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Operations of Bipolar Fuzzy Soft Graph

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***Abstract*—Numerical demonstrating, investigation and processing of issues with uncertainty is one of the most sizzling regions in interdisciplinary research including connected arithmetic, computational insight and choice sciences. It is significant that uncertainty emerges from different areas has altogether different nature and can't be caught inside a solitary numerical system. In this paper we initiate bipolar fuzzy soft graphs, vertex and edge- induced soft graphs and some operation of bipolar soft fuzzy graph are explore.**

Keywords*— Bipolar fuzzy soft graphs, strong bipolar fuzzy soft graphs, complete bipolar fuzzy soft graphs, regular bipolar fuzzy soft graphs.

I. INTRODUCTION

Soft set theory was introduced by Molodtsov[1]. Later Feng, Liu and Fotea combined soft set with fuzzy set and rough set. In 1975[1] Rosenfeld initiated fuzzy graph theory. During the same time various concepts in connectedness with fuzzy graphs was introduced by Yeh and Bang [2]. The concept of bipolar fuzzy graphs was introduced by Akramin. In this paper we initiate bipolar fuzzy soft graphs, vertex and edge- induced soft graphs and some operation of bipolar soft fuzzy graph.

II. PRELIMINARIES

Definition 2.1. If $A \subset$ P, then the fuzzy soft set over the universe is a pair of (M,A) where $M: A \to C^U$, C^U is a gathering of fuzzy subset of U.

Definition 2.2. [1] Let S be the universal set and P is the set of parameter then (M,P) is called soft set over S where M: P→P(S).

Definition 2.2 [15]: If $A \subset$ P, then the fuzzy soft set over the universe is a pair of (M,A) where $M: A \to C^S$, C^S is a gathering of fuzzy subset of S.

Definition 2.3. If $A \subset$ P, then the bipolar fuzzy soft set over the universe is a pair of (M, A) where $M: A \to BF^S$ where BF^S is the collection of all bipolar fuzzy subsets of S. It is defined by

$$(F,A) = F(e_i)$$

$$F(e_i) = \{c_i, \mu^+(c_i), \mu^-(c_i) : \forall (c_i) \in U, \forall (e_i) \in A\}$$

Definition 2.4. An intersection of two bipolar fuzzy soft sets (F, A) and (G, B) is a bipolar fuzzy soft set (H, C), where C = A ∩ B /= ∅ and H : C → BF U is defined by H (e) = F (e) ∩ G (e) ∀ e ∈ C and denoted by (H, C) = (F, A)∩$^{-}$ (G, B).

Definition 2.5[4]: Union of two bipolar fuzzy soft sets over a common universe U is a bipolar fuzzy soft set (H, C), where C=A∪B and $H: C \to BF^U$ is defined by

H(e) = F(e) if e ∈A\B
= G(e) if e ∈B\A
= F(e) ∪G(e) if $e \in A \cap B$

Denoted as $(F,A)\tilde{\cup}(G,B) = (H,C)$

III. MAIN RESULT

Definition 3.1. An bipolar fuzzy soft graph $\tilde{G} = \left(G^*, \tilde{J}_{\varepsilon^P,\eta^N}, \tilde{L}_{\rho^P,\xi^N}, A\right)$ is such that

a) G^* = (V, E) is a simple graph
b) A is a nonempty set of parameters
c) $\left(\tilde{J}_{\varepsilon^P,\eta^N}, A\right)$ is a bipolar fuzzy soft set over V
d) $\left(\tilde{L}_{\rho^P,\xi^N}, A\right)$ is a bipolar fuzzy soft set over E
e) $\left(\tilde{J}_{\varepsilon^P,\eta^N}, \tilde{L}_{\rho^P,\xi^N}\right)$ is a bipolar fuzzy (sub)graph of G^* for all $a \in A$. That is

$$\tilde{L}_{\rho^P}(a)(xy) \le \min\{\tilde{J}_{\varepsilon^P}(a)(x), \tilde{J}_{\varepsilon^P}(a)(y)\}$$

$$\tilde{L}_{\varsigma^N}(a)(xy) \ge \max\{\tilde{J}_{\eta^N}(a)(x), \tilde{J}_{\eta^N}(a)(y)\} \forall a \in A; x, y \in V$$

The bipolar fuzzy soft graph is denoted by $\tilde{B}_{\varsigma,\sigma}(a)$.

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Definition 3.2. An bipolar fuzzy soft graph $\widetilde{G}=\left(G^{*},\widetilde{J}_{\varepsilon^{P},\eta^{N}},\widetilde{L}_{\rho^{P},\xi^{N}},A\right)$ is said to be vertex induced if $\widetilde{B}_{\varsigma,\sigma}(a)=\left(\widetilde{J}_{\varepsilon^{P},\eta^{N}},\widetilde{L}_{\rho^{P},\xi^{N}}\right)=\widetilde{J}_{\varepsilon^{P},\eta^{N}}\quad\forall a\in A.$

Definition 3.3. An bipolar fuzzy soft graph $\widetilde{G}=\left(G^{*},\widetilde{J}_{\varepsilon^{P},\eta^{N}},\widetilde{L}_{\rho^{P},\xi^{N}},A\right)$ is said to be edge induced if $\widetilde{B}_{\varsigma,\sigma}(a)=\left(\widetilde{J}_{\varepsilon^{P},\eta^{N}},\widetilde{L}_{\rho^{P},\xi^{N}}\right)=\widetilde{L}_{\rho^{P},\xi^{N}}\quad\forall a\in A.$

Example3.4.

