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On $\hat{\mu}$-Continuous Functions In Topological Spaces

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ABSTRACT

In this paper, we introduce $\hat{\mu}$-continuous map and their relations with some generalized continuous maps. Various properties and characterizations of $\hat{\mu}$- continuous map are discussed by using $\hat{\mu}$-closure and $\hat{\mu}$-interior under certain conditions.

KEYWORDS :

$\hat{\mu}$-continuous map, $\hat{\mu}$-closure, $\hat{\mu}$-interior.

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1. INTRODUCTION

Many others ([4] [5] [6] [13]) working in the field of general topology have shown more interest in studying the concepts of generalizations of continuous map. A weak form of continuous map called g–continuous map was introduced by Balachandran, Sundaram and Maki [2].

M.K.R.S.Veerakumar has introduced several generalized closed sets namely, $\hat{g}$-closed sets, g*-closed sets, g*p–closed sets, *g–closed sets, α*g–closed sets, *gs-closed sets, μ-closed sets, μp-closed sets and μs-closed sets and their continuity. The concept of, $\hat{\mu}$-closed sets was introduced by S.Pious Missier and E.Sucila [12]. In this paper we introduce the concept of $\hat{\mu}$-continuous map in topological spaces.

2. PRELIMINARIES

Throughout this paper, we consider spaces on which no separation axioms are assumed unless explicity stated . For $A \subset X$, the closure and interior of A is denoted by cl(A) and int(A) respectively. The complement of A is denoted by A^C.

Definition 2.1 :

A subset A of a topological space (X, τ) is called

1. a preopen set [9] if $A \subseteq int(cl(A))$ and preclosed if $cl(int(A)) \subseteq A$.
2. a semiopen set [6] if $A \subseteq cl(int(A))$ and a semiclosed set if $int(cl(A)) \subseteq A$.
3. an α-open set [11] if $A \subseteq int(cl(int(A))$ and α-closed set if $cl(int(cl(A))) \subseteq A$.
4. a semi preopen set [1] if $A \subseteq cl(int(cl(A)))$ and a semipreclosed set if $int (cl(int(A))) \subseteq A$.

The intersection of all semiclosed (resp. preclosed, semipreclosed, α-closed) sets containing a subset A of X is called semiclosure (resp. preclosure, semipreclosure, α-closure) of A is denoted by scl(A) (resp. pcl(A), spcl(A), αcl(A)).

Definition 2.2 :

A subset A of a topological space (X, τ) is called

1. a generalized closed set (briefly g–closed [7] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ).
2. an α-generalized closed set (briefly αg–closed) [8] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ).
3. a $\hat{g}$-closed set [15] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semiopen in (X, τ).
4. a *g–closed set [16] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $\hat{g}$-open in (X, τ).
5. a g*-closed set [16] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is g-open in (X, τ).
6. a g*-preclosed set (briefly g*p–closed) [17] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is g–open in (X, τ).
7. a *g- semiclosed set [20] (briefly *gs-closed) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is $\hat{g}$-open in (X, τ).
8. a α*g-closed set [22] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$, and U is $\hat{g}$-open in (X, τ).
9. a gα*-closed set [8] if $\alpha cl(A) \subseteq int (U)$ whenever $A \subseteq U$ and U is α-open in (X, τ).
10. a ψ-closed set [19] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is sg–open in (X, τ).
11. a g* ψ-closed set [19] if $\psi\, cl(A) \subseteq U$ whenever $A \subseteq U$ and U is g–open in (X, τ).

