ONE MODULO N GRACEFULNESS OF n-POLYGONAL SNAKES, $C_n^{(t)}$ AND $P_{a,b}$

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Abstract

A function f is called a graceful labelling of a graph G with q edges if f is an injection from the vertices of G to the set $\{0,1,2,\ldots,q\}$ such that, when each edge xy is assigned the label $|f(x)-f(y)|$, the resulting edge labels are distinct. A graph G is said to be one modulo N graceful (where N is a positive integer) if there is a function ϕ from the vertex set of G to $\{0,1,N,(N+1),2N,(2N+1),\ldots,N(q-1),N(q-1)+1\}$ in such a way that (i) ϕ is $1-1$ (ii) ϕ induces a bijection ϕ^* from the edge set of G to $\{1,N+1,2N+1,\ldots,N(q-1)+1\}$ where $\phi^*(uv)=|\phi(u)-\phi(v)|$. In this paper we prove that the n-Polygonal snakes , $C_n^{(t)}$ and $P_{a,b}$ are one modulo N graceful for all positive integers N .

Keywords : Graceful, modulo N graceful, n-Polygonal snakes, $C_n^{(t)}$ and $P_{a,b}$.

AMS Subject Classification (2010):05C78

1 Introduction

S.W.Golomb [1] introduced graceful labelling.Odd gracefulness was introduced by R.B.Gnanajothi [2] .C.Sekar [6] introduced one modulo three graceful labelling. In this paper we introduce the concept of one modulo N graceful where N is any positive integer.In the case $N=2$, the labelling is odd graceful and in the case $N=1$ the labelling is graceful.We prove that the n-Polygonal snakes, $C_n^{(t)}$ and $P_{a,b}$ are one modulo N graceful for all positive integers N .

2 Main Results

Definition 2.1. *A graph G with q edges is said to be one modulo N graceful (where N is a positive integer) if there is a function ϕ from the vertex set of G to $\{0,1,N,(N+1),2N,(2N+1),\ldots,N(q-1),N(q-1)+1\}$ in such a way that (i) ϕ is $1-1$ (ii) ϕ induces a bijection ϕ^* from the edge set of G to $\{1,N+1,2N+1,\ldots,N(q-1)+1\}$ where $\phi^*(uv)=|\phi(u)-\phi(v)|$.*

Definition 2.2. *Consider k copies of path P_n .An n-Polygonal snake containing k number of n-Polygons is obtained from a path $v_1,v_2,\ldots,v_k$ by identifying the pendant vertices of ith copy of the path P_n with v_{i-1} and v_i for $i=1,2,\ldots,k$.*

Definition 2.3. *The one point union of t cycles of length n is denoted by $C_n^{(t)}$ .This graph has $t(n-1)+1$ vertices and tn edges.*

Definition 2.4. *Let u and v be two fixed vertices. We connect u and v by means of "b" internally disjoint paths of length "a" each. The resulting graph is denoted by $P_{a,b}$.*

Theorem 2.5. *n-Polygonal snakes for $n\equiv 0(mod4)$ are one modulo N graceful for every positive integer N .*

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Proof: Let $n = 4r, r \geq 1$.Let there be k polygons.This graph has $4rk - (k-1)$ vertices and $4rk$ edges. For $1 \leq j \leq k$ let $u_i^{(j)}, j = 1, 2, \ldots 4r$ be the vertices of ith polygon. Identify $u_1^{(4r)}$ with $u_2^{(1)}$, $u_2^{(4r)}$ with $u_3^{(1)}$, and so on.
Define

$\phi(u_{2i-1}^{(1)}) = \phi(u_{2(i-1)}^{(4r)}) = 4Nr(i-1)$ for $i = 1, 2, 3, 4 \ldots,$

$\phi(u_{2i}^{(1)}) = \phi(u_{(2i-1)}^{(4r)}) = 4Nrk - (N-1) - 2Nr + N - 4Nr(i-1)$ for $i = 1, 2, 3, 4 \ldots,$

$\phi(u_{2i-1}^{(j)}) = 4Nrk - (N-1) - \frac{N(j-2)}{2} - 4Nr(i-1)$ for $j = 2, 4, 6, 8 \ldots, 4r-2$ and i=1,2,3,4...

$\phi(u_{2i-1}^{(j)}) = N + \frac{N(j-3)}{2} + 4Nr(i-1)$ for $j = 3, 5, 7, 9 \ldots, 2r-1$ and i=1,2,3,4...

$\phi(u_{2i-1}^{(j)}) = 2N + \frac{N(j-3)}{2} + 4Nr(i-1)$ for $j = 2r+1, 2r+3, \ldots, 4r-1$ and i=1,2,3,4...

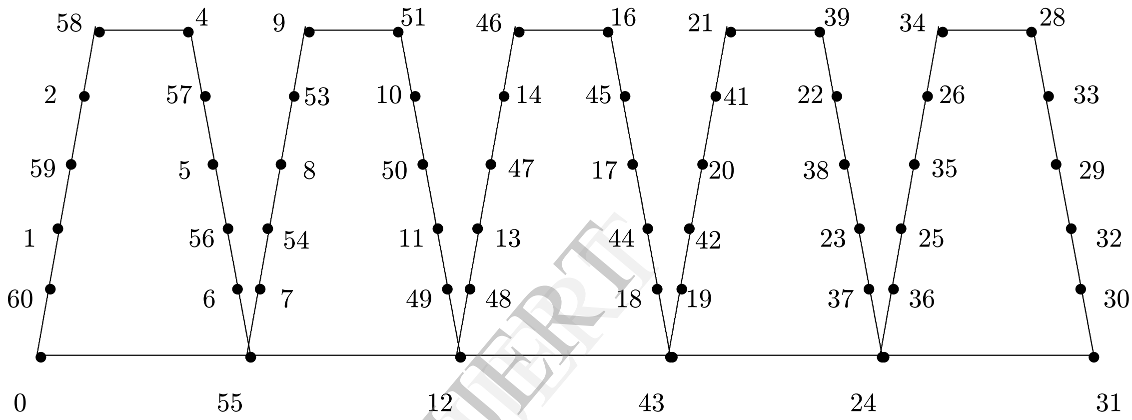
$\phi(u_{2i}^{(j)}) = N(2r+1) + \frac{N(j-2)}{2} + 4Nr(i-1)$ for $j = 2, 4, 6, 8 \ldots, 4r-2$ and i=1,2,3,4...

$\phi(u_{2i}^{(j)}) = 4Nrk - (N-1) - 2Nr - \frac{N(j-3)}{2} - 4Nr(i-1)$ for $j = 3, 5, 7 \ldots, 2r-1$ and i=1,2,3,4...