Consider the bipolar fuzzy graph G^{*}.The parameter set is denoted by $A=\{e_1,e_3,e_5\},\left(\widetilde{J}_{\varepsilon^{P},\eta^{N}},A\right)$ and $\left(\widetilde{L}_{\rho^{P},\xi^{N}},A\right)$ is a bipolar fuzzy soft set over V and E respectively, with bipolar fuzzy approximation function $\widetilde{J}_{\varepsilon^{P},\eta^{N}}:A\to BF^{U}$ and $\widetilde{L}_{\rho^{P},\xi^{N}}:A\to BF^{U}$

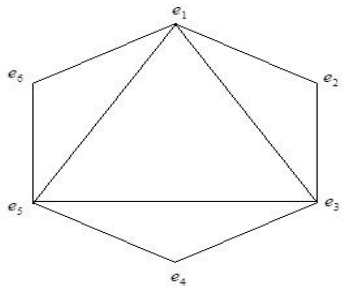

Fig.3.1. Simple Graph G^{*}

$$\widetilde{J}_{\varepsilon^{P},\eta^{N}}(a)=\left\{b\in V/aRb\Leftrightarrow\widetilde{d}_{\varepsilon^{P},\eta^{N}}(a,b)=(1,-1)\right\}$$
$\forall a\in A$

i.e. $\widetilde{J}_{\varepsilon^{P},\eta^{N}}(e_1)=\{e_2,e_3,e_5,e_6\},$
$\widetilde{J}_{\varepsilon^{P},\eta^{N}}(e_3)=\{e_1,e_2,e_5,e_6\}$ and
$\widetilde{J}_{\varepsilon^{P},\eta^{N}}(e_5)=\{e_1,e_3,e_4,e_6\}$

The edges $\widetilde{L}_{\rho^{P},\xi^{N}}(a)=\left\{uv\in E/\{u,v\}\subseteq\widetilde{J}_{\varepsilon^{P},\eta^{N}}(a)\right\}$

$$\widetilde{L}_{\rho^{P},\xi^{N}}(e_1)=\{e_2e_3,e_3e_5,e_5e_6\},$$
$$\widetilde{L}_{\rho^{P},\xi^{N}}(e_3)=\{e_2e_1,e_1e_5,e_5e_4\}\text{ and}$$
$$\widetilde{L}_{\rho^{P},\xi^{N}}(e_5)=\{e_4e_3,e_3e_1,e_1e_6\}$$

Thus,

$$\widetilde{B}_{\varsigma^{P},\sigma^{N}}(e_1)=\left(\widetilde{J}_{\varepsilon^{P},\eta^{N}}(e_1),\widetilde{L}_{\rho^{P},\xi^{N}}(e_1)\right),$$
$$\widetilde{B}_{\varsigma^{P},\sigma^{N}}(e_3)=\left(\widetilde{J}_{\varepsilon^{P},\eta^{N}}(e_3),\widetilde{L}_{\rho^{P},\xi^{N}}(e_3)\right)and$$
$$\widetilde{B}_{\varsigma^{P},\sigma^{N}}(e_5)=\left(\widetilde{J}_{\varepsilon^{P},\eta^{N}}(e_5),\widetilde{L}_{\rho^{P},\xi^{N}}(e_5)\right)$$

are bipolar fuzzy soft graphs of G^{*}.Tabular representation of vertex and edges of bipolar fuzzy soft graphs.

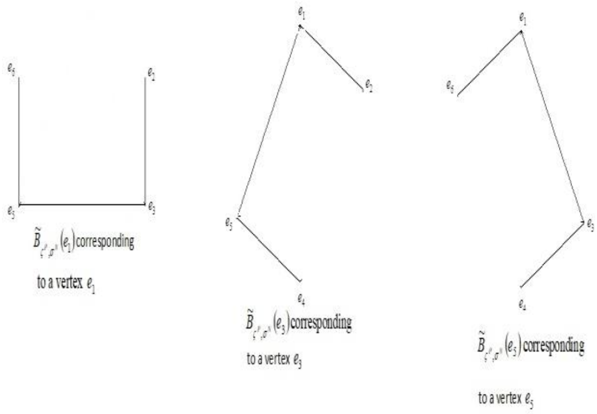

Fig.3.2.Subgraphs $\widetilde{B}_{\varsigma^{P},\sigma^{N}}(e_1)$, $\widetilde{B}_{\varsigma^{P},\sigma^{N}}(e_3)$, $\widetilde{B}_{\varsigma^{P},\sigma^{N}}(e_5)$

A/V	e_1	e_2	e_3	e_4	e_5	e_6
e_1	(0,0)	(1,-1)	(1,-1)	(0,0)	(1,-1)	(1,-1)
e_3	(1,-1)	(1,-1)	(0,0)	(1,-1)	(1,-1)	(0,0)
e_5	(1,-1)	(0,0)	(1,-1)	(1,-1)	(0,0)	(1,-1)

A/E	e_1e_2	e_2e_3	e_3e_4	e_4e_5	e_5e_6	e_6e_1
e_1	(0,0)	(1,-1)	(0,0)	(0,0)	(1,-1)	(0,0)
e_3	(1,-1)	(0,0)	(0,0)	(1,-1)	(0,0)	(0,0)
e_5	(0,0)	(0,0)	(1,-1)	(0,0)	(0,0)	(1,-1)

A/E	e_1e_3	e_3e_5	e_5e_1
e_1	(0,0)	(1,-1)	(0,0)
e_3	(0,0)	(0,0)	(1,-1)
e_5	(1,-1)	(0,0)	(0,0)

Definition3.5.Let $\widetilde{G}_1=\left\langle\widetilde{J}_{\varepsilon_1^{P},\eta_1^{N}},\widetilde{L}_{\rho_1^{P},\xi_1^{N}},A\right\rangle$ and $\widetilde{G}_2=\left\langle\widetilde{J}_{\varepsilon_2^{P},\eta_2^{N}},\widetilde{L}_{\rho_2^{P},\xi_2^{N}},B\right\rangle$ be the two bipolar fuzzy soft graph .Then $\widetilde{G}_2$ is the bipolar fuzzy soft subgraph of $\widetilde{G}_1$.

