

Optimal Power Distortion using AWGN In Sensor Network

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Abstract

In this paper, We have investigated the estimation of a memory less source by a wireless sensor network when the underlying random process is Gaussian where 'L' distributed sensors transmit noisy observation of the source through a Gaussian multiple access channel to a single fusion center. In this paper, we consider the problem of source channel coding with a Fidelity criterion. Fidelity is measured by a distortion function. Simulation result shows that the gap between upper and lower bounds of distortion D is small as compared with the existing implementation[1]. This paper analyze the fundamental tradeoff between number of sensors, total transmit power, number of degrees of freedom of source and distortion. This paper shows that as the number of sensor increases distortion is exactly optimal in case of symmetric environment and almost optimal in asymmetric case.

Keywords: Asymmetric and Symmetric Gaussian Sensor Network, Power- distortion tradeoff, Analog Forwarding, Rate distortion

I. INTRODUCTION

As Power (energy) is the main source of interest in developing any network because of transmission cost constraint. The major challenge in communication system is to transmit an information from a source to a destination over a noisy channel such that the destination can reconstruct the source message with

the highest fidelity. In other words, we can associate a cost for using the channel and define the fidelity of the reconstruction by a distortion function. Inherently, there is a tradeoff between the available budget for transmission and the achievable distortion at the destination. Then the problem is to find the minimum budget (per channel use) required to achieve a target distortion requirement for the channel with minimum power. The solution characterizes the power-distortion tradeoff for the given system. There are upper and lower bounds to these functions including the famous Shannon lower bound, if the gap between the provided upper bound and the lower bound of the distortion D is small, to tighten the gap between two provided bounds, provide the optimal power allocation in the uncoded transmission that minimizes the total power consumption for a given distortion D. With the recent advancement in hardware technology, small cheap nodes with sensing, computing and communication capabilities have now become available. In this paper, we have studied the source channel communication in an asymmetric Gaussian sensor network for the achievability of power distortion tuples $(P,D)=(P_1,P_2,\dots,P_L,D)$ where L distributed sensors transmit noisy observation of source through an additive white Gaussian multiple access channel to a single collector node. Each encoder is subject to a transmission cost constraint. This constraint comes from restriction on the resources such as power and bandwidth at each sensor in a network. The fusion (collector) center wants to reconstruct the main source X with an average distortion D at smallest cost in communication link. Our interest lies in determining the power distortion region while the fidelity of

estimation is measured by minimum mean square error (MSE) distortion [1-2]. The key goal of this paper is to characterize the relationship between the number of underlying sources, distortion, the total sensor power and number of sensors in a network.

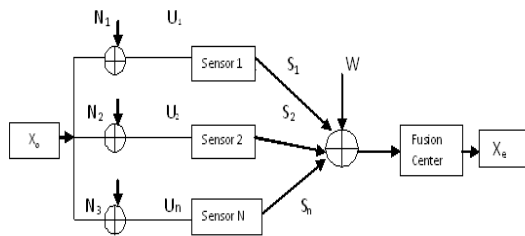
II. Gaussian Sensor Network

Wireless sensor network is said to be Gaussian when the statistics used are Gaussian. This is an intuitively pleasing behavior corresponds to decay of MSE. There is distortion due to fact that channel is noisy and leads to interference. This distortion can be decreased by allowing more power. In order to optimized the power, Gaussian sensor network is classified in two parts. First, Symmetric Gaussian sensor network, where sensors observation have considered to be same noise level and transmitting terminal are subject to same average power on each sensor in a network corresponds that as the number of sensor increases, uncoded transmission achieves the smallest possible distortion. Second, Asymmetric Gaussian sensor network, where sensor observation have differing noise variances and transmitted power are not the same. Hence, uncoded transmission (analog forwarding) is optimal in symmetric case and almost optimal in an asymmetric environment.

III. Sensor Network Model

The goal of the sensor network is for the sensor nodes to code and transmit the measured samples to a collector node over a Gaussian multiple-access channel and for the collector node to reconstruct the entire random process with minimum expected distortion [2].

Figure 1: Sensor Network Model



we assume that $P_X(x)$ is Gaussian with variance σ^2 , and samples of the signal X are independent and identically distributed where a team of 'N'

sensors observe noisy observation of source data sequence. [6]

$$R(D) = \begin{cases} \frac{1}{2} \log_2 (\sigma_x^2 / D), & \text{if } D \leq \sigma x^2 \\ 0 & , \text{if } D > \sigma x^2 \end{cases}$$

(P, D) is admissible if there is a coding scheme that can achieve a distortion close to D while satisfying the transmission cost constraints.