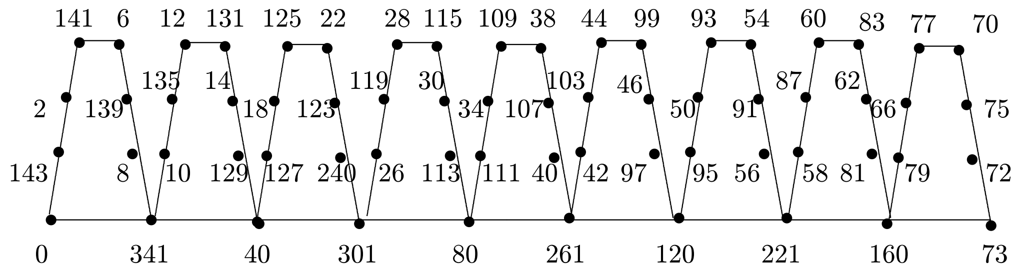
$\phi(u_{2i}^{(j)}) = 4Nrk - (N-1) - 2Nr - N - \frac{N(j-3)}{2} - 4Nr(i-1)$ for $j = 2r+1, 2r+3, \ldots, 4r-1$ and i=1,2,3,4...

Clearly ϕ is $1-1$ and ϕ defines a one modulo N graceful labelling of the n-Polygonal snakes for $n \equiv 0(mod 4)$.

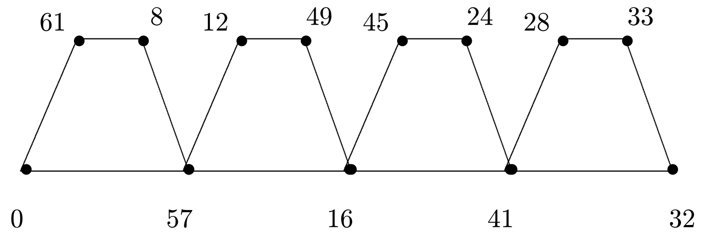
Example 2.6. *Graceful labelling of the* 12*-Polygonal snake. (No.of polygons* $= 5$*)*



Example 2.7. *Odd graceful labelling of the* 8*-Polygonal snake. (No.of polygons* $= 9$*)*



Example 2.8. *One modulo* 4 *graceful labelling of the* 4*-Polygonal snake. (No.of polygons* $= 4$*)*



Theorem 2.9. *n-Polygonal snakes for $n \equiv 2(mod4)$ are one modulo N graceful for every positive integer N if the number of polygons is even .*

Proof: Let $n = 4r+2$ and let there be $k = 2s$ polygon.This graph has $4rk-(k-1)$ vertices and $4rk$ edges. For $1 \leq j \leq k$ let $u_i^{(j)}, j = 1,2,\ldots 4r$ be the vertices of ith polygon. Identify $u_1^{(4r)}$ with $u_2^{(1)}$, $u_2^{(4r)}$ with $u_3^{(1)}$, and so on.
Define

$\phi(u_{2i-1}^{(1)}) = \phi(u_{2(i-1}^{(4r+2)}) = N(4r+3)(i-1)$ for $i = 1,2,3,4\ldots,s$

$\phi(u_{2i}^{(1)}) = \phi(u_{(2i-1}^{(4r+2)}) = N(4r+2)k-(N-1)-4Nr-N(4r+1)(i-1)$ for $i = 1,2,3,4\ldots,s$

$\phi(u_{2i-1}^{(j)}) = N + \frac{N(j-3)}{2} + N(4r+3)(i-1)$ for $j = 3,5,7,9\ldots,4r+1$ and i=1,2,3,4...,s

$\phi(u_{2i-1}^{(j)}) = N(4r+2)k-(N-1)-\frac{N(j-2)}{2}-N(4r+1)(i-1)$ for $j = 2,4,6,8\ldots,4r$ and i=1,2,3,4...,s

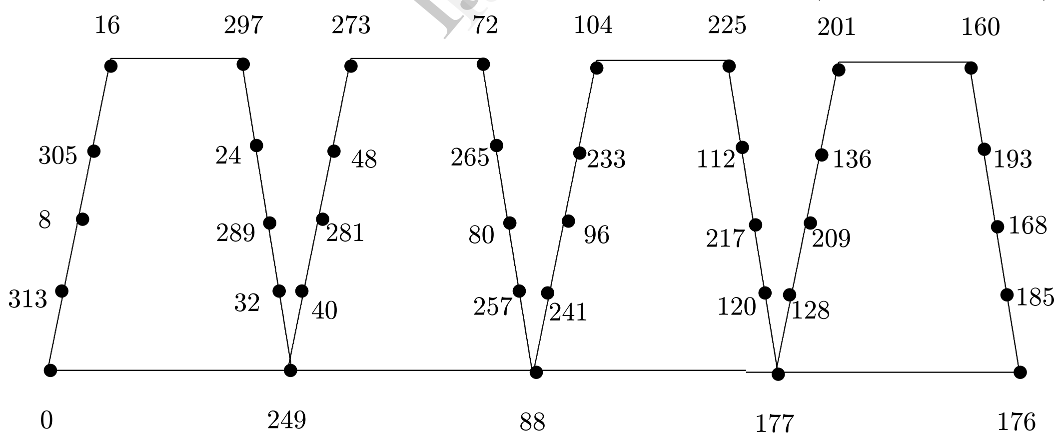
$\phi(u_{2i}^{(j)}) = N(4r+2)k-(N-1)-2Nr-\frac{N(j-3)}{2}-N(4r+1)(i-1)$ for $j = 3,5,7\ldots,4r+1$ and i=1,2,3,4...,s

$\phi(u_{2i}^{(j)}) = N(2r+1) + \frac{N(j-2)}{2} + N(4r+3)(i-1)$ for $j = 2,4,6,\ldots,2r$ and i=1,2,3,4..., s

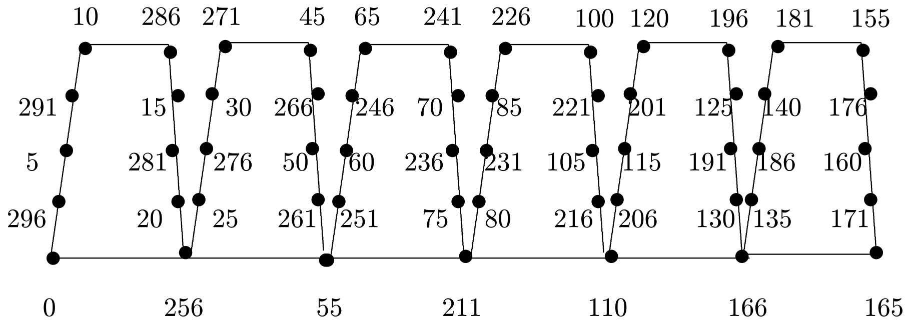
$\phi(u_{2i}^{(j)}) = N(2r+1)+2N+\frac{N(j-2)}{2}+N(4r+3)(i-1)$ for $j = 2r+2,2r+4,\ldots,4r$ and i=1,2,3,4,..., s

Clearly ϕ is $1-1$ and ϕ defines a one modulo N graceful labelling of the n-Polygonal snakes for $n \equiv 0(mod2)$.