$i, B \subseteq A$

$ii, B_{\beta_2^P,\delta_2^N}(x)$ is the subgraph of $B_{\beta_1^P,\delta_1^N}(x) \forall x \in B$

Example3.6.Consider the bipolar fuzzy graph G^*.The parameter of two set is denoted by $A = \{e_1, e_2, e_5\}$ and $B = \{e_1, e_2\}, (\widetilde{J}_{\varepsilon^P,\eta^N}, A)$ and $(\widetilde{L}_{\rho^P,\xi^N}, A)$ is a bipolar fuzzy soft set over V and E respectively, with bipolar fuzzy approximation function $\widetilde{J}_{\varepsilon^P,\eta^N} : A \to BF^U$ and $\widetilde{L}_{\rho^P,\xi^N} : A \to BF^U$

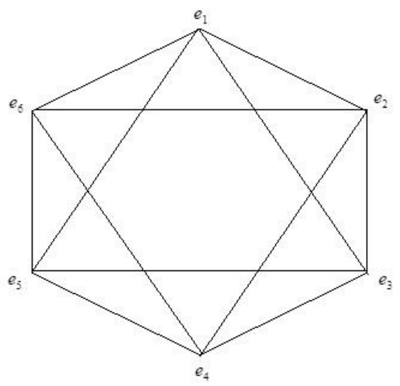

Fig.3.3. Simple Graph G^*

$$\widetilde{J}_{\varepsilon^P,\eta^N}(a) = \{b \in V / aRb \Leftrightarrow \widetilde{d}_{\varepsilon^P,\eta^N}(a,b) \leq (1,-1)\}$$
$\forall a \in A$

i.e., $\widetilde{J}_{\varepsilon^P,\eta^N}(e_1) = \{e_1, e_2, e_3, e_5, e_6\}$,

$\widetilde{J}_{\varepsilon^P,\eta^N}(e_2) = \{e_1, e_2, e_3, e_4, e_6\}$ and

$\widetilde{J}_{\varepsilon^P,\eta^N}(e_5) = \{e_1, e_3, e_4, e_5, e_6\}$

The edges

$$\widetilde{L}_{\rho^P,\xi^N}(a) = \{uv \in E / \{u,v\} \subseteq \widetilde{J}_{\varepsilon^P,\eta^N}(a)\} \forall a \in A$$

$\widetilde{L}_{\rho^P,\xi^N}(e_1) = \{e_1e_2, e_2e_3, e_3e_5, e_5e_6, e_6e_2, e_6e_1, e_5e_1, e_1e_3\}$,

$\widetilde{L}_{\rho^P,\xi^N}(e_2) = \{e_1e_2, e_2e_3, e_3e_4, e_4e_6, e_6e_1, e_1e_3, e_2e_4, e_2e_6\}$ and

$\widetilde{L}_{\rho^P,\xi^N}(e_5) = \{e_1e_3, e_3e_4, e_4e_5, e_5e_6, e_6e_1, e_1e_5, e_5e_3, e_4e_6\}$

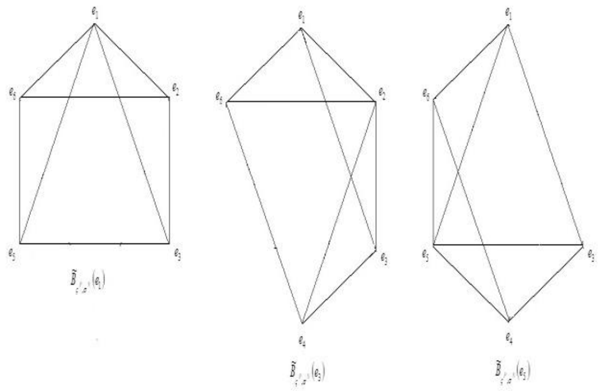

Fig.3.4. Subgraphs $\widetilde{B}_{\varsigma^P,\sigma^N}(e_1)$, $\widetilde{B}_{\varsigma^P,\sigma^N}(e_2)$, $\widetilde{B}_{\varsigma^P,\sigma^N}(e_5)$

Now the bipolar fuzzy approximation function $\widetilde{J}_{\varepsilon^P,\eta^N} : B \to BF^U$ and $\widetilde{L}_{\rho^P,\xi^N} : B \to BF^U$

$$\widetilde{J}_{\varepsilon^P,\eta^N}(a) = \{b \in V / aRb \Leftrightarrow \widetilde{d}_{\varepsilon^P,\eta^N}(a,b) = (1,-1)\}$$
$\forall a \in B$

Then $\widetilde{J}_{\varepsilon^P,\eta^N}(e_1) = \{e_2, e_3, e_5, e_6\}$,

$\widetilde{J}_{\varepsilon^P,\eta^N}(e_2) = \{e_1, e_3, e_4, e_6\}$

The edges

$$\widetilde{L}_{\rho^P,\xi^N}(a) = \{uv \in E / \{u,v\} \subseteq \widetilde{J}_{\varepsilon^P,\eta^N}(a)\} \forall a \in B$$

$\widetilde{L}_{\rho^P,\xi^N}(e_1) = \{e_2e_3, e_3e_5, e_5e_6, e_6e_2\}$ and

$\widetilde{L}_{\rho^P,\xi^N}(e_2) = \{e_4e_6, e_6e_1, e_1e_3, e_3e_4\}$. Hence

$B \subseteq A$ and

$B_{\beta_2^P,\delta_2^N}(a)$ is the subgraph of $B_{\beta_1^P,\delta_1^N}(a) \forall a \in B$

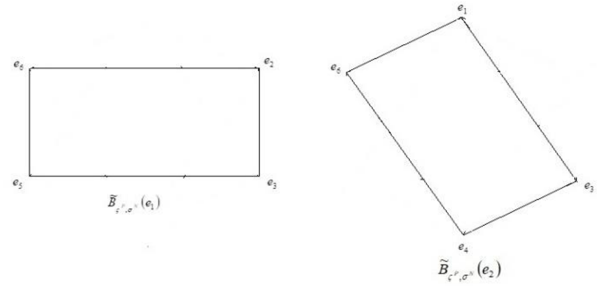

Fig.5 Subgraphs $\widetilde{B}_{\varsigma^P,\sigma^N}(e_1)$, $\widetilde{B}_{\varsigma^P,\sigma^N}(e_2)$

Theorem 3.7. Let $\widetilde{G}_1 = \langle \widetilde{J}_{\varepsilon_1^P,\eta_1^N}, \widetilde{L}_{\rho_1^P,\xi_1^N}, A \rangle$ and $\widetilde{G}_2 = \langle \widetilde{J}_{\varepsilon_2^P,\eta_2^N}, \widetilde{L}_{\rho_2^P,\xi_2^N}, B \rangle$ be the two bipolar fuzzy soft graph .Then $\widetilde{G}_2$ is the bipolar fuzzy soft subgraph of $\widetilde{G}_1$ if

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and only if $\widetilde{J}_{\varepsilon_2^P,\eta_2^N}(a) \subseteq \widetilde{J}_{\varepsilon_1^P,\eta_1^N}(a)$ and $\widetilde{L}_{\rho_2^P,\xi_2^N}(a) \subseteq \widetilde{L}_{\rho_1^P,\xi_1^N}(a)\ \forall a \in B$.