12. a μ-closed set [21] if cl(A) ⊆ U whenever A ⊆ U and U is gα*-open in (X, τ).
13. a μ-preclosed set (briefly μp–closed) [22] if pcl(A) ⊆ U whenever A ⊆ U and U is gα*- open in (X, τ) .
14. a μ-semiclosed set (briefly μs–closed) [23] if scl(A) ⊆ U whenever A ⊆ U and U is gα*-open in (X, τ).
15. a $\hat{\mu}$-closed set [12] if scl(A) ⊆ U whenever A ⊆ U and U is μ-open in (X, τ). The complement of $\hat{\mu}$-closed set is called $\hat{\mu}$- open set. The class of all $\hat{\mu}$-open (resp. $\hat{\mu}$-closed) subsets of X is denoted by $\hat{\mu}$o(X, τ), (resp. $\hat{\mu}$c(X, τ)).

Definition 2.3:

A map f : (X, τ) → (Y, σ) is called

1. semicontinuous [6] if $f^{-1}(V)$ is semiclosed in (X, τ) for every closed set V in (Y, σ).
2. g-continuous [2] if $f^{-1}(V)$ is g-closed in (X, τ) for every closed set V in (Y, σ).
3. α-continuous [10] if $f^{-1}(V)$ is α-closed in (X,τ) for every closed set V in (Y, σ).
4. αg–continuous [8] if $f^{-1}(V)$ is αg-closed in (X, τ) for every closed set V in (Y, σ).
5. $\hat{g}$-continuous [15] if $f^{-1}(V)$ is $\hat{g}$-closed in (X, τ) for every closed set V in (Y, σ).
6. *g-continuous [15] if $f^{-1}(V)$ is *g-closed in (X, τ) for every closed set V in (Y, σ).
7. α*g- continuous [15] if $f^{-1}(V)$ is α*g-closed in (X, τ) for every closed set V in (Y, σ).
8. g*- continuous [16] if $f^{-1}(V)$ is g*-closed in (X, τ) for every closed set V in (Y, σ).
9. g*p- continuous [17] if $f^{-1}(V)$ is g*p-closed in (X, τ) for every closed set V in (Y, σ).
10. *gs- continuous [20] if $f^{-1}(V)$ is *gs-closed in (X, τ) for every closed set V in (Y, σ).
11. g*ψ- continuous [19] if $f^{-1}(V)$ is g*ψ-closed in (X, τ) for every closed set V in (Y, σ).
12. μ-continuous [21] if $f^{-1}(V)$ is μ-closed in (X, τ) for every closed set V in (Y, σ).
13. μp- continuous [22] if $f^{-1}(V)$ is μp-closed in (X, τ) for every closed set V in (Y, σ).
14. μs- continuous [23] if $f^{-1}(V)$ is μs-closed in (X, τ) for every closed set V in (Y, σ).

Definition 2.4:

A topological space (X, τ) is called a

1. T$\hat{\mu}$-space [12] if every $\hat{\mu}$-closed set is closed.

2. αT$\hat{\mu}$-space [12] if every $\hat{\mu}$-closed set is α-closed.

3. sT$\hat{\mu}$-space [12] if every $\hat{\mu}$-closed set is semiclosed.

4. pT$\hat{\mu}$-space [12] if every $\hat{\mu}$-closed set is preclosed.

5. spT$\hat{\mu}$-space [12] if every $\hat{\mu}$-closed set is semipreclosed.

6. μT$\hat{\mu}$-space [12] if every $\hat{\mu}$-closed set is μ-closed.

3. $\hat{\mu}$-CONTINUITY

We introduce the following definition.

Definition 3.1:

A function f : (X, τ) → (Y, σ) is called $\hat{\mu}$-continuous if f^{-1}(V) is $\hat{\mu}$-closed subset of (X, τ) for every closed subset V of (Y, σ).

Proposition 3.2 :

Every continuous (resp. semicontinuous) map is $\hat{\mu}$-continuous but not conversely.

Proof :

The proof follows from the fact that every closed (resp. semiclosed) set is $\hat{\mu}$-closed.