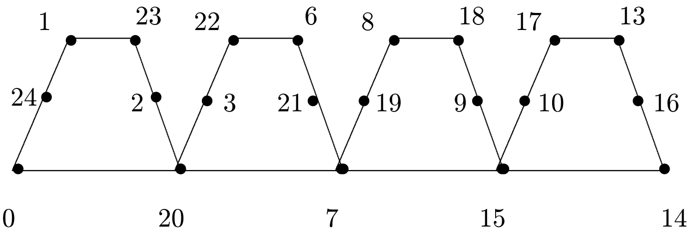
Example 2.10. *One modulo 8 graceful labelling of the 10-Polygonal snake. (No.of polygons$=4$)*



Example 2.11. *One modulo 5 graceful labelling of the 10-Polygonal snake. (No.of polygons$=6$)*



Example 2.12. *Graceful labelling of the* 6*-Polygonal snake. (No.of polygons*$=4$*)*



Theorem 2.13. *Let* $C_n^{(t)}$ *denote the one point union of* t *cycles of length* n. $C_n^{(t)}$ *is one modulo* N *graceful when* $n = 4, 8, t > 2$ *and* $n = 6, t$ *even and* $t \geq 4$ *for every positive integer* $N > 1$.

Proof: Case (i) $n = 4, t > 2$
For $1 \leq i \leq t$. Let $u_i^{(j)}, j = 1, 2, 3, 4$ be the vertices of the ith cycle with the one point identification of $u_1^{(1)}, u_2^{(1)}, \ldots, u_t^{(1)}$ at u_0.
Define

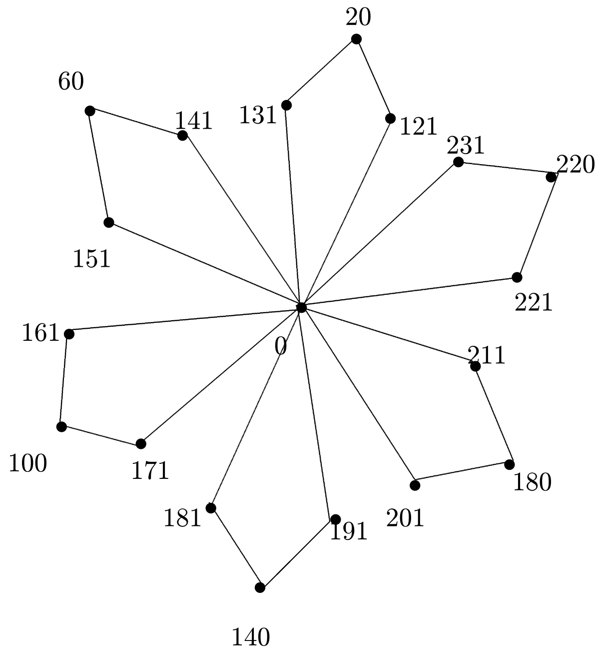
$\phi(u_0) = 0$

$\phi(u_i^{(j)}) = 4Nt - (N-1) - \frac{N(j-2)}{2} - 2N(i-1)$ for $j = 2, 4$ and i=1,2,3,4...,t

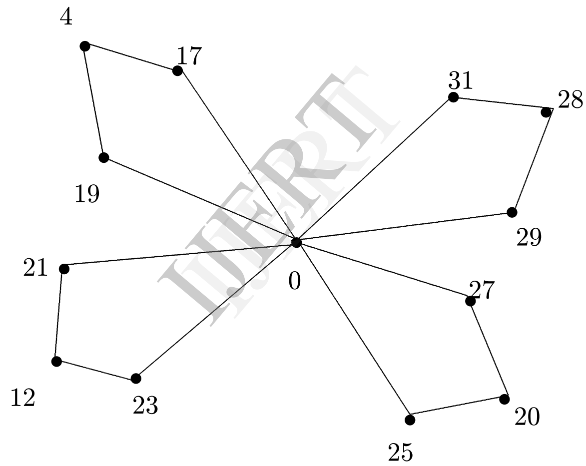
$\phi(u_i^{(3)}) = 4Nt - 2N - 4N(i-1)$ for $i = 1, 2, 3, 4 \ldots, t$

Clearly ϕ is $1-1$ and ϕ defines a one modulo N graceful labelling of $C_n^{(t)}$ when $n = 4, t > 2$.

Example 2.14. *One modulo* 10 *graceful labelling of* $C_4^{(6)}$



Example 2.15. *Odd graceful labelling of* $C_4^{(4)}$



Case (ii) $n = 8, t > 2$

For $1 \leq i \leq t$. Let $u_i^{(j)}, j = 1, 2, \ldots, 8$ be the vertices of the ith cycle with the one point identification of $u_1^{(1)}, u_2^{(1)}, \ldots, u_t^{(1)}$ at u_0.

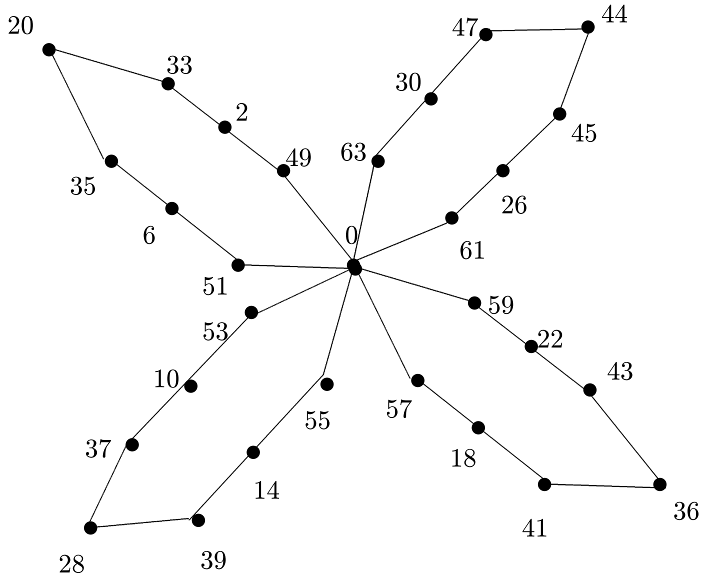
Define

$$\phi(u_0) = 0$$

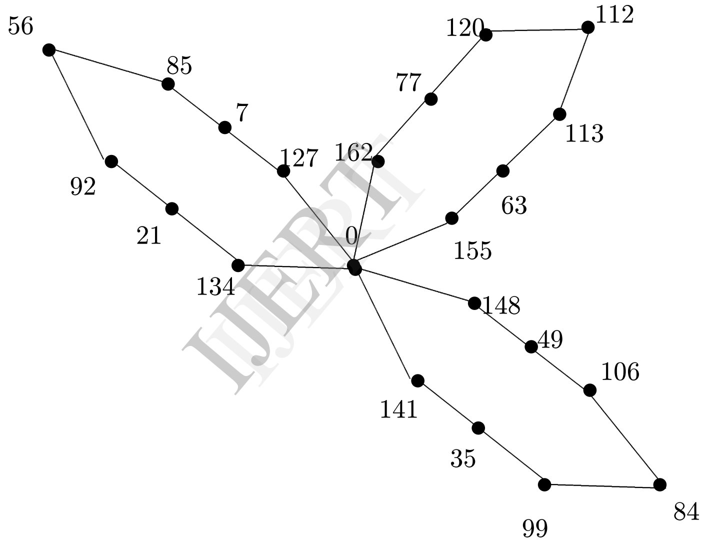
$$\phi(u_i^{(j)}) = \begin{cases} 8Nt - (N-1) - 2N(i-1) - \frac{N(j-2)}{6} & \text{if } i = 1, 2, 3, \ldots, t \text{ and } j = 2, 8 \\ 6Nt - (N-1) - 2N(i-1) - \frac{N(j-4)}{2} & \text{if } i = 1, 2, 3, \ldots, t \text{ and } j = 4, 6 \\ 4Nt - N - 4N(i-1) - \frac{2N(j-3)}{4} & \text{if } i = 1, 2, 3, \ldots, t \text{ and } j = 3, 7 \\ 6Nt - 2N - 4N(i-1) & \text{if } i = 1, 2, 3, \ldots, t \text{ and } j = 5 \end{cases}$$