Proof:

Suppose $\widetilde{G}_2$ is the bipolar fuzzy soft subgraph of $\widetilde{G}_1$.

$$i, B \subseteq A$$
$$ii, \widetilde{B}_{\varsigma^P,\sigma^N}(e_2) = \left(\widetilde{J}_{\varepsilon^P,\eta^N}(e_2), \widetilde{L}_{\rho^P,\xi^N}(e_2)\right)$$
$$\text{is the subgraph of}$$
$$\widetilde{B}_{\varsigma^P,\sigma^N}(e_1) = \left(\widetilde{J}_{\varepsilon^P,\eta^N}(e_1), \widetilde{L}_{\rho^P,\xi^N}(e_1)\right) \forall a \in B$$

i.e., $\widetilde{J}_{\varepsilon_2^P,\eta_2^N}(a) \subseteq \widetilde{J}_{\varepsilon_1^P,\eta_1^N}(a)$ and $\widetilde{L}_{\rho_2^P,\xi_2^N}(a) \subseteq \widetilde{L}_{\rho_1^P,\xi_1^N}(a)\ \forall a \in B$, since $B_{\beta_2^P,\delta_2^N}(a)$ *is the subgraph of* $B_{\beta_1^P,\delta_1^N}(a)$.

Conversely, Assume that $\widetilde{J}_{\varepsilon_2^P,\eta_2^N}(a) \subseteq \widetilde{J}_{\varepsilon_1^P,\eta_1^N}(a)$ and $\widetilde{L}_{\rho_2^P,\xi_2^N}(a) \subseteq \widetilde{L}_{\rho_1^P,\xi_1^N}(a)\ \forall a \in B$. Since $\widetilde{G}_1$ is the bipolar fuzzy soft graph of G^*. $B_{\beta_1^P,\delta_1^N}(a)$ is a bipolar fuzzy subgraph of G^* for all $a \in A$.

Since $\widetilde{G}_2$ is the bipolar fuzzy soft graph of G^*. $B_{\beta_2^P,\delta_2^N}(a)$ is a bipolar fuzzy subgraph of G^* for all $a \in B$.Thus $B_{\beta_2^P,\delta_2^N}(a)$ is a bipolar fuzzy subgraph of $B_{\beta_1^P,\delta_1^N}(a)$ for all $a \in B$.Hence $\widetilde{G}_2$ is the bipolar fuzzy soft subgraph of $\widetilde{G}_1$.

Definition 3.8. Let $\widetilde{G}_1 = \left\langle \widetilde{J}_{\varepsilon_1^P,\eta_1^N}, \widetilde{L}_{\rho_1^P,\xi_1^N}, A \right\rangle$ and $\widetilde{G}_2 = \left\langle \widetilde{J}_{\varepsilon_2^P,\eta_2^N}, \widetilde{L}_{\rho_2^P,\xi_2^N}, B \right\rangle$ be the two bipolar fuzzy soft graphs. Extended union of two bipolar fuzzy soft graphs over a common universe U is a bipolar fuzzy soft graph (B_{β^P,δ^N},C), where C = A∪B and $B_{\beta^P,\delta^N} : C \to BF^U$ is defined by

$$\widetilde{J}_{\varepsilon^P,\eta^N}(e) = \begin{cases} \widetilde{J}_{\varepsilon_1^P,\eta_1^N}(e)\ if\ e \in A \setminus B \\ \widetilde{J}_{\varepsilon_2^P,\eta_2^N}(e) if\ e \in B \setminus A \\ \widetilde{J}_{\varepsilon_1^P,\eta_1^N}(e) \cup \widetilde{J}_{\varepsilon_2^P,\eta_2^N}(e)\ if\ e \in A \cap B \end{cases}$$

$$\widetilde{L}_{\rho^P,\xi^N}(e) = \begin{cases} \widetilde{L}_{\rho_1^P,\xi_1^N}(e) if\ e \in A \setminus B \\ \widetilde{L}_{\rho_2^P,\xi_2^N}(e) if\ e \in B \setminus A \\ \widetilde{L}_{\rho_1^P,\xi_1^N}(e) \cup \widetilde{L}_{\rho_2^P,\xi_2^N}(e) if\ e \in A \cap B \end{cases}$$

i.e.,
$$\widetilde{G}_1 \cup_E \widetilde{G}_2 = \left\{ B_{\beta^P,\delta^N}(e) = \left(\widetilde{J}_{\varepsilon^P,\eta^N}(e), \widetilde{L}_{\rho^P,\xi^N}(e)\right) / e \in C \right\}$$

Example3.9.Consider the bipolar fuzzy graph G^*.The parameter of two set is denoted by $A = \{e_2, e_5\}$ and $B = \{e_1, e_4\}$, $\left(\widetilde{J}_{\varepsilon^P,\eta^N}, A\right)$ and $\left(\widetilde{L}_{\rho^P,\xi^N}, A\right)$ is a bipolar fuzzy soft set over V and E respectively, with bipolar fuzzy approximation function $\widetilde{J}_{\varepsilon^P,\eta^N} : A \to BF^U$ and $\widetilde{L}_{\rho^P,\xi^N} : A \to BF^U$

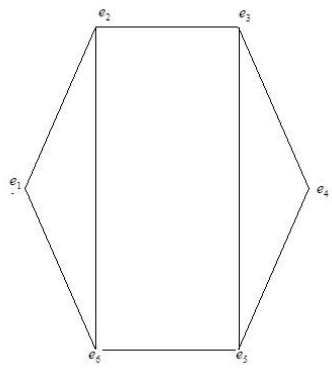

Fig.3.6. Simple Graph G^*

$$\widetilde{J}_{\varepsilon^P,\eta^N}(a) = \left\{ b \in V / aRb \Leftrightarrow \widetilde{d}_{\varepsilon^P,\eta^N}(a,b) \le (1,-1) \right\} \forall a \in A$$

i.e., $\widetilde{J}_{\varepsilon^P,\eta^N}(e_2) = \{e_1, e_2, e_3, e_6\}$ and
$\widetilde{J}_{\varepsilon^P,\eta^N}(e_5) = \{e_3, e_4, e_5, e_6\}$