Example 3.3 :

Let X = Y = {a, b, c}, τ = {X, ϕ, {a}, {b, c}} and σ = {Y, ϕ, {b}}. Define a map f : X → Y by f(a) = a, f(b) = b and f(c) = c is $\hat{\mu}$-continuous. However f is neither continuous nor semicontinuous, since for the closed set U = {a, c} in Y, f^{-1}(U) = {a, c} which is neither closed nor semiclosed in X.

Proposition 3.4 :

Every α - continuous map is $\hat{\mu}$-continuous but not conversely.

Proof : The proof follows from the fact that every α-closed set is $\hat{\mu}$-closed.

Example 3.5:

Let X = Y = {a, b, c}, τ = {X, φ, {b}, {c, a}} and σ = {Y, ϕ, {c}, {b, c}}.Define a map f : X → Y by f(a) = a, f(b) = b and f(c) = c. This map is $\hat{\mu}$-continuous but not α-continuous, since for the closed set U = {a} in Y, f^{-1} (U) = {a} is not α-closed in X.

Thus the class of all $\hat{\mu}$-continuous maps properly contains the classes of continuous maps, semicontinuous maps and α-continuous maps.

Remark 3.6 :

The following examples shows that $\hat{\mu}$-continuity is independent of μ-continuity.

Example 3.7 :

Let X = Y= {a, b, c}, τ = {X, ϕ, {b}, {a, c}} and σ = {Y, ϕ, {b, c}}. Define f : X $\rightarrow$ Y by f(a) = a, f(b) = c, f(c) = b is $\hat{\mu}$-continuous but not μ-continuous, since for the closed set U = {a} in Y, f^{-1}(U) = {a} is not μ-closed in X.

Example 3.8:

Let X = Y = {a, b, c}, τ = {X, φ, {b}, {a, b}} and σ = {Y, ϕ, {a}}. Let f : X $\rightarrow$ Y be an identity map. Here the map f is μ-continuous but not $\hat{\mu}$-continuous, since for the closed set U = {b, c} in Y, f^{-1}(U) = {b, c} is not $\hat{\mu}$-closed in X.

Remark 3.9 :

The following examples shows that $\hat{\mu}$-continuous is independent of μp – continuous and μs-continuous.

Example 3.10 :

Let X = Y = {a, b, c}, τ = {X, ϕ, {a}, {a, b}} and σ = {Y, ϕ, {c}}. Let f : X $\rightarrow$ Y be an identity map. Then f is μp – continuous and μs – continuous but not $\hat{\mu}$-continuous, since for the closed set U = {a, b} in Y, f^{-1}(U) = {a, b} is not $\hat{\mu}$-closed in X.

Example 3.11:

Let X = Y = {a, b, c}, τ = {X, ϕ, {a}, {c}, {a, c}} and σ = {Y, ϕ, {b, c}}. Define a map f : X $\rightarrow$ Y by f(a) = a, f(b) = c and f(c) = b. Here the map f is $\hat{\mu}$-continuous but not μp-continuous, since for the closed set U = {a} in Y, f^{-1}(U) = {a} is not μp-closed in X.

Example 3.12 :

Let X = Y = {a, b, c}, τ = {X, ϕ, {a}, {b, c}} and σ = {Y, ϕ, {b}}. Let f : X $\rightarrow$ Y be an identity map. Then the map f is $\hat{\mu}$-continuous but not μs – continuous, since for the closed set U = {a, c} in Y, f^{-1}(U) = {a, c} is not μs –closed in X.

Remark 3.13:

The following examples shows that $\hat{\mu}$-continuous is independent of *g-continuous, α*g-continuous and *gs-continuous.

Example 3.14:

Let X = Y = {a, b, c}, τ = {X, ϕ, {a}, {b, c}} and σ = {Y, ϕ, {c}}. Define a map f : X $\rightarrow$ Y by f(a) = a, f(b) = c and f(c) = b. Here the map f is $\hat{\mu}$-continuous but it is not *g-continuous, α*g–continuous and *gs–continuous. Since for the closed set U = {a, b} in Y, f^{-1}(U) = {a,c} which is not *g-closed, α*g-closed and *gs-closed in X.