Clearly ϕ is $1-1$ and ϕ defines a one modulo N graceful labelling of $C_n^{(t)}$ when $n = 8, t > 2$.

Example 2.16. *Odd graceful labelling of* $C_8^{(4)}$



Example 2.17. *One modulo* 7 *graceful labelling of* $C_8^{(3)}$



Case (iii) $n = 6$, t is even $t \geq 4$ let $t = 2s$

For $1 \leq i \leq 2s$. Let $u_i^{(j)}, j = 1, 2, \ldots, 6$ be the vertices of the ith cycle with the one point identification of $u_1^{(1)}, u_2^{(1)}, \ldots, u_t^{(1)}$ at u_0.

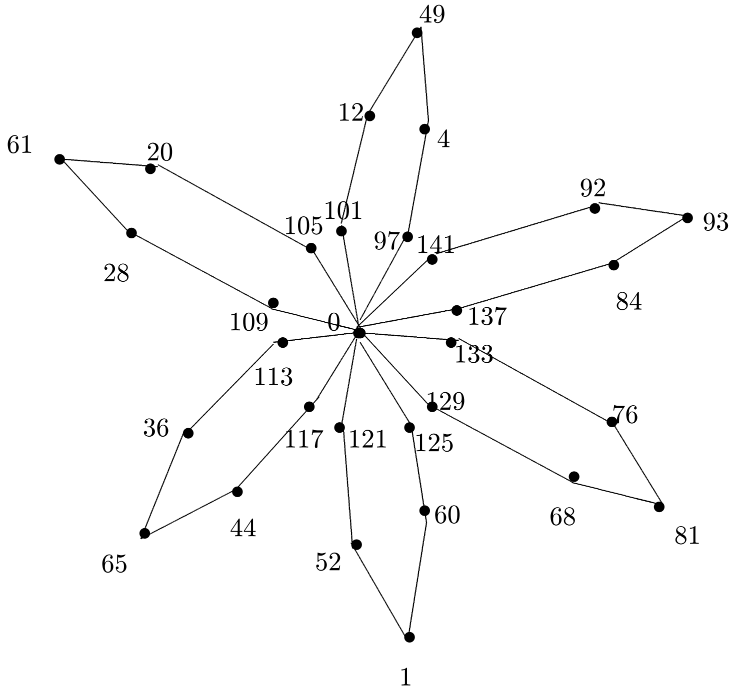
Define

$\phi(u_0) = 0$

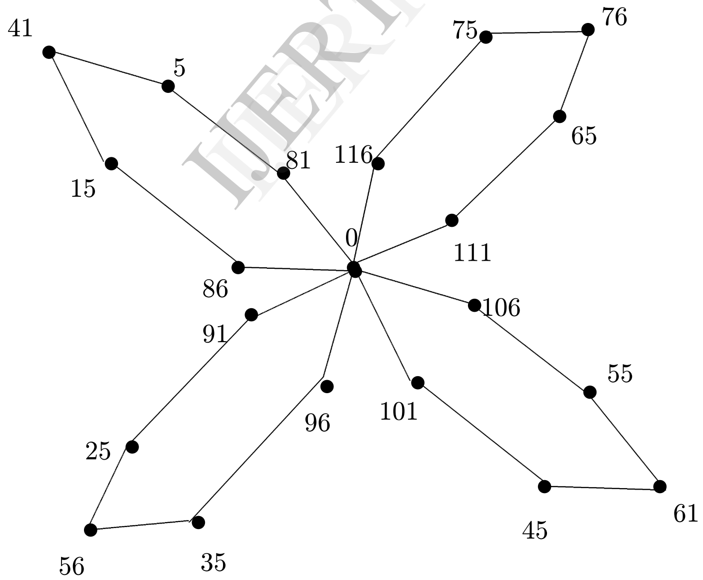
$$\phi(u_i^{(j)}) = \begin{cases} 6Nt - (N-1) - 2N(i-1) - \frac{N(j-2)}{4} & \text{if } i = 1, 2, 3, \ldots, 2s \text{ and } j = 2, 6 \\ 4Nt - N - 4N(i-1) - \frac{2N(j-3)}{2} & \text{if } i = 1, 2, 3, \ldots, 2s \text{ and } j = 3, 5 \\ 4Nt - (N-1) - \frac{4N(j-1)}{2} & \text{if } i = 1, 3, 5, \ldots, 2s-1 \text{ and } j = 4 \\ 4Nt - (4N-1) - \frac{4N(j-2)}{2} & \text{if } i = 2, 4, 6, \ldots, 2s \text{ and } j = 4 \end{cases}$$

Clearly ϕ is $1-1$ and ϕ defines a one modulo N graceful labelling of $C_n^{(t)}$ when $n = 6, t$ is even and $t \geq 4$.

Example 2.18. *One modulo 4 graceful labelling of* $C_6^{(6)}$



Example 2.19. *One modulo 5 graceful labelling of* $C_6^{(4)}$



Theorem 2.20. *$P_{a,b}$ for all $a \geq 2$ and for all odd b is one modulo N graceful for every positive integer N. Here P_m is a path of length $m-1$.*

Proof: Let $b = 2r+1, r \geq 1$

Define

$$X(t) = \begin{cases} 1 & \text{if } t \leq r \\ 0 & \text{if } t > r \end{cases}$$

Define

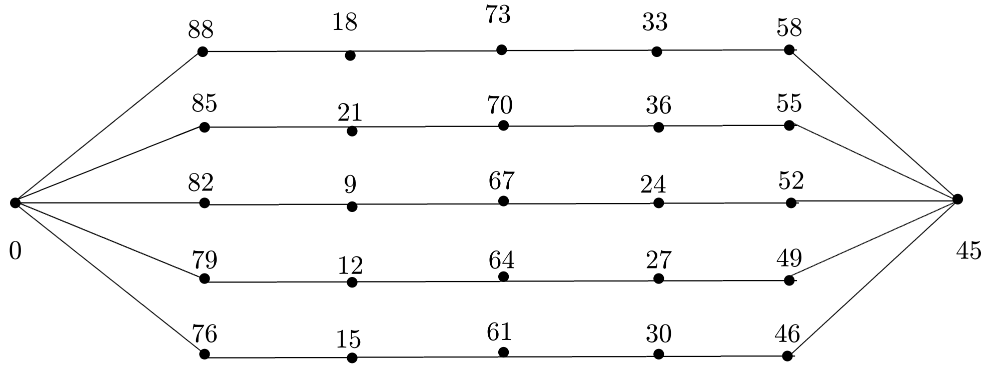
$$\phi(u) = 0$$

$$\phi(v) = \begin{cases} \frac{Na(2r+1)}{2} & \text{if } a \text{ is even} \\ \frac{N(ab-1)}{2} + 1 & \text{if } a \text{ is odd} \end{cases}$$