The edges
$$\widetilde{L}_{\rho^P,\xi^N}(a) = \left\{ uv \in E / \{u,v\} \subseteq \widetilde{J}_{\varepsilon^P,\eta^N}(a) \right\} \forall a \in A$$
$$\widetilde{L}_{\rho^P,\xi^N}(e_2) = \{e_1e_2, e_2e_3, e_6e_1, e_2e_6\} \text{ and}$$
$$\widetilde{L}_{\rho^P,\xi^N}(e_5) = \{e_3e_4, e_4e_5, e_5e_6, e_5e_3\}$$

$$\widetilde{B}_{\varsigma^P,\sigma^N}(e_2)=\left(\widetilde{J}_{\varepsilon^P,\eta^N}(e_3),\widetilde{L}_{\rho^P,\xi^N}(e_3)\right) and$$
$$\widetilde{B}_{\varsigma^P,\sigma^N}(e_5)=\left(\widetilde{J}_{\varepsilon^P,\eta^N}(e_5),\widetilde{L}_{\rho^P,\xi^N}(e_5)\right)$$

Now the bipolar fuzzy approximation function $\widetilde{J}_{\varepsilon^P,\eta^N}: B \to BF^U$ and $\widetilde{L}_{\rho^P,\xi^N}: B \to BF^U$

$$\widetilde{J}_{\varepsilon^P,\eta^N}(a)=\left\{b\in V / aRb \Leftrightarrow \widetilde{d}_{\varepsilon^P,\eta^N}(a,b)=(1,-1)\right\}\forall a\in B$$

Then $\widetilde{J}_{\varepsilon^P,\eta^N}(e_1)=\{e_2,e_6\}$ and $\widetilde{J}_{\varepsilon^P,\eta^N}(e_4)=\{e_3,e_5\}$

The edges

$$\widetilde{L}_{\rho^P,\xi^N}(a)=\left\{uv\in E/\{u,v\}\subseteq \widetilde{J}_{\varepsilon^P,\eta^N}(a)\right\}\forall a\in B$$

$\widetilde{L}_{\rho^P,\xi^N}(e_1)=\{e_6e_2\}$ and $\widetilde{L}_{\rho^P,\xi^N}(e_4)=\{e_3e_5\}$

$$\widetilde{B}_{\varsigma^P,\sigma^N}(e_1)=\left(\widetilde{J}_{\varepsilon^P,\eta^N}(e_1),\widetilde{L}_{\rho^P,\xi^N}(e_1)\right) and$$
$$\widetilde{B}_{\varsigma^P,\sigma^N}(e_4)=\left(\widetilde{J}_{\varepsilon^P,\eta^N}(e_4),\widetilde{L}_{\rho^P,\xi^N}(e_4)\right)$$

Extended union of two bipolar fuzzy soft graphs

$$\widetilde{G}_1\cup_E \widetilde{G}_2=\left\{B_{\beta^P,\delta^N}(e)=\left(\widetilde{J}_{\varepsilon^P,\eta^N}(e),\widetilde{L}_{\rho^P,\xi^N}(e)\right)/ e\in C\right\}$$

Where C = A∪B $=\{e_1,e_2,e_4,e_5\}$

$\widetilde{J}_{\varepsilon^P,\eta^N}(e_1)=\{e_2,e_6\}, \widetilde{L}_{\rho^P,\xi^N}(e_1)=\{e_6e_2\}$

$\widetilde{J}_{\varepsilon^P,\eta^N}(e_2)=\{e_1,e_2,e_3,e_6\}$,

$\widetilde{L}_{\rho^P,\xi^N}(e_2)=\{e_1e_2,e_2e_3,e_6e_1,e_2e_6\}$

$\widetilde{J}_{\varepsilon^P,\eta^N}(e_4)=\{e_3,e_5\}, \widetilde{L}_{\rho^P,\xi^N}(e_4)=\{e_3e_5\}$

$\widetilde{J}_{\varepsilon^P,\eta^N}(e_5)=\{e_3,e_4,e_5,e_6\}$,

$\widetilde{L}_{\rho^P,\xi^N}(e_5)=\{e_3e_4,e_4e_5,e_5e_6,e_5e_3\}$

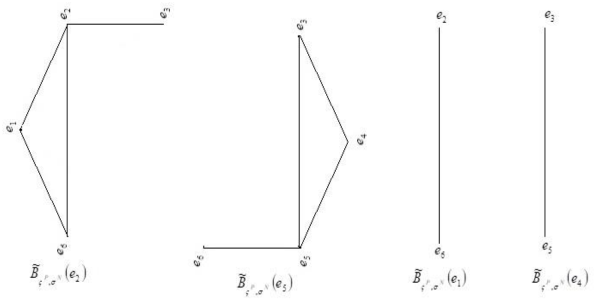

Fig.3.7. Subgraphs

$$\widetilde{B}_{\varsigma^P,\sigma^N}(e_2),\widetilde{B}_{\varsigma^P,\sigma^N}(e_5),\widetilde{B}_{\varsigma^P,\sigma^N}(e_1),\widetilde{B}_{\varsigma^P,\sigma^N}(e_4)$$

Then subgraphs of G^*

$$\widetilde{B}_{\varsigma^P,\sigma^N}(e_1)=\left(\widetilde{J}_{\varepsilon^P,\eta^N}(e_1),\widetilde{L}_{\rho^P,\xi^N}(e_1)\right),$$
$$\widetilde{B}_{\varsigma^P,\sigma^N}(e_2)=\left(\widetilde{J}_{\varepsilon^P,\eta^N}(e_2),\widetilde{L}_{\rho^P,\xi^N}(e_2)\right),$$
$$\widetilde{B}_{\varsigma^P,\sigma^N}(e_4)=\left(\widetilde{J}_{\varepsilon^P,\eta^N}(e_4),\widetilde{L}_{\rho^P,\xi^N}(e_4)\right) and$$
$$\widetilde{B}_{\varsigma^P,\sigma^N}(e_5)=\left(\widetilde{J}_{\varepsilon^P,\eta^N}(e_5),\widetilde{L}_{\rho^P,\xi^N}(e_5)\right)$$

