

# Optimal Co-Operative Beamforming Design for Enhanced Information Rate and Reduced Computing Time with Large Number of Antennas

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**Abstract-**Multiple input multiple output (MIMO) relay channels has great application potentials in wireless communication systems and therefore attracts both academia and industrial attentions recently. In this paper, we consider the multiple relay channels where both source node and relay node are equipped with multiple antennas and the destination node has a single antenna and a transmit beamforming design for multiple-input-multiple output decode-and-forward (DF) half duplex two-hop relay channels with a direct source-destination link. We formulate and solve the optimal beamforming vectors for different system configurations. The mathematical derivation, based on which we develop a systematic approach to arrive at the optimal beamforming vectors are also mentioned. The method of improving information rate and an optimal Beamforming scheme to plot the average computational time versus the number of antennas N is carried out using numerical results.

**Keywords-**Beamforming Design, Multiple Input Multiple Output, Decode and Forward Relay channels

## I. INTRODUCTION

Recently, the potentials of relay techniques in terms of coverage extension and spectral efficiency improvement are being actively explored for wireless communications systems. Standardization organizations, such as 3GPP and IEEE 802.16 task group, have incorporated relay transmission mode into their standard proposals for next generation wireless systems. In general, there are two popular relaying strategies: i) decode-and-forward (DF) strategy, where the relay decodes the signal that it received and then transmits the re-encoded information; ii) amplify and-forward (AF) strategy, where the relay simply amplifies its received signal. These two strategies correspond, respectively, to Type I and Type II relaying in LTE Advanced and non-transparency and transparency relaying in WiMax standards. The DF strategy demands relay nodes with greater signal processing capabilities and outperform AF strategy when the source-relay channel is statistically better than the source-destination and relay-destination channels.

Meanwhile, deploying multiple antennas at wireless terminals can bring many desirable benefits into wireless communication systems, such as boosting the channel capacity via transmit spatial multiplexing and/or enhancing the transmission reliability through spatial modulation and diversity combining. The resulting so-called multiple-input multiple-output (MIMO) technology has been widely investigated over the past decades. MIMO schemes have also been included in standard proposals for next generation mobile broadband communication systems, e.g., 3GPP LTE-Advanced and WiMAX. As such, there has been a growing interest in MIMO relay channels, where the source node, the relay node, and/or the destination node may have multiple antennas.

Smart antenna technology offers a significantly improved solution to reduce interference levels and improve the system space. With this technology, each user's signal is transmitted and received by the base station only in the direction of that precise user. This drastically reduces the overall obstruction in the system. A smart antenna system as shown in Fig.1 consists of an array of antennas that together direct different transmission/reception beams toward each user in the system. This method of communication is called beamforming and is made possible through smart (advanced) signal processing at the baseband. In spatial filtering, each user's signal is aggregated with complex weights that adjust the magnitude and phase of the signal to and from each antenna. This causes the array output of antennas to form a transmit/receive beam in the desired direction and minimizes the output in other directions.

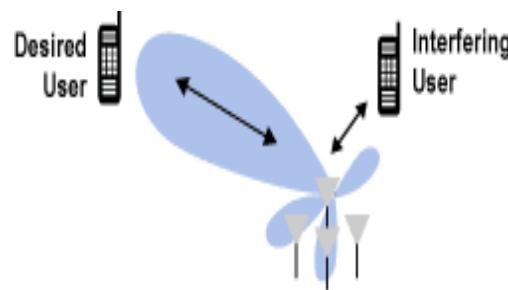


Figure 1.1 Smart Antenna System—Beamforming

### A. Existing Work

Earlier works on MIMO relay channels focused on the network capacity, but recently, MIMO relay beamforming design has started to focus for MIMO AF and DF relay channels. Most work on the DF MIMO relay beamforming assumed that no direct link exists between the source and the destination. This assumption leads to a simpler joint source-relay beamforming design.

Several recent work addressed the joint source and relay beamforming design in two-hop DF MIMO relay channels with the consideration of source-destination direct link. When a direct link is involved, the received signals via the source-to-destination link and the relay-to-destination link will be combined at destination to strengthen the received signal. As such, the beamforming vector design at the source has to consider both links. A balanced linear precoding (BLP) algorithm for MIMO DF relay channels is designed, where the source, the relay and the destination are equipped with multiple antennas at the same time. By using this algorithm, the complexity of the beamforming design can be reduced from  $O(N^6)$  of traditional method to  $O(N^3)$ . This algorithm is iterative, and non-explicit expressions on the optimal solutions were derived. In this iterative algorithm, to achieve simplicity, the complex coefficients of the linear combination were approximated by real numbers.