Example 3.15:

Let X = Y = {a, b, c}, τ = {X, ϕ, {a}, {a, b}} and σ = {Y, ϕ, {c}}. Let f : X $\rightarrow$ Y be a map defined by f(a) = b, f(b) = c and f(c) = a. Here the map f is *g-continuous, α*g-continuous and *gs-continuous but not $\hat{\mu}$-continuous, since for the closed set U = {a, b} in Y, f^{-1}(U) = {a, c} which is not $\hat{\mu}$-closed in X.

Remark 3.16:

The following examples shows that $\hat{\mu}$-continuous is independent of g*-continuous, g-continuous, αg – continuous and g*p-continuous.

Example 3.17:

Let X = Y = {a, b, c}, τ = {X, ϕ, {b}, {c}, {b, c}} and σ = {Y, ϕ, {a, b}}. Let f : X $\rightarrow$ Y be an identity map. Here the map f is $\hat{\mu}$-continuous but not g*-continuous, g-continuous, αg-continuous and g*p-continuous. Since for the closed set U = {c} in Y, f^{-1}(U) = {c} which is not g*-closed, g-closed, α g-closed and g*p-closed in X.

Example 3.18:

Let X = Y = {a, b, c}, τ = {X, ϕ, {a}, {a, c}} and σ = {Y, ϕ, {c}}. Let f : X $\rightarrow$ Y be an identity map. Then the map f is g*-continuous, g-continuous, αg-continuous and g*p-continuous but not $\hat{\mu}$-continuous. Since for the closed set U = {a, b} in Y, f^{-1}(U) = {a, b} is not $\hat{\mu}$-closed in X.

Remark 3.19:

The following examples shows that $\hat{\mu}$-continuous is independent of g*ψ -continuous.

Example 3.20:

Let X = Y = {a, b, c}, τ = {X, ϕ, {a}, {b, c}} and σ = {Y, ϕ, {b, c}}. Define a map f : X $\rightarrow$ Y by f(a) = c, f(b) = a and f(c) = b. Here f is $\hat{\mu}$-continuous but not g*ψ - continuous, since for the closed set U = {a} in Y, f^{-1}(U) = {b} which is not g*ψ-closed in X.

Example 3.21:

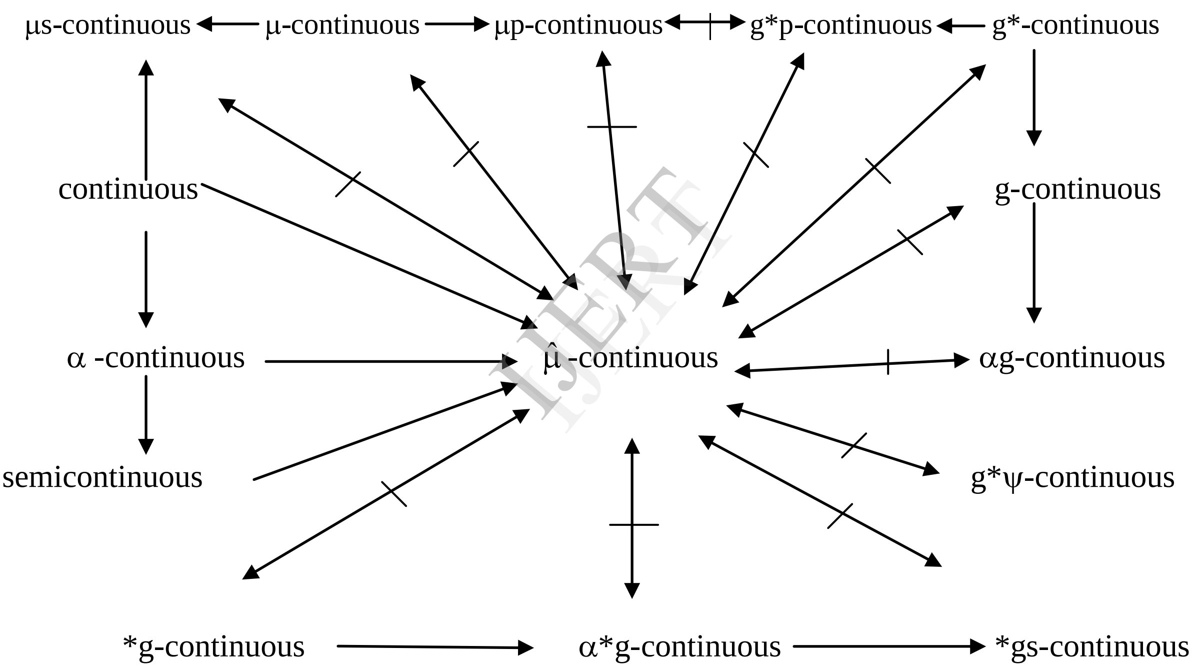
Let X = Y = {a, b, c}, τ = {X, ϕ, {b}, {a, b}} and σ = {Y, ϕ, {a}}. Let f : X $\rightarrow$ Y be an identity map. Then f is g*ψ - continuous but not $\hat{\mu}$-continuous, since for the closed set U = {b, c} in Y, f^{-1}(U) = {b, c} is not $\hat{\mu}$-closed in X.