Therefore

$$\widetilde{G}_1\cup_E \widetilde{G}_2=\left\{B_{\beta^P,\delta^N}(e_1),B_{\beta^P,\delta^N}(e_2),B_{\beta^P,\delta^N}(e_4),B_{\beta^P,\delta^N}(e_5)\right\}$$

Theorem 3.10. Let $\widetilde{G}_1=\left\langle \widetilde{J}_{\varepsilon_1^P,\eta_1^N},\widetilde{L}_{\rho_1^P,\xi_1^N},A\right\rangle$ and $\widetilde{G}_2=\left\langle \widetilde{J}_{\varepsilon_2^P,\eta_2^N},\widetilde{L}_{\rho_2^P,\xi_2^N},B\right\rangle$ be the two bipolar fuzzy soft graphs with $A\cap B\neq\phi$ and $\widetilde{J}_{\varepsilon_1^P,\eta_1^N}(a)\cap \widetilde{J}_{\varepsilon_2^P,\eta_2^N}(a)\neq\phi$ for all $a\in A\cap B$. Then their union is the bipolar fuzzy soft graph of G^*.

Proof:

Let $\widetilde{G}_1=\left\langle \widetilde{J}_{\varepsilon_1^P,\eta_1^N},\widetilde{L}_{\rho_1^P,\xi_1^N},A\right\rangle$ and $\widetilde{G}_2=\left\langle \widetilde{J}_{\varepsilon_2^P,\eta_2^N},\widetilde{L}_{\rho_2^P,\xi_2^N},B\right\rangle$ be the two bipolar fuzzy soft graphs. Extended union of two bipolar fuzzy soft graphs

$$\widetilde{G}_1\cup_E \widetilde{G}_2=\left\{B_{\beta^P,\delta^N}(e)=\left(\widetilde{J}_{\varepsilon^P,\eta^N}(e),\widetilde{L}_{\rho^P,\xi^N}(e)\right)/ e\in C\right\}$$

Where C = A∪B

$$\widetilde{J}_{\varepsilon^P,\eta^N}(e)=\begin{cases}\widetilde{J}_{\varepsilon_1^P,\eta_1^N}(e)\ if\ e\in A\setminus B\\ \widetilde{J}_{\varepsilon_2^P,\eta_2^N}(e) if\ e\in B\setminus A\\ \widetilde{J}_{\varepsilon_1^P,\eta_1^N}(e)\cup \widetilde{J}_{\varepsilon_2^P,\eta_2^N}(e)\ if\ e\in A\cap B\end{cases}$$

$$\widetilde{L}_{\rho^P,\xi^N}(e)=\begin{cases}\widetilde{L}_{\rho_1^P,\xi_1^N}(e) if\ e\in A\setminus B\\ \widetilde{L}_{\rho_2^P,\xi_2^N}(e) if\ e\in B\setminus A\\ \widetilde{L}_{\rho_1^P,\xi_1^N}(e)\cup \widetilde{L}_{\rho_2^P,\xi_2^N}(e) if\ e\in A\cap B\end{cases}$$

Since $\widetilde{G}_1$ is the bipolar fuzzy soft graph of G^*. $B_{\beta_1^P,\delta_1^N}(a)$ is a connected bipolar fuzzy subgraph of G^* for all $a\in A\setminus B$. Since $\widetilde{G}_2$ is the bipolar fuzzy soft graph of G^*. $B_{\beta_2^P,\delta_2^N}(a)$ is a connected bipolar fuzzy subgraph of G^* for all $a\in B\setminus A$.Let $a\in A\cap B$ and

$$\left(\widetilde{J}_{\varepsilon^P,\eta^N}(a),\widetilde{L}_{\rho^P,\xi^N}(a)\right)=\begin{pmatrix}\widetilde{J}_{\varepsilon_1^P,\eta_1^N}(a)\cup \widetilde{J}_{\varepsilon_2^P,\eta_2^N}(a),\\ \widetilde{L}_{\rho_1^P,\xi_1^N}(a)\cup \widetilde{L}_{\rho_2^P,\xi_2^N}(a)\end{pmatrix}.$$

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Since $\left(\tilde{J}_{\varepsilon_1^P,\eta_1^N}(a), \tilde{L}_{\rho_1^P,\xi_1^N}(a)\right)$ and $\left(\tilde{J}_{\varepsilon_2^P,\eta_2^N}(a), \tilde{L}_{\rho_2^P,\xi_2^N}(a)\right)$ are connected bipolar fuzzy subgraph of G^* by assumption

$\tilde{J}_{\varepsilon_1^P,\eta_1^N}(a) \cap \tilde{J}_{\varepsilon_2^P,\eta_2^N}(a) \neq \phi$ for all $a \in A \cap B$.

$\left(\tilde{J}_{\varepsilon^P,\eta^N}(a), \tilde{L}_{\rho^P,\xi^N}(a)\right)$ is a connected bipolar fuzzy subgraph of G^* and

$$\tilde{G}_1 \cup_E \tilde{G}_2 = \left\{B_{\beta^P,\delta^N}(e) = \left(\tilde{J}_{\varepsilon^P,\eta^N}(e), \tilde{L}_{\rho^P,\xi^N}(e)\right) / e \in C\right\}$$

is a bipolar soft fuzzy graph of G^*.