### B. Proposed Work

This paper considers the joint source-relay beamforming design for the three-node MIMO DF relay network with source-destination link. We assume that both the source and relay nodes are equipped with multiple antennas while the destination node is only deployed with multiple antenna. Our design aims to fully explore the special diversity advantage of MIMO DF relay channel to enhance system throughput to the destination node.

The contributions of our work are summarized as follows. First, we formulate an optimization problem on the joint source and relay beamforming design for the MIMO DF relay channel, which actually is a max-min fairness optimization problem. We first examine the properties of the optimal solutions and effectively separate the phase angle design and real norm design problems for the optimal vectors. Based these properties, we solve the optimal beamforming design problem in three cases. For the first and the second cases, we derive the explicit expressions for the optimal solutions. For the third case, as it is hard to drive the unified explicit result, we further divided it into three different sub cases in terms of the number of antennas deployed at the source and relay nodes. For the scenario that two antennas are deployed at both the source and the relay nodes and single antenna is deployed at the destination node, i.e., 2:2:1 scenario, we derived the explicit expression of the

optimal solution. For the scenario where  $N_S > 1$  antennas are deployed at the source and only one antenna is equipped on the relay and the destination nodes, i.e.,  $N_S:1:1$  scenario, we present a non-iterative numerical method to calculate the optimal solution. For the general  $N_S: N_R:1$  scenarios, we first fix the SNR of the source-relay link and design the optimal beamforming vector to maximize the SNR of source-destination link. Then, we design a Bisection based algorithm to find the optimal solution. Finally, we arrive at an optimal beamforming design solution. Extensive simulation results show that our proposed solution can achieve the optimal beamforming design for the MIMO DF relay channel with low complexity.

## II. SYSTEM DESIGN

Consider a network model consisting of a source S, a relay R and a destination D. It is assumed that the direct link between S and D exists in the system and the relay R helps the Information transmission from S to D.

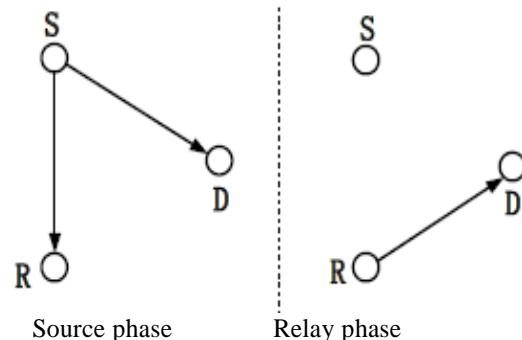


Figure 2.1 Two-phase DF relay transmission

Multiple antennas are deployed both at S and R, and only one antenna is equipped at D. Half duplex mode is adopted so that R cannot transmit and receive signals at the same time. Therefore, each round of information transmission from S to D can be divided into two phases, i.e., a source phase and a relay phase, as illustrated in Figure 2.1. In the source phase, S broadcasts its information to both R and D, while in the relay phase, R decodes the received information and then forwards the decoded information to D. Thus, D can obtain the desired information by decoding the combined signals received over the aforementioned two phases. We assume that all channel state information (CSI) for each round of transmission is known at the transmitters, by using techniques such as channel training, feedback and channel reciprocity exploiting, etc. As a result, S and R can configure their beamforming vectors accordingly to achieve the best transmission performance. Without loss of generality, the transmitted information from S and R can be represented by symbols  $X_S$  and  $X_R$ , respectively. Assume that  $N_S$  antennas and  $N_R$  are deployed at S and R, respectively. Then, the information transmission process mentioned above can be specifically described as follows.

