One - Dimensional Stabilization of a Multi-rotor Helicopter

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Abstract—We propose a systematic controller synthesis procedure for a small battery-operated multirotor helicopter whose technical specifications are not given. We construct a dynamic model of the multirotor helicopter from a set of simple experiments and then design a simple controller to validate the identified model. Experimental results suggest that our approach can serve as an efficient way to stabilize multirotor systems with little technical data.

Keywords—Multirotor Helicopter; Drone; Thrust Dynamics

I. INTRODUCTION

In spite of extensive popularity of small battery-operated multirotor helicopters usually called as *drones*, in public and mass media, it is essentially a difficult task to control those helicopters [1-4].

Multi-rotor helicopters have six degrees of freedom in three-dimensional space with a quite complicated rigid body dynamics. In addition, those helicopters are generally equipped with more than three independent actuators, belonging to the unstable nonlinear multi-input multi-output (MIMO) systems. Actually, controller synthesis methodologies for such MIMO systems are highly complicated and beyond the scope of undergraduate control courses.

Furthermore, in synthesizing a controller for a multirotor helicopter, one frequently meets a situation that technical specifications of various components of the helicopter are disclosed for end-users. Even when technical details of some components are available, we are still in need of a proper dynamic modeling of the combined components. For an example, even with datasheets of a certain propeller and a motor, it is far from feasible to characterize the thrust dynamics of the propeller-motor combination in a theoretical way. This sort of difficulty arises as a significant obstacle when teachers adopt low-cost helicopters for educational purposes in higher education.

It seems that the most critical step in finding a dynamic model of a multi-rotor helicopter is how to model the dynamic properties of an actuator subsystem. Generally, an actuator subsystem of a helicopter is composed of three components: (i) a brushless DC (BLDC) motor, (ii) a motor driver board composed of a microcontroller and current amplifiers (iii) a propeller. Jung-Ho Moon Department of Electrical Engineering Gangneung-Wonju National University, South Korea

The dynamics of an actuator subsystem depends not only on a complicated and highly nonlinear aerodynamics but also on a programmable firmware of a microcontroller in a motor driver board. This means that the dynamics of a known actuator subsystem can be modified by a new firmware whose detailed information is usually unknown.

This situation motivated us to propose an experimental identification method for the actuator dynamics in [5]. Later we replaced the manual procedures in [5] with an automatic thrust measurement system in [6]. In addition, we validate the thrust transfer function from our approach in [5, 6], by a comparison to a FFT (fast Fourier transform)-based transfer function measurement result in [7].

In addition, our experimental thrust dynamic model was applied to a dynamic modelling of a propeller-driven pendulum for which a stabilizing controller was tested in [8]. We observed a stable motion of the pendulum but the performance of the closed loop system in [8] was not very satisfactory.

In this paper, we consider a stabilization control of a onedimensional multi-rotor helicopter. A key aim of this work is to justify that our experimental thrust modeling method proposed in [5-7] can be an effective approach for a control of a multirotor system whose technical information is not available.

II. SYSTEM MODEL

Our test-bed system shown in Fig.1 is composed of two pairs of propeller actuators. The BLDC motor is driven by the ESC and a speed command to the ESC (electronic speed controller) board has a form of PWM (pulse width modulation) digital pulses.

Traditionally the driving frequency of PWM signals for most commercial ESC's are fixed to be about 50 Hz but recent ESC's, especially designed for multi-rotor helicopters, can also accommodate driving frequencies of up to several hundred hertz.

The speed of a common BLDC motor combined with an ESC, is determined by the duration of a high signal. For an example, both 5% duty ratio with a 50 Hz signal (a period of 20 millisecond) and 10% duty ratio with 100 Hz (a period of 10 millisecond) signal have the same one millisecond duration of high signal and thus will give the same motor speed. More detailed discussions on ESC signals can be found in [8].



Fig. 1. Multirotor Helicopter

A. Thrust Model

In order to obtain a transfer function from an ESC command to a thrust force, we designed a measurement system as shown in Fig. 2. The measurement system is composed of a load cell for a thrust measurement and an optical sensor for measuring a propeller speed.

By changing the duration of high signal with a fixed 200 Hz PWM frequency, we have obtained the results shown in Fig. 3 and Fig. 4. A relation between the duration of high signal and the thrust force is shown in Fig. 3.

We chose the next operating (equilibrium) point

$$(u_e, T_e) = (1.4 \text{ msec}, 2.38 \text{ N})$$
 (1)

where u_e denotes an equilibrium ESC command (highduration of 1.4 millisecond) and T_e is the corresponding thrust. Then, around (u_e, T_e) , the linearized relation between an input perturbation Δu and a thrust perturbation ΔT could be determined from the slope of a straight-line tangent to the operating point. This gives the next *static* or DC relation

$$\frac{\Delta T(0)}{\Delta u(0)} = 8.78 \text{ (N/msec)}$$
 (2)

The experiment results in Fig. 4 show that a thrust force is approximately a quadratic function of a propeller speed. A quadratic interpolation (curve fitting) of experimental data gives the next relation

$$T = 1.3215 \times 10^{-7} \,\omega^2 \tag{3}$$



Fig. 2. Thrust Measurement System



Fig. 3. Thrust versus Duration of High Signal



Fig. 4. Thrust versus Rotor Speed

where T is the thrust force (N) and ω denotes the angular speed in RPM of the propeller.

