. ASSUMPTIONS AND NOTATIONS

- All the components in each stage are assumed to be identical, i.e., all the components have the same Reliability.
- The components are assumed to be statistically independent i.e., failure of component does not affect the performance of the other components in any system.
- A component is either in working condition or non working condition.

R_s = System reliability

R_j = Reliability of stage j, $0< R_j < 1$

r_j = Reliability of each component in stage j, 0< rj < 1

x_j = Number of components in stage j

c_j = Cost coefficient of each component in stage j

w_j = Weight coefficient of each component in stage j

v_j = Volume coefficient of each component in stage j

C_o = Maximum allowable system cost

W_o = Maximum allowable system weight

V_o = Maximum allowable system volume

a_j = Constant ;b_j = Constant ;p_j = Constant

q_j = Constant ;u_j = Constant ;v_j = Constant

3. MATHEMATICAL MODEL:

The objective function and the constraints of the model

$$\text{Max } R_s(t) = 1 - \prod_{i=1}^{k} \left[1 - \prod_{j=1}^{n} R_{ij} \right] \quad \textbf{(1)}$$

subject to the constraints

$$\sum_{j=1}^{n} c_j . x_j \le C_0 \quad (2)$$

$$\sum_{j=1}^{n} w_j . x_j \le W_0 \quad (3)$$

$$\sum_{j=1}^{n} v_j . x_j \le V_0 \quad (4)$$

non-negative restriction that x_j is an integer and r_j, $R_j > 0$

4. MATHEMATICAL FUNCTION

To establish the mathematical model, the most commonly used function is considered for the purpose of reliability design and analysis. The proposed mathematical function

$$r_j = \left[\frac{c_j}{b_j} \right]^{\frac{1}{d_j}} \quad (5)$$

System reliability for the given cost function

$$R_s = 1 - \prod_{i=1}^{n} (1 - \prod_{j=1}^{m} R_{ij}) \quad (6)$$

The transformed equations through the relation are

$$x_j = \frac{\ln(R_j)}{\ln(r_j)} \quad (7)$$

The problem under consideration is

$$\text{Maximize } R_s = 1 - \prod_{j=1}^{n} [1 - (r_j)^{x_j}] \quad (8)$$

subject to the constraints

$$\sum_{j=1}^{n} \left[b_j . r_j^{d_j} . \frac{\ln(R_j)}{\ln(r_j)} \right] - C \le O \quad (9)$$

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$$\sum_{j=1}^{n}\left[p_j . r_j^{q_j} \frac{\ln(R_j)}{\ln(r_j)} \right] - W \le 0$$

(10)

$$\sum_{j=1}^{n}\left[v_j . r_j^{l_j} \frac{\ln(R_j)}{\ln(r_j)} \right] - V_o \le 0$$

(11)

Non-negativity restriction $x_j \geq 0$

5. THE LAGRANGIAN METHOD

A Lagrangean function is formulated as

$$F = R_s + \lambda_1 \left[\sum_{j=1}^{n} \left\{ c_j . r_j^{d_j} \frac{\ln(R_j)}{\ln(r_j)} \right\} - C_0 \right] + \lambda_2 \left[\sum_{j=1}^{n} \left\{ p_j . r_j^{q_j} \frac{\ln(R_j)}{\ln(r_j)} \right\} - W_0 \right] + \lambda_3 \left[\sum_{j=1}^{n} \left\{ v_j . r_j^{l_j} \frac{\ln(R_j)}{\ln(r_j)} \right\} - V_0 \right]$$

(12)

where $\lambda 1$, λ_2, λ_3 are Langrangean multipliers.

To determine the optimum component reliability (r_j), stage reliability (R_j), Number of components in each stage (x_j) and the system reliability (R_s) by using the Lagrangean Method. The method provides a real (valued) solution with reference to cost, weight, and volume.

i. RELIABILITY DESIGN RELATING TO COST:

Stage	r_j	R_j	x_j	c_j	$c_j . x_j$
01	0.9404	0.9343	1.1	88.44	98
02	0.9604	0.9311	1.77	44.29	78
03	0.9741	0.9874	0.48	49.52	24
TOTAL COST					**200**

ii. RELIABILITY DESIGN RELATING TO WEIGHT:

Stage	r_j	R_j	X_j	W_j	$W_j . X_j$
01	0.9404	0.9343	1.1	88.44	98
02	0.9604	0.9311	1.77	132.88	235
03	0.9741	0.9874	0.48	139.56	67
TOTAL WEIGHT					**400**

iii. RELIABILITY DESIGN RELATING TO VOLUME:

Stage	r_j	R_j	X_j	V_j	$V_j . X_j$
01	0.9404	0.9343	1.1	265.31	293
02	0.9604	0.9311	1.77	132.88	235
03	0.9741	0.9874	0.48	148.56	72
TOTAL VOLUME					**600**

SYSTEM RELIABILITY =R_S=0.8589

6. OPTIMIZATION OF PARALLEL-SERIES INTEGRATED REDUNDANT RELIABILITY MODEL WITH MULTIPLE CONSTRAINTS – DYNAMIC PROGRAMING APPROACH:

- In Dynamic Programming problems, decisions regarding a certain problem are typically optimized at subsequent stages rather than simultaneously. This implies that if a program is to be solved using Dynamic Programming, it must be separated in 'N' sub problems.
- Dynamic Programming deals with problems in which choices or decisions are to be made at each stage. The set of all

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possible choices is reflected and or governed by the state at each stage.