In the source phase, the information symbol  $X_S$  is first multiplied with a beamforming vector  $W_S = [W_{S,1}, W_{S,2}, \dots, W_{S,N_S}]^T$  before being transmitted from the  $N_S$  transmit antennas of source S and assume that symbol  $X_S$  is Gaussianly coded with average power  $P_S$  and  $W_S$  is a unit-norm vector, where

$$\|W_S\|^2 = \|W_{S,1}\|^2 + \|W_{S,2}\|^2 + \dots + \|W_{S,N_S}\|^2 = 1$$

Thus, the complex baseband received symbol at D in the source phase is mathematically given by

$$y_{D,S} = h_{D,S}^T W_S X_S + n_{D,S} \quad (1)$$

Where  $h_{D,S}$  denotes the channel gain vector from S to D and  $n_{D,S}$  is scalar additive Gaussian noise with unit variance. The received signal to noise ratio (SNR) at D during the source phase is given by

$$S_{D,S} = \|h_{D,S}^T W_S\|^2 P_S \quad (2)$$

Meanwhile, the complex baseband signal vector received at N antennas of node R can be mathematically given by

$$y_{R,S} = H_{R,S} W_S X_S + n_{R,S} \quad (3)$$

Where  $H_{R,S}$  denotes the channel gain matrix from source S to relay R. Applying singular value decomposition (SVD), we can rewrite  $H_{R,S}$  as

$$H_{R,S} = U \Sigma V^H \quad (4)$$

Where  $\Sigma$  is an  $N_R \times N_S$  diagonal matrix with non-negative real numbers on the diagonal and U and V are two complex unitary matrices with the dimensions of  $N_R \times N_R$  and  $N_S \times N_S$ .

The effective received SNR at R is

$$S_{R,S} = \|H_{R,S} W_S\|^2 P_S \quad (5)$$

So the achievable information rate from S to R is

$$C_R = \log_2 (1 + S_{R,S}) \quad (6)$$

In the relay phase, R forwards its decoded symbol  $x_r$  to D using beamforming transmission over its  $N_R$  antennas. Similar to the source phase, symbol  $x_r$  is also Gaussianly coded, but with average power  $P_r$ . Specifically, the symbol  $x_r$  is multiplied with a unit-norm beamforming vector  $W_r = [W_{r,1}, W_{r,2}, \dots, W_{r,N_r}]^T$  satisfying

$$\|W_r\|^2 = \|W_{r,1}\|^2 + \|W_{r,2}\|^2 + \dots + \|W_{r,N_r}\|^2 = 1$$

and then transmitted via  $N_R$  antennas. The complex symbol received at D in the relay phase is given by

$$y_{D,R} = h_{D,R}^T W_r x_r + n_{D,R} \quad (7)$$

Where  $n_{D,R}$  is scalar additive Gaussian noise with unit variance and  $h_{D,R}$  denotes the channel gain vector from R to D. The received SNR at D in the relay phase is given by

$$S_{D,R} = \|h_{D,R}^T W_r\|^2 P_r \quad (8)$$

The achievable information rate at D in the relay phase is

$$C_D = \log_2 (1 + S_{D,S} + S_{D,R}) \quad (9)$$

The maximum information rate from source S to destination D over the half-duplex DF relay channel is bounded by  $C_R$  and  $C_D$ , the total achievable information transmission rate at D from S over the MIMO relay channel can be given by

$$C_{DF} = \frac{1}{2} \min \{C_R, C_D\} = \frac{1}{2} \min \{\log_2 (1 + S_{R,S}), \log_2 (1 + S_{D,S} + S_{D,R})\} \quad (10)$$

Where the pre-log parameter, 1/2, actually captures the time-division feature of the half-duplex relay system.

### III. OPTIMAL BEAMFORMING DESIGN

In this section, we try to simplify the computational complexity of optimal beamforming design for MIMO DF relay channels. We do this from a perspective of communication scenario, i.e., the deployed number of antennas. After this, we shall present the final framework of our proposed optimal beamforming design scheme.

#### A. Optimal Beamforming Design for 2:2:1 Scenario

This section considers the beamforming design for a special scenario where both the source and the relay are equipped with two antennas, i.e., 2:2:1 scenario. The deployment of two antennas is more realistic than deploying two more antennas in many practical cases. Especially on some size-limited devices such as notebook PC.

Secondly, the 2:2:1 MIMO scenario also applies to the situation that only two antennas are selected to transmit signals at the source and relay nodes and 2x2 MIMO channel has attracted much attention.

#### B. Optimal Beamforming Design for Ns: 1:1 Scenario

We now consider another special case where  $N_S$  antennas are deployed at the source and a single antenna is employed at both the relay node and the destination, i.e.,  $N_S: 1:1$  scenario.

#### IV. SIMULATION RESULTS

In this section, we show the effectiveness of our proposed optimal beamforming design for MIMO DF relay channels through numerical examples. Without loss of generality, we assume  $P_s = P_r = P_{in}$  in all simulations.