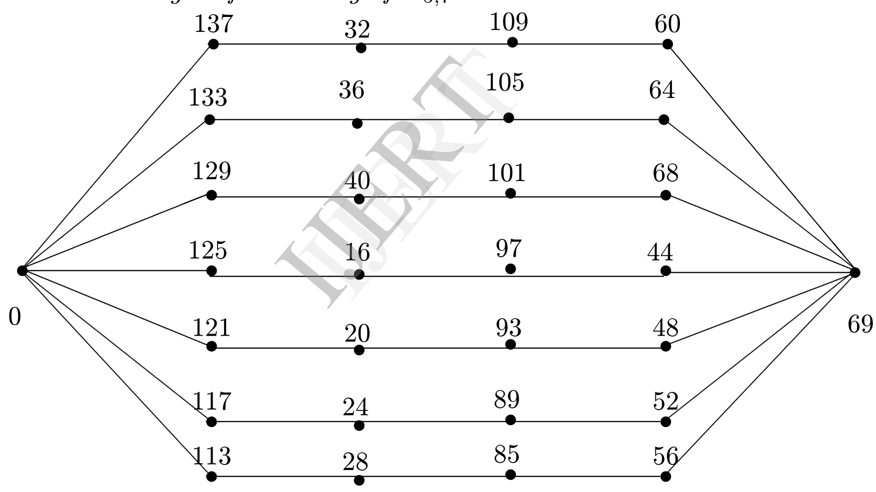
For $j = 1, 3, 5, \ldots$
$\phi(v_j^{(i)}) = N(a(2r+1)-1)+1-N(i-1)-(2r+1)(j-1)$ if $i = 1, 2, 3, \ldots, 2r+1$
For $j = 2, 4, 6, \ldots$
$\phi(v_j^{(i)}) = X(i)\{2N(r+1)+N(i-1)+(2r+1)(j-2)\}+(1-X(i))\{N+N(i-1)+(2r+1)(j-2)\}$ if $i = 1, 2, 3, \ldots, 2r+1$

Clearly ϕ is $1-1$ and ϕ defines a one modulo N graceful labelling of $P_{a,b}$ for all $a \geq 2$ and for all odd b .

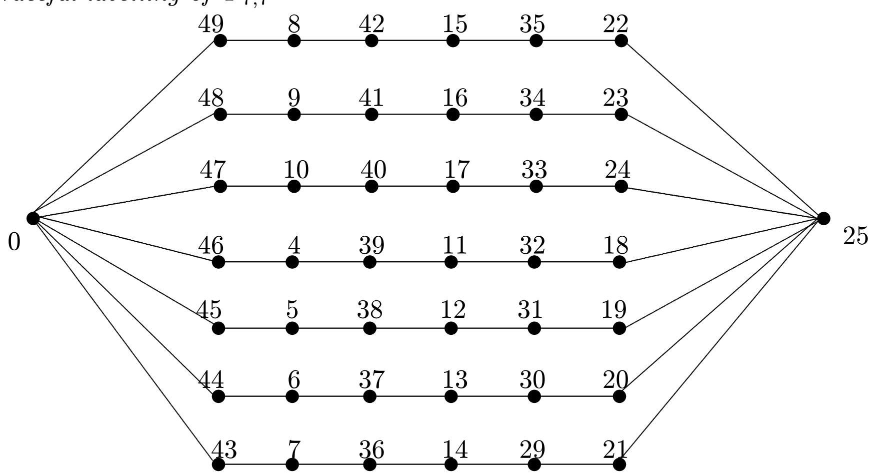
Example 2.21. *One modulo 3 graceful labelling of* $P_{6,5}$



Example 2.22. *One modulo 4 graceful labelling of* $P_{5,7}$



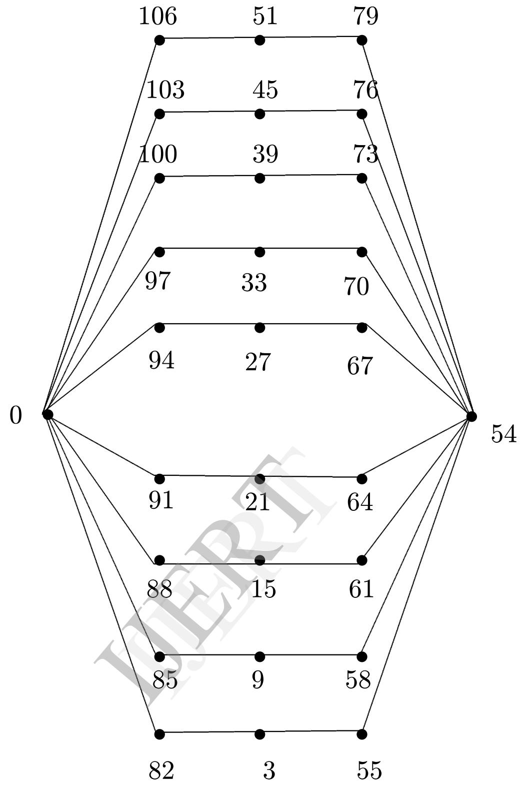
Example 2.23. *Graceful labelling of* $P_{7,7}$



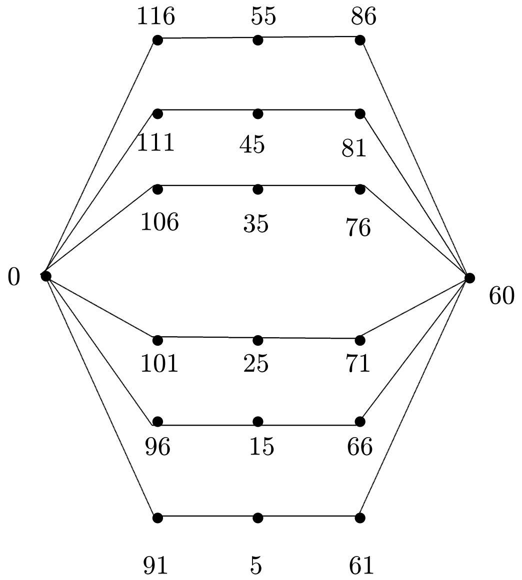
Theorem 2.24. *$P_{4,b}$ for all $b \geq 2$ is one modulo N graceful for every positive integer N.*