Definition 3.11. Let $\tilde{G}_1 = \left\langle \tilde{J}_{\varepsilon_1^P,\eta_1^N}, \tilde{L}_{\rho_1^P,\xi_1^N}, A\right\rangle$ and $\tilde{G}_2 = \left\langle \tilde{J}_{\varepsilon_2^P,\eta_2^N}, \tilde{L}_{\rho_2^P,\xi_2^N}, B\right\rangle$ be the two bipolar fuzzy soft graphs. Restricted union of two bipolar fuzzy soft graphs over a common universe U is a bipolar fuzzy soft graph (B_{β^P,δ^N},C), where C = $A \cap B$ and $B_{\beta^P,\delta^N} : C \to BF^U$ is defined by

$$\left(\tilde{J}_{\varepsilon^P,\eta^N}(a), \tilde{L}_{\rho^P,\xi^N}(a)\right) = \begin{pmatrix} \tilde{J}_{\varepsilon_1^P,\eta_1^N}(a) \cup \tilde{J}_{\varepsilon_2^P,\eta_2^N}(a), \\ \tilde{L}_{\rho_1^P,\xi_1^N}(a) \cup \tilde{L}_{\rho_2^P,\xi_2^N}(a) \end{pmatrix}$$

Definition 3.12. Let $\tilde{G}_1 = \left\langle \tilde{J}_{\varepsilon_1^P,\eta_1^N}, \tilde{L}_{\rho_1^P,\xi_1^N}, A\right\rangle$ and $\tilde{G}_2 = \left\langle \tilde{J}_{\varepsilon_2^P,\eta_2^N}, \tilde{L}_{\rho_2^P,\xi_2^N}, B\right\rangle$ be the two bipolar fuzzy soft graphs. Extended intersection of two bipolar fuzzy soft graphs over a common universe U is a bipolar fuzzy soft graph (B_{β^P,δ^N},C), where C = $A \cup B$ and $B_{\beta^P,\delta^N} : C \to BF^U$ is defined by

$$\tilde{J}_{\varepsilon^P,\eta^N}(e) = \begin{cases} \tilde{J}_{\varepsilon_1^P,\eta_1^N}(e) \text{ if } e \in A \setminus B \\ \tilde{J}_{\varepsilon_2^P,\eta_2^N}(e) \text{ if } e \in B \setminus A \\ \tilde{J}_{\varepsilon_1^P,\eta_1^N}(e) \cap \tilde{J}_{\varepsilon_2^P,\eta_2^N}(e) \text{ if } e \in A \cap B \end{cases}$$

$$\tilde{L}_{\rho^P,\xi^N}(e) = \begin{cases} \tilde{L}_{\rho_1^P,\xi_1^N}(e) \text{ if } e \in A \setminus B \\ \tilde{L}_{\rho_2^P,\xi_2^N}(e) \text{ if } e \in B \setminus A \\ \tilde{L}_{\rho_1^P,\xi_1^N}(e) \cap \tilde{L}_{\rho_2^P,\xi_2^N}(e) \text{ if } e \in A \cap B \end{cases} \quad \text{i.e.,}$$

$$\tilde{G}_1 \cap_E \tilde{G}_2 = \left\{B_{\beta^P,\delta^N}(e) = \left(\tilde{J}_{\varepsilon^P,\eta^N}(e), \tilde{L}_{\rho^P,\xi^N}(e)\right) / e \in C\right\}$$

Example3.13.Consider the bipolar fuzzy graph G^*.The parameter of two set is denoted by $A = \{e_1, e_2, e_5\}$ and $B = \{e_1, e_2\}$, $\left(\tilde{J}_{\varepsilon^P,\eta^N}, A\right)$ and $\left(\tilde{L}_{\rho^P,\xi^N}, A\right)$ is a bipolar fuzzy soft set over V and E respectively, with bipolar fuzzy approximation function $\tilde{J}_{\varepsilon^P,\eta^N} : A \to BF^U$ and $\tilde{L}_{\rho^P,\xi^N} : A \to BF^U$

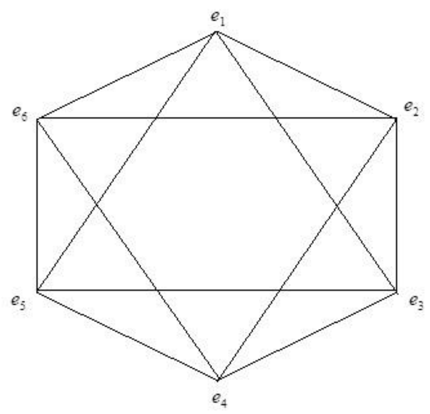

Fig.3.8. Simple Graph G^*

$$\tilde{J}_{\varepsilon^P,\eta^N}(a) = \left\{b \in V / aRb \Leftrightarrow \tilde{d}_{\varepsilon^P,\eta^N}(a,b) = rad(G^*)\right\} \forall a \in A$$

i.e.,

$\tilde{J}_{\varepsilon^P,\eta^N}(e_1) = \{e_2, e_3, e_5, e_6\}$,

$\tilde{J}_{\varepsilon^P,\eta^N}(e_2) = \{e_1, e_3, e_4, e_6\}$ and

$\tilde{J}_{\varepsilon^P,\eta^N}(e_5) = \{e_1, e_3, e_4, e_6\}$

The edges

$$\tilde{L}_{\rho^P,\xi^N}(a) = \left\{uv \in E / \{u,v\} \subseteq \tilde{J}_{\varepsilon^P,\eta^N}(a)\right\} \forall a \in A$$

$\tilde{L}_{\rho^P,\xi^N}(e_1) = \{e_2e_3, e_3e_5, e_5e_6, e_6e_2\}$,

$\tilde{L}_{\rho^P,\xi^N}(e_2) = \{e_3e_4, e_4e_6, e_6e_1, e_1e_3\}$ and

$\tilde{L}_{\rho^P,\xi^N}(e_5) = \{e_1e_3, e_3e_4, e_6e_1, e_4e_6\}$

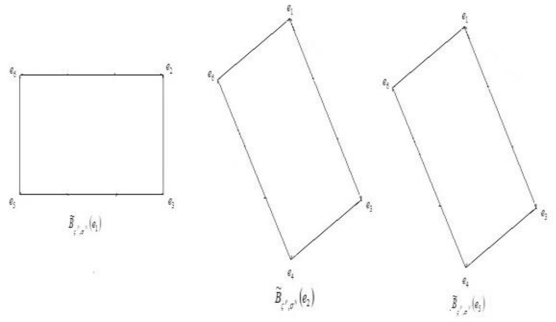

Fig.3.9. Subgraphs $\widetilde{B}_{\varsigma^P,\sigma^N}(e_1)$, $\widetilde{B}_{\varsigma^P,\sigma^N}(e_2)$, $\widetilde{B}_{\varsigma^P,\sigma^N}(e_5)$