Remark 3.22:

The following diagram shows the relationship established between $\hat{\mu}$-continuous function and some other continuous functions. A $\rightarrow$ B (resp.A ↮ B) represents A implies B but not conversely (resp. A and B are independent of each other).

From the above Propositions and Examples, we have the following diagram.

DIAGRAM



Remark 3.23:

The composition of two $\hat{\mu}$-continuous maps need not be $\hat{\mu}$-continuous.

Example 3.24:

Let X = Y = Z = {a, b, c}, τ = {X, ϕ, {a}, {a, c}}, σ = {Y, ϕ, {b, c}} and η = {Z, ϕ, {c}}. Define a map f : X $\rightarrow$ Y by f(a) = b, f(b) = c and f(c) = a. Let g : Y $\rightarrow$ Z be an identity map. Then both f and g are $\hat{\mu}$-continuous, but $g \circ f$ is not $\hat{\mu}$-continuous. Since for the closed set U = {a, b} in Z, $(g \circ f)^{-1}$ (U) = f^{-1} (g^{-1}(U)) = f^{-1}({a, b}) = {c, a} which is not $\hat{\mu}$-closed in X.

Proposition 3.25:

If $f : X \to Y$ is $\hat{\mu}$-continuous and $g : Y \to Z$ is continuous then their composition $g \circ f : X \to Z$ is $\hat{\mu}$- continuous.

Proof :

Clearly follows from the definitions.

Proposition 3.26:

A map $f : (X, \tau) \to (Y, \sigma)$ is $\hat{\mu}$-continuous if and only if $f^{-1}(U)$ is $\hat{\mu}$-open in (X, τ), for every open set U in (Y, σ).

Proof :

Let $f : X \to Y$ be $\hat{\mu}$-continuous and U be an open set in Y. Then $f^{-1}(U^c)$ is $\hat{\mu}$-closed in X. But $f^{-1}(U^c) = (f^{-1}(U))^c$ and $f^{-1}(U)$ is $\hat{\mu}$-open in X. Converse is similar.

4. $\hat{\mu}$- CLOSURE AND $\hat{\mu}$- INTERIOR

Definition 4.1 :

For every set $E \subset X$ we define the $\hat{\mu}$-closure of E to be the intersection of all $\hat{\mu}$-closed sets containing E. In symbols, $\hat{\mu}\,cl(E) = \cap\{A : E \subset A, A \in \hat{\mu}C(X, \tau)\}$.

Lemma 4.2 :

For any $E \subset X$, $E \subset \hat{\mu}\,cl(E) \subset cl(E)$.