Proof:
Define
$\phi(u) = 0$
$\phi(v) = 2Nb$
$\phi(v_j^{(i)}) = N(ab-1) + 1 - b(j-1) - N(i-1)$ for $i = 1, 2, \ldots, b$ and $j = 1, 3$
$\phi(v_2^{(i)}) = 2Nb - N - 2N(i-1)$ for $i = 1, 2, \ldots, b$
Clearly ϕ is $1-1$ and ϕ defines a one modulo N graceful labelling of $P_{4,b}$ for all $b \geq 2$

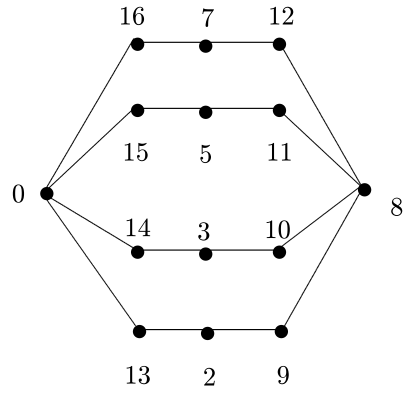
Example 2.25. *One modulo 3 graceful labelling of $P_{4,9}$*



Example 2.26. *One modulo 5 graceful labelling of $P_{4,6}$*



Example 2.27. *Graceful labelling of* $P_{4,4}$



Theorem 2.28. $P_{2,b}$ *for all* $b \geq 2$ *is one modulo* N *graceful for every positive integer* N *.*

Proof:
Define

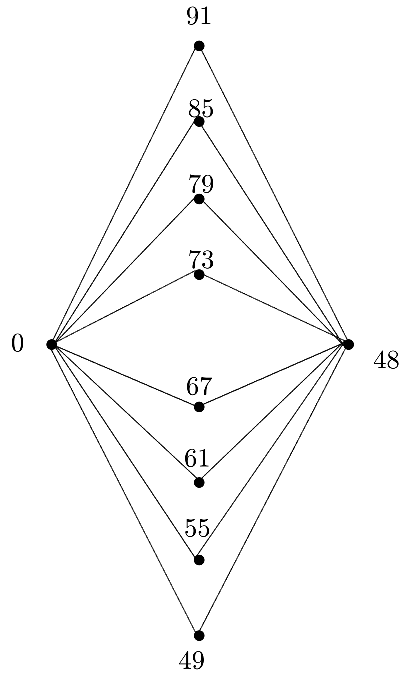
$\phi(u) = 0$

$\phi(v) = Nb$

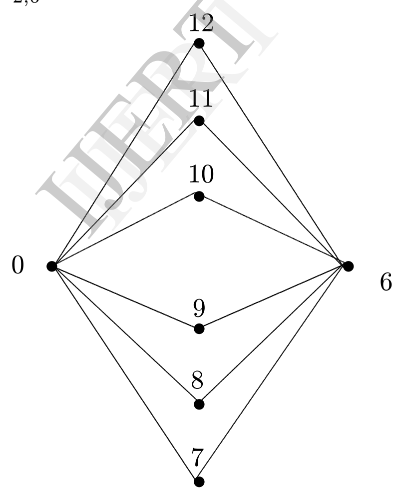
$\phi(v_1^{(i)}) = 2Nb - (N-1) - N(i-1)$ for $i = 1, 2, \ldots, b$

Clearly ϕ is $1-1$ and ϕ defines a one modulo N graceful labelling of $P_{2,b}$ for all $b \geq 2$

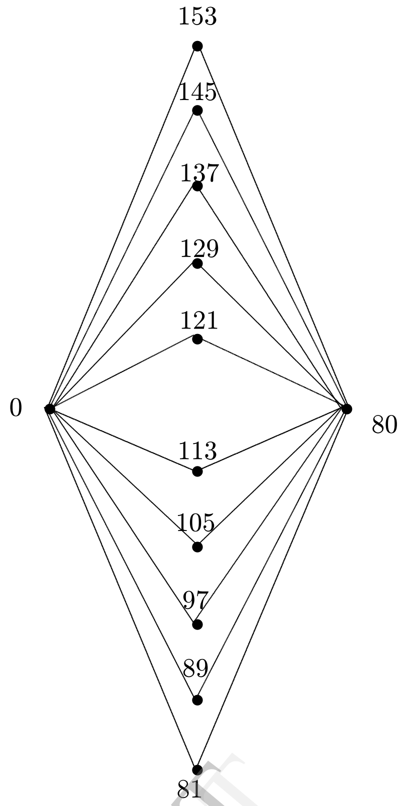
Example 2.29. *One modulo 6 graceful labelling of* $P_{2,8}$



Example 2.30. *Graceful labelling of* $P_{2,6}$



Example 2.31. *One modulo* 8 *graceful labelling of* $P_{2,10}$



Theorem 2.32. $P_{4r-1,4r}$ *for all* $r \geq 1$ *is one modulo* N *graceful for every positive integer* N *.*

Proof:
Define

$\phi(u) = 0$

$$\phi(v) = \begin{cases} N[6 + 16\{\frac{r^2-1}{2}\}] + 1 & \text{if } r \text{ is odd} \\ N[14 + 16\{\frac{r^2}{2} - 1\}] + 1 & \text{if } r \text{ is even} \end{cases}$$

For $i = 2, 3, 4, \ldots, 4r$

$$\phi(v_j^{(i)}) = \begin{cases} N(4r-1)4r - (N-1) - N(i-2) - \frac{N}{2}(4r-1)(j-1) & \text{if } j = 1, 3, 5, \ldots, 2r-1 \\ N(4r-1)4r - (2N-1) - N(i-2) - \frac{N}{2}(4r-1)(j-1) & \text{if } j = 2r+1, 2r+3, 5, \ldots, 4r-3 \end{cases}$$

For $j = 2, 3, 4, \ldots, 2r$

$\phi(v_j^{(i)}) = 4Nr + N(i-2) + \frac{N}{2}(4r-1)(j-2)$ if $i = 2, 4, \ldots, 4r-2$

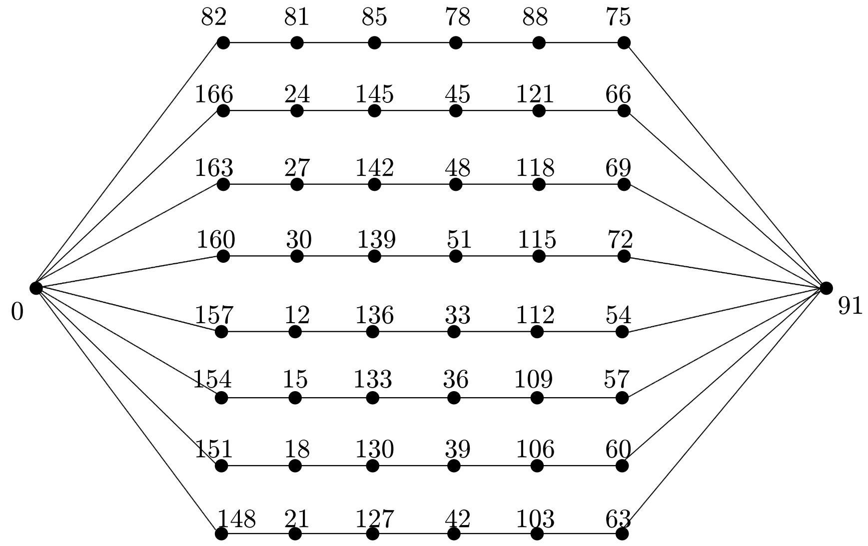
For $i = 2r+1, 2r+2, \ldots, 4r$

$\phi(v_j^{(i)}) = 2Nr + N(i-2r-1) + \frac{N}{2}(4r-1)(j-2)$ if $i = 2, 4, \ldots, 4r-2$