Now the bipolar fuzzy approximation function $\widetilde{J}_{\varepsilon^P,\eta^N}: B \rightarrow BF^U$ and $\widetilde{L}_{\rho^P,\xi^N}: B \rightarrow BF^U$

$$\widetilde{J}_{\varepsilon^P,\eta^N}(a)=\{b \in V / aRb \Leftrightarrow \widetilde{d}_{\varepsilon^P,\eta^N}(a,b) \leq rad(G^*)\}$$
$\forall a \in B$

Then

$$\widetilde{J}_{\varepsilon^P,\eta^N}(e_1)=\{e_1,e_2,e_3,e_5,e_6\},$$
$$\widetilde{J}_{\varepsilon^P,\eta^N}(e_2)=\{e_1,e_2,e_3,e_4,e_6\}$$

The edges

$$\widetilde{L}_{\rho^P,\xi^N}(a)=\{uv \in E / \{u,v\} \subseteq \widetilde{J}_{\varepsilon^P,\eta^N}(a)\} \forall a \in B$$
$$\widetilde{L}_{\rho^P,\xi^N}(e_1)=\{e_1e_2,e_2e_3,e_3e_5,e_5e_6,e_6e_2,e_6e_1,e_5e_1,e_1e_3\}$$
and
$$\widetilde{L}_{\rho^P,\xi^N}(e_2)=\{e_1e_2,e_2e_3,e_4e_3,e_4e_6,e_6e_1,e_2e_4,e_2e_6,e_1e_3\}$$

The extended intersection of two bipolar soft fuzzy graph is

$$\widetilde{G}_1 \cap_E \widetilde{G}_2 = \{B_{\beta^P,\delta^N}(e)=(\widetilde{J}_{\varepsilon^P,\eta^N}(e),\widetilde{L}_{\rho^P,\xi^N}(e)) / e \in C\}$$

where C = $A \cup B$

$$\widetilde{J}_{\varepsilon^P,\eta^N}(e_1)=\{e_1,e_2,e_3,e_5,e_6\},$$
$$\widetilde{L}_{\rho^P,\xi^N}(e_1)=\{e_1e_2,e_2e_3,e_3e_5,e_5e_6,e_6e_2,e_6e_1,e_5e_1,e_1e_3\}$$

$$\widetilde{J}_{\varepsilon^P,\eta^N}(e_2)=\{e_1,e_2,e_3,e_4,e_6\},$$
$$\widetilde{L}_{\rho^P,\xi^N}(e_2)=\{e_1e_2,e_2e_3,e_4e_3,e_4e_6,e_6e_1,e_2e_4,e_2e_6,e_1e_3\}$$
and

$$\widetilde{J}_{\varepsilon^P,\eta^N}(e_5)=\{e_1,e_3,e_4,e_6\},$$
$$\widetilde{L}_{\rho^P,\xi^N}(e_5)=\{e_1e_3,e_3e_4,e_6e_1,e_4e_6\}$$ Finally, complete

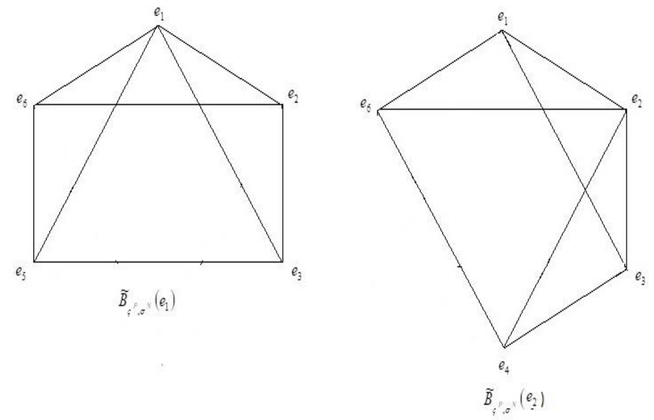

Fig.3.10. Subgraphs $\widetilde{B}_{\varsigma^P,\sigma^N}(e_1)$, $\widetilde{B}_{\varsigma^P,\sigma^N}(e_2)$

Then subgraphs of G^*

$$\widetilde{B}_{\varsigma^P,\sigma^N}(e_1)=(\widetilde{J}_{\varepsilon^P,\eta^N}(e_1),\widetilde{L}_{\rho^P,\xi^N}(e_1)),$$
$$\widetilde{B}_{\varsigma^P,\sigma^N}(e_2)=(\widetilde{J}_{\varepsilon^P,\eta^N}(e_2),\widetilde{L}_{\rho^P,\xi^N}(e_2)) \text{ and}$$
$$\widetilde{B}_{\varsigma^P,\sigma^N}(e_5)=(\widetilde{J}_{\varepsilon^P,\eta^N}(e_5),\widetilde{L}_{\rho^P,\xi^N}(e_5))$$

Therefore

$$\widetilde{G}_1 \cap_E \widetilde{G}_2 = \{B_{\beta^P,\delta^N}(e_1), B_{\beta^P,\delta^N}(e_2), B_{\beta^P,\delta^N}(e_5)\}$$

Definition 3.14. Let $\widetilde{G}_1=\langle \widetilde{J}_{\varepsilon_1^P,\eta_1^N}, \widetilde{L}_{\rho_1^P,\xi_1^N}, A\rangle$ and $\widetilde{G}_2=\langle \widetilde{J}_{\varepsilon_2^P,\eta_2^N}, \widetilde{L}_{\rho_2^P,\xi_2^N}, B\rangle$ be the two bipolar fuzzy soft graphs. Restricted intersection of two bipolar fuzzy soft graphs over a common universe U is such that $A \cap B = \phi$ bipolar fuzzy soft graph (B_{β^P,δ^N} ,C), where C = $A \cap B$ and $B_{\beta^P,\delta^N}: C \rightarrow BF^U$ is defined by

$$(\widetilde{J}_{\varepsilon^P,\eta^N}(a),\widetilde{L}_{\rho^P,\xi^N}(a))=\begin{pmatrix} \widetilde{J}_{\varepsilon_1^P,\eta_1^N}(a) \cap \widetilde{J}_{\varepsilon_2^P,\eta_2^N}(a), \\ \widetilde{L}_{\rho_1^P,\xi_1^N}(a) \cap \widetilde{L}_{\rho_2^P,\xi_2^N}(a) \end{pmatrix}$$

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