Minimize P (D) Subject to (P,D)=(P1, P2, P3.....PL, D)

IV. Optimal Power Allocation

In a Wireless Sensor Network, many sensor devices are battery-powered and thus power constrained. Thus, minimizing power consumption is critical in maximizing the lifetime of the individual sensor nodes and the entire network. A sum-power constraint has direct impact on the network lifetime since it is imposed on the power consumption of all sensors taken together. However, in order to prevent excessive power consumption for individual sensors, or due to differing power supply capabilities at the sensors, individual power constraints for each sensor may also be required. This motivates us to combine cost coefficients with the sum-power constraint and consider a complexity constraint consisting of linear constraint power. These cost constraints might depend on the sensors' location and battery lifetime and can be assigned by the fusion center. In this paper, we consider an optimal power allocation for uncoded transmission in order to minimize the Mean square error distortion under sum power constraint.

V. Results and Discussions:

The goal is to minimize the mean squared error

$$D = \frac{1}{L} \sum_{l=1}^L E [|X_o - X_e|^2]$$

As Power depends on the distortion occurs in sensor network

$$\sum_{j=1}^j \sum_{l=1}^L E [|Xm[j] |^2] \leq J \frac{P_{tot}}{K}$$

P_{tot} = is the average total sensor power available per observation vector

For the symmetric Case: $N_1=N_2=N, P_1=P_2=P$

$$\frac{Du - Dl}{Du} = \left[(\sigma_x^2 (\sigma_z^2 + \sum_{i=1}^L P_i N_i) / (\frac{P_i N_i}{\sigma_x^2 + N_i})) / (\sigma_z^2 + \varphi) - D^* (1 + (\sigma_x^2 \sigma_z^2 \sum_{i=1}^L \frac{1}{N_i})) / (\sigma_z^2 + \varphi) \right] / (\sigma_x^2 (\sigma_z^2 + \sum_{i=1}^L P_i N_i) / (\frac{P_i N_i}{\sigma_x^2 + N_i})) / (\sigma_z^2 + \varphi)$$

Where , $D^* = (\frac{1}{\sigma_x^2} + \sum_{i=1}^L \frac{1}{N_i})^{-1}$

$$\Phi = \sum_{i=1}^L P_i + 2 \sigma_x^2 \sum_{i=1}^L \sum_{j>i}^L (P_i P_j) / (\sigma_x^2 + N_i) N_j$$

For the asymmetric Case: $N_1 \neq N_2, P_1 \neq P_2$

$$\frac{Du - Dl}{Du} = \frac{\sigma_x^2}{N} \frac{Ptot}{\sigma_z^2} / (1 + 2 \frac{\sigma_x^2}{N}) (1 + \frac{\sigma_x^2}{N} + \frac{Ptot}{\sigma_z^2}) (1 - 2\tau(1-\tau))$$

Where

σ_x^2 = samples of Source 'X' Which are i.i.d over time 't'

σ_z^2 = Zero mean variance

X(t) = is source data sequence for t=1,2,3

$X(t) \sim \mathfrak{N}(0, \sigma_x^2)$

In Figure1 we have analyzed the performance of Gaussian sensor network by plotting the percentage of the relative distortion gap between the upper and the lower bounds versus the value of $\delta=N_2-N_1$, parametrized by the value of N_1 . For $\delta = 0$, the gap coincide and the system becomes symmetric and the gap approaches zero. In Figure2, we analyzed by plotting the percentage of the relative distortion gap between the upper and the lower bounds versus the value of $\delta=N_2-N_1$, parametrized by the value of P_1 and keeping the noise at same level by taking $N_1=N_2=29, \sigma_x^2=100, \sigma_z^2=20$. In Figure 3, we analyzed by using $N_1=20, N_2=25, P_1=25, P_2=28, L=20, \sigma_x^2=100, \sigma_z^2=20$ and found that when noise variances and power are parametrized results in almost optimal performance as the number of sensors increases. while

in Figure 4 observed that by keeping the noise variance and power at same level $N_1=N_2=25, P=50, L=20, \sigma_x^2=100, \sigma_z^2=20$ found that as the number of sensor increases results in decrease in distortion Gap and completely optimal performance in symmetric environment.