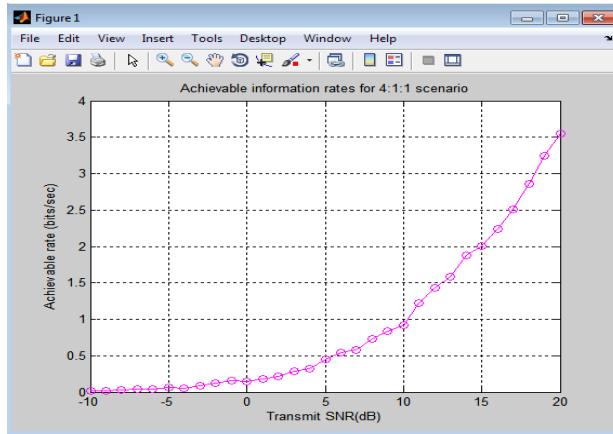


Figure 3.1 Plot of achievable transmission rate versus transmits SNR for the 4:1:1 scenario.

In Figure 3.1, we plotted for  $N_s: 1: 1$  scenario, where  $N$  is 4. The results are averaged over 1000 independent simulations. From the figure 3.1 we can see notable performance gains can be achieved. Moreover, it can be observed that, more antennas lead to higher achievable information rates and larger performance gains for MIMO DF relaying system.

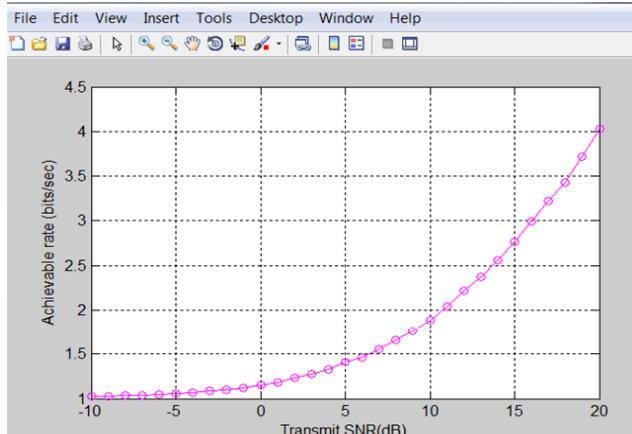


Figure 3.2 Plot of achievable transmission rate versus transmit SNR for the 2:2:1 scenario.

In Figure 3.2, 3.3, 3.4, 3.5 plot the achievable information rates of our proposed optimal beamforming scheme for  $N_s: N_r: 1$  scenarios, where in the simulation of Figure 3.3  $N_s = N_r = 2$ , in Figure 3.4  $N_s = N_r = 4$ , in Figure 3.5  $N_s = 2$ ,  $N_r = 4$  and In Figure 3.6  $N_s = 4$  and  $N_r = 2$ . The results presented in the four figures are also obtained by averaging 1000 independent channel realizations.

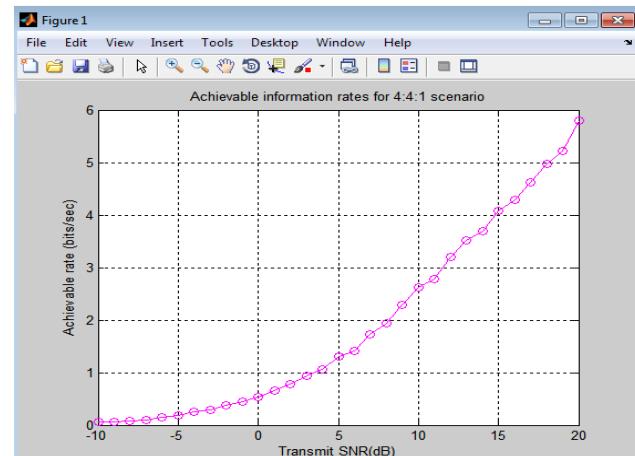


Figure 3.3 Plot of achievable transmission rate versus transmits SNR for the 4:4:1 scenario

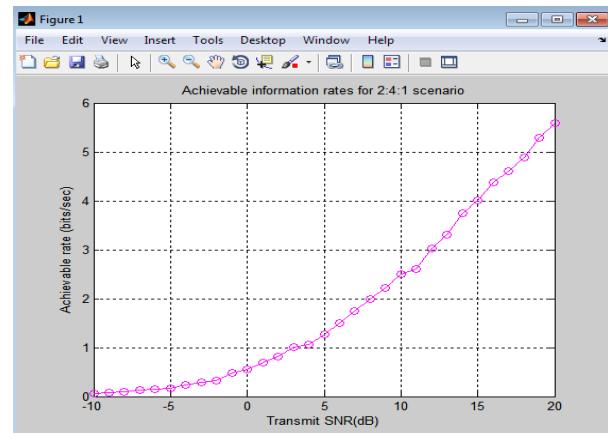


Figure 3.4 Plot of achievable transmission rate versus transmits SNR for the 2:4:1 scenario.