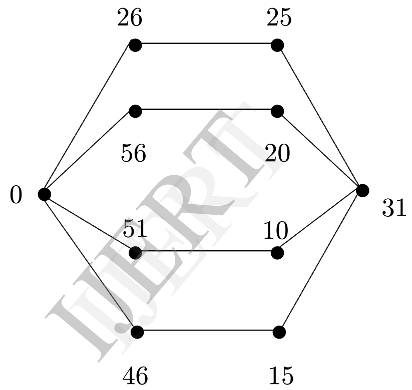
$$\phi(v_1^{(i)}) = \begin{cases} 2Nr(4r-1) - (N-1) + (j-1) & \text{if } j = 1, 3, 5, \ldots, 4r-3 \\ 2Nr(4r-1) - N - (j-2) & \text{if } j = 2, 4, 6, \ldots, 4r-2 \end{cases}$$

Clearly ϕ is $1-1$ and ϕ defines a one modulo N graceful labelling of $P_{4r-1,4r}$ for all $a \geq 2$ and for all odd b .

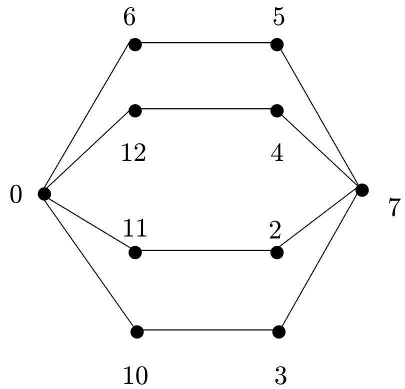
Example 2.33. *One modulo* 3 *graceful labelling of* $P_{7,8}$



Example 2.34. *One modulo 5 graceful labelling of* $P_{3,4}$



Example 2.35. *Graceful labelling of* $P_{3,4}$



Theorem 2.36. $P_{a,b}$ *for all even* $a \geq 4$ *is one modulo* N *graceful and for all even* $b \geq 4$ *for every positive integer* N *.*

Proof: **Case (i)** Let $a = 4r,\ r \geq 1$
Let $b = 2m$,
$Define$

$$x(t) = \begin{cases} 1 & \text{if } t \leq m \\ 0 & \text{if } t > m \end{cases}$$

$$y(j) = \begin{cases} 1 & \text{if } j \equiv 1(mod\ 2) \\ 0 & \text{if } j \equiv 1(mod\ 2) \end{cases}$$

$Define$

$\phi(u) = N(r-1)$

$\phi(v) = 4Nrm - N(r+1)$

$$\phi(v_j^{(1)}) = \begin{cases} y(j)\{8Nrm - (N-1) - \frac{N(2r-1-j)}{2}\} + y(j+1)\{Nr - N - \frac{Nj}{2}\} & \text{if } j = 1, 2, 3, \ldots, 2r-1 \\ y(j)\{4Nrm - (N-1) - \frac{N(j-2r-1)}{2}\} + y(j+1)\{4Nrm - N - \frac{N(j-2r)}{2}\} & \text{if } j = 2r, 2r+1, \ldots, 4r-1 \end{cases}$$

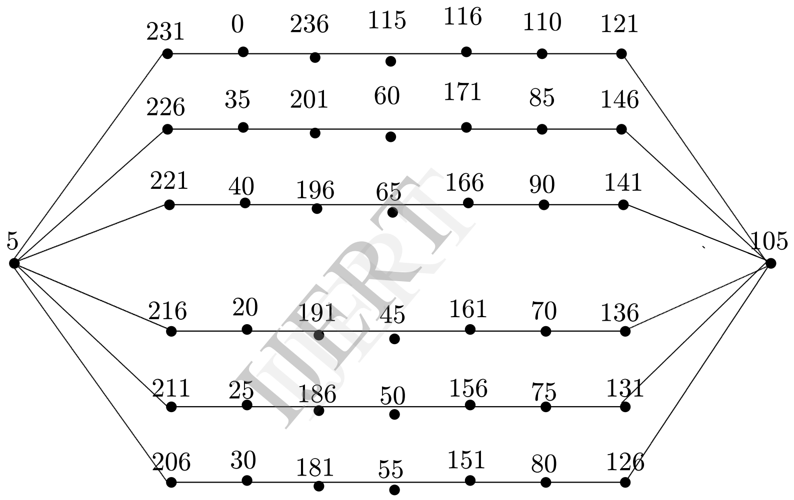
For $i = 2, 3, \ldots, 2m$

$$\phi(v_j^{(i)}) = \begin{cases} 8Nrm - (N-1) - Nr - N(i-2) - \frac{N(j-1)(2m-1)}{2} & \text{if } j = 1, 3, \ldots, 2r-1 \\ 8Nrm - (2N-1) - Nr - N(i-2) - \frac{N(j-1)(2m-1)}{2} & \text{if } j = 2r+1, 2r+3, \ldots, 4r-1 \end{cases}$$

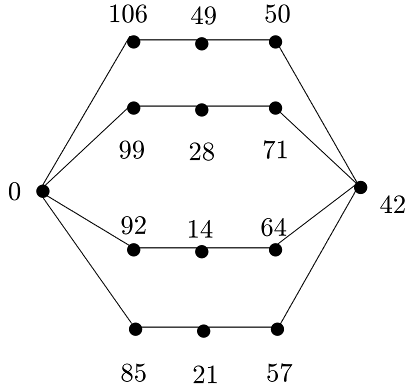
$\phi(v_j^{(i)}) = x(i)\{N(2m+r-1) + N(i-2) + \frac{N(j-2)(2m-1)}{2}\} + (1-x(i))\{N(m+r-1) + N(i-m-1) + \frac{N(j-2)(2m-1)}{2}\}$ if $j = 1, 2, \ldots, 4r-2$

Clearly ϕ is $1-1$ and ϕ defines a one modulo N graceful labelling of $P_{a,b}$ for for all even $a \geq 4$ and for all even $b \geq 4$.