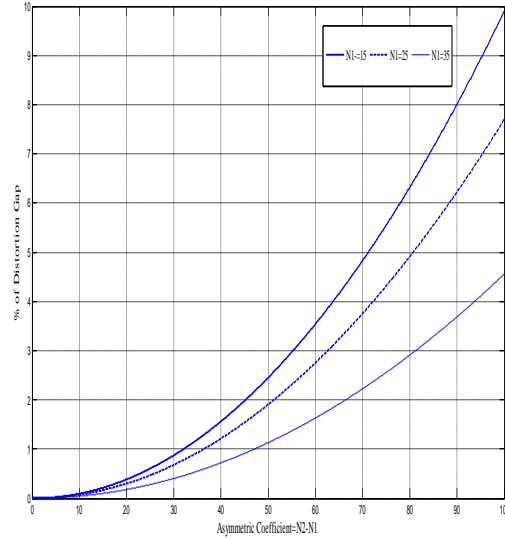


Figure2:Relative % of Distortion Gap vs Asymmetric Coefficient in which N_1 is parametrized

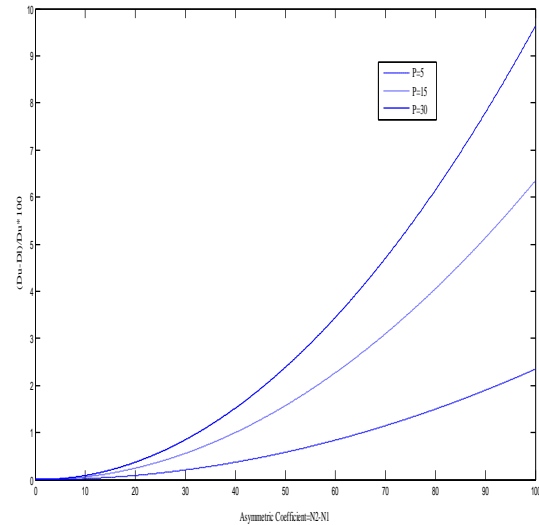


Figure 3:Relative % of Distortion Gap vs Asymmetric Coefficient in which 'P' is parametrized

.Simulation result shows the superiority of our work in terms of power utilization.

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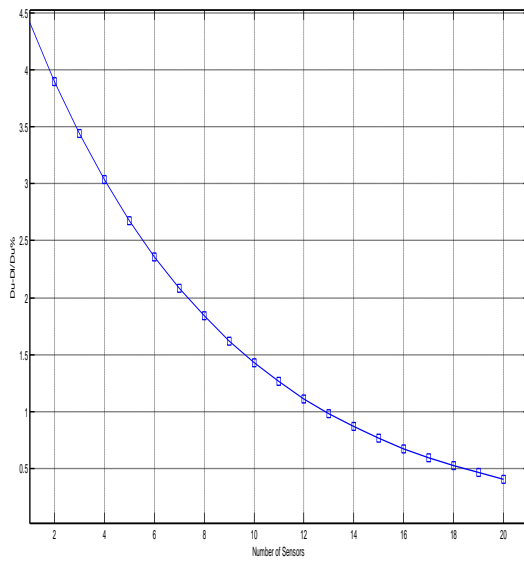


Figure 4:Relative % of Distortion Gap vs Number of Sensors'L' for asymmetric environment and achieves the almost optimality

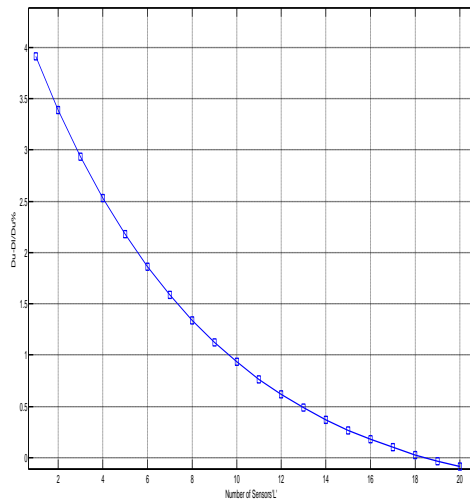


Figure 4:Relative % of Distortion Gap vs Number of Sensors'L' for symmetric environment is completely Optimal.

VI.Conclusion

We consider the asymmetric environment in which sensors have differing noise variances and transmission power. Our goal wants to reconstruct the main source with smallest possible power (energy) with an average distortion (D) with respect to mean square error distortion. We have closed a small gap between upper and lower bounds