- Associated with each decision at every stage is a return function that evaluates the choice made at each decision in terms of the contributed that the decision can make to the overall objective (Maximum or Minimum).
- Each stage 'n' the total decision process is related to its adjoining stages by a quantitative relationship called a transition function. This transition function can either reflect discrete quantities or continuous quantities depending on the nature of the problem.
- Given the current state, an optimal policy for the remaining stages in terms of a possible input state is independent of the policy adopted in previous stages.
- The solution procedure always proceeds by finding the optimal policy for each possible input state at the present stage.
- A recursive relationship is always used to relate the optimal policy at stage 'n' to the (n-1) stage that follows. This relationship is given by

$$f_n*(S_n)=\text{opt}_{d_n}\ \{r_n(d_n)*f_{n-1}*(S_n*d_n)\}$$

Here the symbol ∗ denotes any mathematical relationship between S_n and d_n; including addition, subtraction, multiplication and root operations.

- By using this recursive relation, the solution procedure moves form stage to stage each time finding an optimal policy for each state at that stage until the optimal policy for the last stage is found. Once the N-stage optimal policy has been discovered N-component decision vector can be recovered by tracking back through the N-stage transition function.

i. DYNAMIC PROGRAMMING TABLE – STAGE 1:

No. of Components	STAGE RELIABILITY
x_j	R_j
01	0.9404
02	0.8844
03	0.8316
04	0.7820
05	0.7355

ii. DYNAMIC PROGRAMMING TABLE – STAGE 2:

No.of Components	STAGE RELIABILITY					
x_j	R_j					
02	0.9976					
03	0.9954	0.9954				
04	0.9933	0.9911	0.9932			
05	0.9914	0.9861	0.9868	0.9941		
06	0.9895	0.9831	0.9808	0.9828	0.9928	

iii. DYNAMIC PROGRAMMING TABLE – STAGE 3:

No. of Components	STAGE RELIABILITY					
x_j	R_j					
03	0.9999					
04	0.9999	0.9998				

05	0.9998	0.9996	0.9997			
06	0.9998	0.9995	0.9995	0.9996		
07	0.9998	0.9995	0.9992	0.9993	0.9996	

i. RELIABILITY DESIGN RELATING TO COST:

STAGE	r_j	R_j	x_j	c_j	$c_j.x_j$
01	0.9404	0.9404	1	88.44	88.44
02	0.9604	0.9954	2	44.29	88.58
03	0.9741	0.9999	1	49.52	49.52
TOTAL COST					**226.54**

ii. RELIABILITY DESIGN RELATING TO WEIGHT:

STAGE	r_j	R_j	x_j	c_j	$c_j.x_j$
01	0.9404	0.9404	01	88.44	88.44
02	0.9604	0.9954	02	132.88	265.76
03	0.9741	0.9999	01	139.56	139.56
TOTAL WEIGHT					493.76

iii. RELIABILITY DESIGN RELATING TO VOLUME:

STAGE	r_j	R_j	x_j	c_j	$c_j.x_j$
01	0.9404	0.9404	01	88.44	88.44
02	0.9604	0.9954	02	132.88	265.76
03	0.9741	0.9999	01	139.56	139.56
TOTAL VOLUME					679.63

SYSTEM RELIABILITY R_S = 0.9359

VARIATION IN SYSTEM RELIABILITY = 6.41%

7. SENSITIVITY ANALYSIS:

It is observed that when the input data of constraints is increased by 10% variation in constraints through Sensitivity analysis conforms that there is no significant effect of this change on the developed model. the variation in the system reliability is as shown in Table.

Variation in factors		System Reliability
Cost	10% decrease	No change
	10% increase	No change
Weight	10% decrease	No change
	10% increase	No change
Volume	10% decrease	No change
	10% increase	No change

SENSITIVITY ANALYSIS TABLE

8. DISCUSSION:

The Integrated Reliability Models for redundant systems with multiple constraints for the mathematical function is established by applying Dynamic Programming. The inputs for the case problem for the Dynamic Programming are taken from the Lagrangean method. The results of the problem inform that the prime advantage of Dynamic programming is that the values of number of Components in each stage i.e. x_j will be in the form of integer values which are highly useful for practical applications to real life problems. Further the 10 %variation in constraints through Sensitivity analysis conforms that there is no significant effect of this change on the developed model.

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