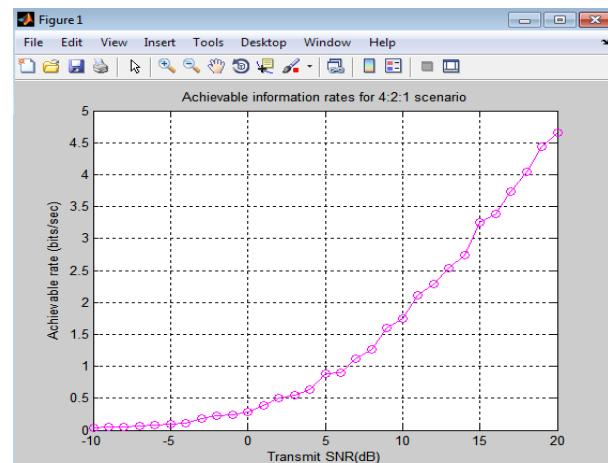


Figure 3.5 Plot of achievable transmission rate versus transmits SNR for the 4:2:1 scenario

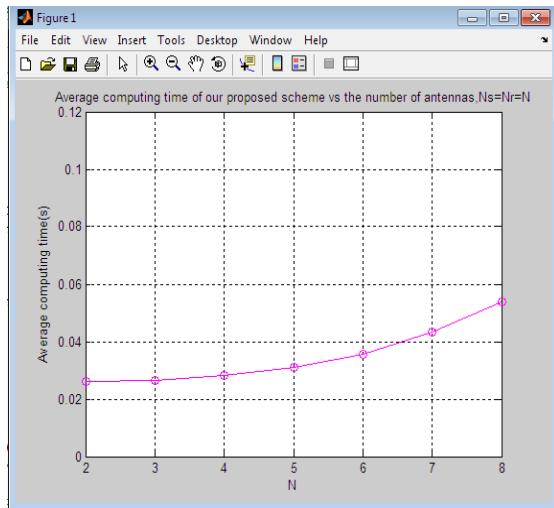


Figure 3.6 Average computing time of our proposed scheme vs. the number of antennas NS=Nr=N

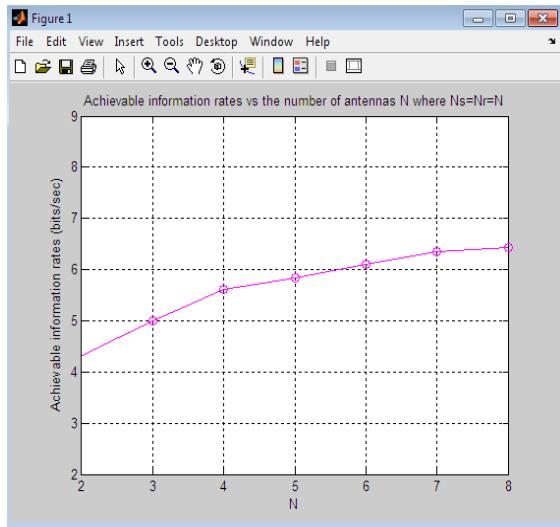


Figure 3.7 Achievable information rate vs. the number of antennas N where NS=Nr=N

In figure 3.6, 3.7 average computing time and achievable information rate is shown according to the number of antennas.

## V. CONCLUSION

In this paper, the Beamforming design for MIMO DF relay channels, where both the source node and relay node are equipped with multiple antennas are considered and developed an efficient scheme to solve the optimization problem and determine the optimal Beamforming vector for MIMO DF relay networks. The Beamforming design for MIMO DF channels, which were derived here, was based on the exact capacity formulation. An extension to the Beamforming design to the scenario where all the three nodes are equipped with multiple antennas to increase the achievable information rate and an optimal Beamforming scheme to plot the

average computational time versus the number of antennas N and the results is averaged over 1000 independent simulations and to show that the scheme achieves much lower complexity compared with BLP and optimal design for single antennas at destination.

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