For a dynamic model of the thrust subsystem, we assume a first-order dynamic relation

$$\frac{\Delta T(s)}{\Delta u(s)} = \frac{k}{s+1/\tau} \tag{3}$$

where two unknown parameters $\{k, a\}$ are to be identified from the static relation (2), that is, $k\tau = 8.78$, and a step response of the thrust subsystem. For this, we applied a step ESC command ranging from 1.2 to 1.8 millisecond to the same measurement system in Fig. 2 and obtained a dynamic thrust response shown in Fig. 5. The magnitude of thrust forces are normalized in Fig. 5 for better readability and the time constant turns out to be around 0.07 second, i.e., $\tau =$ 0.07 in (3).

This result and the previous fact $k\tau = 8.78$ finally give us a transfer function of the thrust system given as

$$\frac{\Delta T(s)}{\Delta u(s)} = \frac{k}{s+1/\tau} = \frac{8.78}{0.07 \, s+1} \tag{4}$$

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Fig. 5. Step response of actuator subsystem

B. Overall Dynamic Model

From mass distributions and geometry of all components in our multi-rotor system shown in Fig. 1, we could roughly estimate the overall moment of inertia of the moving parts as

$$J = 9.4 \times 10^{-3} \,(\text{kg} \cdot \text{m}^2). \tag{5}$$

In addition, from the geometry of our multirotor, we found that the rotational moment M from two thrust forces is given by

$$M = \alpha \left(T^R - T^L \right)$$
 (Nm) (6)

where $\alpha = 0.208$ and $T^{R,L}$ denotes the two thrusts of the right and left motors.

Let us define the right and left ESC commands as

$$u^R = u_e + \Delta u, \qquad u^L = u_e - \Delta u. \tag{7}$$

Then, from (4) and (6), it follows that our one-dimensional multirotor pendulum has a dynamics

$$\frac{\Delta\theta(s)}{\Delta u(s)} = \frac{2\alpha k}{Js^2 \left(s + \frac{1}{\tau}\right)} = \frac{3.652}{s^2 (0.0094s + 0.1343)} \tag{8}$$

where $\Delta\theta(s)$ denotes a perturbation of the multirotor angle near an equilibrium angle $\theta_e = 0$ and Δu is a control input in the unit of millisecond for two ESCs around the chosen equilibrium point $u_e = 1.4$ milliseconds.

III. EXPERIMENT

A. Controller Design

As our key aim of this paper is to validate the transfer function experimentally identified, we chose the following simple *lead* controller

$$\frac{\Delta u(s)}{\Delta e(s)} = 0.85 \frac{s+1}{s+15} \tag{9}$$

where the tracking error $\Delta e(s)$ is defined as

$$\Delta e(s) = \theta_{ref} - \Delta \theta(s). \tag{10}$$

This analog controller is converted to the next discretetime controller with a sampling $f_{samp} = 300$ (Hz)

$$\frac{\Delta u(z)}{\Delta e(z)} = 0.85 \ \frac{z - 0.9967}{z - 0.9512}.$$
(11)

This digital controller is implemented with an *Arduino Due*[©] microcontroller [10].

B. Angle Measurement

The rotational angle of our multirotor helicopter is measured with a *MPU6050* acceleration/gyroscope sensor [11].

It is widely known that the raw angle data of the sensor is unacceptably noisy particularly when motors are running. As a result, it was essential to adopt either a low pass filtering or a sensor fusion technique with gyroscope data.

In this paper, for simplicity, we used a 9th order FIR filter with a cutoff frequency 10 Hz and a sampling rate $f_{samp} = 300$ (Hz).

C. Stabilization Experiment

For stabilization experiments, we made an experimental setup in Fig. 6 in which the multirotor is suspended to a fixed structure by two wires and therefore has only one-dimensional rotational freedom.

Moreover, we used an arbitrary signal generator *Agilent* 33220A to generate an analog reference angle signal that the multirotor is expected to follow.

The voltage signal from the signal generator was captured by an AD (analog to digital) converter inside the controller board. At the same time, the digital angle data measured by the acceleration sensor was converted to an analog signal with a DA (digital to analog) converter inside the same controller board for an easy monitoring with an oscilloscope.

For a square reference signal of a frequency 1/20 Hz, we obtained the tracking performance shown in Fig. 7. Even though the steady state response shows fluctuations, in overall, the closed loop response is acceptable. For a ramp reference signal, we obtained a similar performance in Fig. 8.



Fig. 6. Experiment Setup



Fig. 7. Tracking performance (square reference, 0.05Hz)



Fig. 8. Tracking performance (ramp reference, 0.05 Hz)

Experimental results suggest that our transfer function model experimentally obtained is quite reliable.

IV. CONCLUSION

As a case study, we have shown that a transfer function model of a small multi-rotor helicopter, whose technical specifications are unknown, can be easily obtained from a sequence of elementary experiments. The identified dynamic model was combined with a simple controller to render a closed loop system model. The good performance of the closed loop system suggests that our experimental model identification methodology is valid and effective.

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