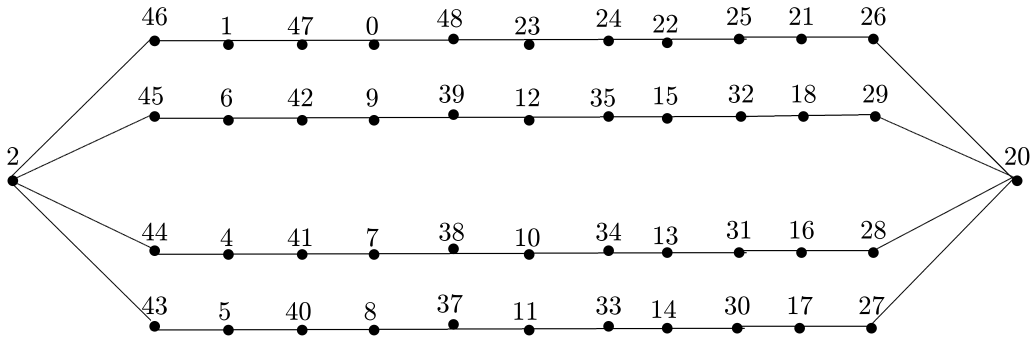
Example 2.37. *One modulo* 5 *graceful labelling of* $P_{8,6}$



Example 2.38. *One modulo* 7 *graceful labelling of* $P_{4,4}$



Example 2.39. *Graceful labelling of* $P_{12,4}$



Case (ii) Let $a = 4r + 2, \ r \geq 1$
Let $b = 2m$,
Define

$$x(t) = \begin{cases} 1 & \text{if } t \leq m \\ 0 & \text{if } t > m \end{cases}$$

$$y(j) = \begin{cases} 1 & \text{if } j \equiv 1(mod\ 2) \\ 0 & \text{if } j \equiv 1(mod\ 2) \end{cases}$$

Define

$\phi(u) = Nr$

$\phi(v) = N(4r + 2)m - Nr$

$\phi(v_j^{(1)}) = y(j)\{2N(4r + 2)m - (N - 1) - \frac{N(2r+1-j)}{2}\} + y(j + 1)\{\frac{N(2r-j)}{2}\}$ if $j = 1, 2, 3, \ldots, 2r + 1$.

$\phi(v_j^{(1)}) = y(j)\{N(4r+2)m+1+\frac{N(j-2r-3)}{2}\}+y(j+1)\{N(4r+2)m-\frac{N(j-2r-2)}{2}\}$ if $j = 2r + 2, 2r + 3, \ldots, 4r + 1$

For $i = 2, 3, \ldots, 2m$

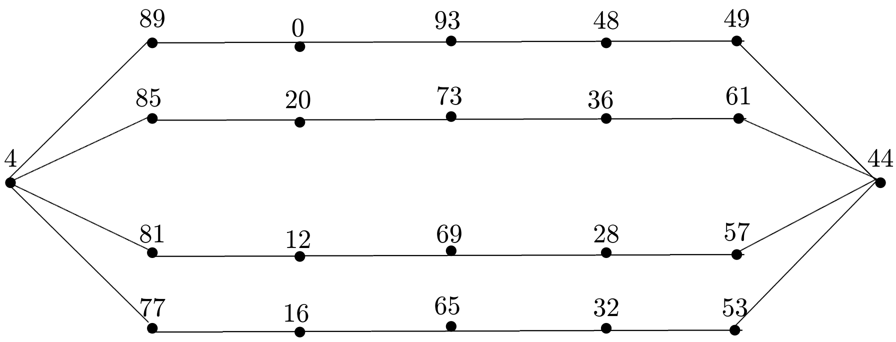
$\phi(v_j^{(i)}) = 2Nm(4r + 2) - (N - 1) - N(r + 1) - N(i - 2) - \frac{N(j-1)(2m-1)}{2}$ if $j = 1, 3, \ldots, 4r + 1$.

$\phi(v_j^{(i)}) = x(i)\{N(r + 2m) + N(i - 2) + \frac{N(j-2)(2m-1)}{2}\} + (1 - x(i))\{N(r + m) + N(i - m - 1) + \frac{N(j-2)(2m-1)}{2}\}$ if $j = 2, 4, \ldots, 2r$

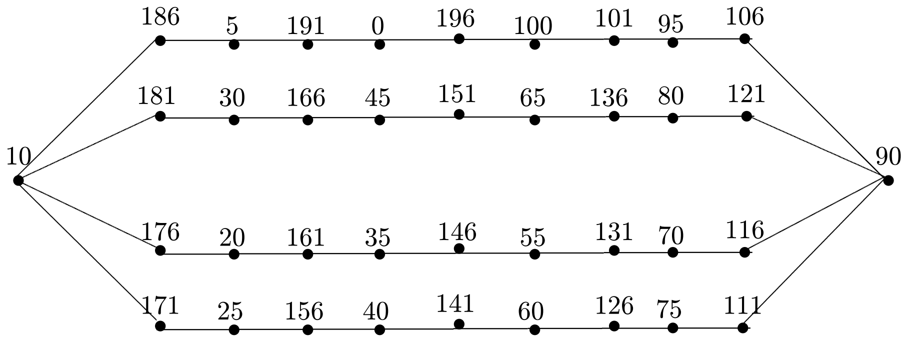
$\phi(v_j^{(i)}) = x(i)\{N(r + 2m + 1) + N(i - 2) + \frac{N(j-2)(2m-1)}{2}\} + (1 - x(i))\{N(r + m + 1) + N(i - m - 1) + \frac{N(j-2)(2m-1)}{2}\}$ if $j = 2r + 2, 2r + 4, \ldots, 4r$

Clearly ϕ is $1 - 1$ and ϕ defines a one modulo N graceful labelling of $P_{a,b}$ for for all even $a \geq 4$ and for all even $b \geq 4$.

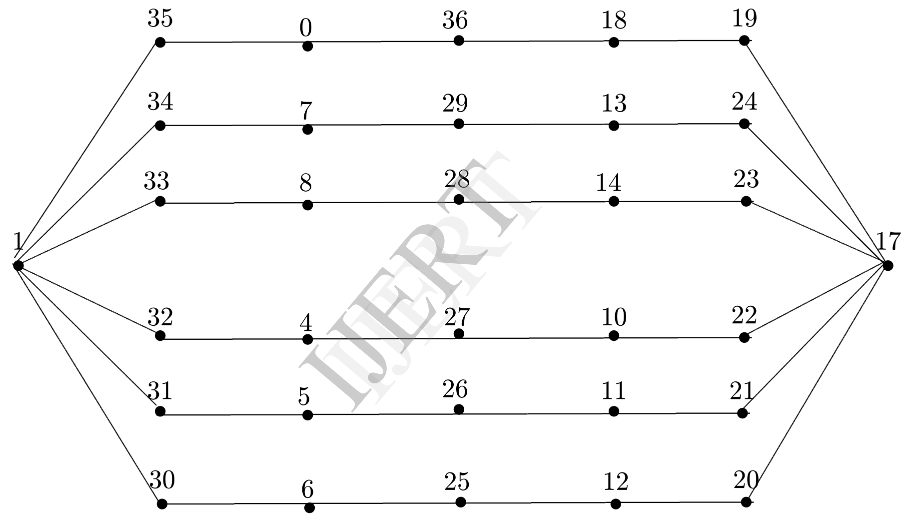
Example 2.40. *One modulo* 4 *graceful labelling of* $P_{6,4}$



Example 2.41. *One modulo* 5 *graceful labelling of* $P_{10,4}$



Example 2.42. *Graceful labelling of* $P_{6,6}$



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