Proof :

Since every closed set is $\hat{\mu}$-closed but not conversely.

Lemma 4.3 :

If $A \subset B$, then $\hat{\mu}\,cl(A) \subseteq \hat{\mu}\,cl(B)$

Proof :

Clearly follows from Definition 4.1.

Lemma 4.4:

If E is $\hat{\mu}$-closed, then $\hat{\mu}\,cl(E) = E$.

Proof :

Clearly follows from Definition 4.1.

Proposition 4.5 :

Let A be a subset of a topological space X. For any $x \in X$, $x \in \hat{\mu}\,cl(A)$ if and only if $U \cap A \neq \phi$ for every $\hat{\mu}$-open set U containing x.

Proof :

Necessity : Suppose that $x \in \hat{\mu}cl(A)$. Let U be an $\hat{\mu}$-open set containing x such that $U \cap A = \phi$ and so $A \subset U^c$. But U^c is $\hat{\mu}$-closed and hence $\hat{\mu}cl(A) \subset U^c$. Since $x \notin U^c$ we obtain $x \notin \hat{\mu}cl(A)$ which is contrary to the hypothesis.

Sufficiency: Suppose that every $\hat{\mu}$-open set of X containing x meets A. If $x \notin \hat{\mu}cl(A)$, then there exists an $\hat{\mu}$-closed F of X such that $A \subset F$ and $x \notin F$. Therefore, $x \in F^c$ and F^c is an $\hat{\mu}$-open set containing x. But $F^c \cap A = \phi$. This is contrary to the hypothesis.

Definition 4.6 :

For any $A \subset X$, $\hat{\mu}int(A)$ is defined as the union of all $\hat{\mu}$-open sets contained in A. That is, $\hat{\mu}int(A) = U\{U : U \subset A \text{ and } U \in \hat{\mu}o(X, \tau)\}$

Lemma 4.7:

For any set $A \subset X$, $int(A) \subset \hat{\mu}\ int(A)$.

Proof :

For any two subsets A_1 and A_2 of X,

(i) If $A_1 \subset A_2$, then $\hat{\mu}int(A_1) \subseteq \hat{\mu}int(A_2)$.

(ii) $\hat{\mu}\ int(A_1 \cup A_2) \supset \hat{\mu}int(A_1) \cup \hat{\mu}int(A_2)$.

Lemma 4.9 :

If A is $\hat{\mu}$-open, then $A = \hat{\mu}int(A)$.

Proof :

Clearly follows from the Definition 4.6.

Proposition 4.9 :

Let A be a subset of a space X, then the following are true.

(i) $(\hat{\mu}int(A))^c = \hat{\mu}cl(A^c)$

(ii) $\hat{\mu}int(A) = (\hat{\mu}cl(A^c))^c$

(iii) $\hat{\mu}cl(A) = (\hat{\mu}int(A^c))^c$

Proof :

(i) Let $x \in (\hat{\mu}int(A))^c$. Then $x \notin \hat{\mu}int(A)$. That is, every $\hat{\mu}$-open set U containing x such that $U \not\subset A$. Thus every $\hat{\mu}$-open set U containing x is such that $U \cap A^c \neq \phi$. By Proposition 4.5, $x \in \hat{\mu}cl(A^c)$ and therefore, $(\hat{\mu}int(A))^c \subset \hat{\mu}cl(A^c)$.

Conversely, let $x \in \hat{\mu}\,cl(A^c)$. Then by Proposition 4.5, every $\hat{\mu}$-open set U containing x is such that $U \cap A^c \neq \phi$. By definition 4.6, $x \notin \hat{\mu}\,int(A)$, hence $x \in (\hat{\mu}\ int(A))^c$ and so $\hat{\mu}\,cl(A^c) \subset (\hat{\mu}\ int(A))^c$. Thus $\hat{\mu}\,cl(A^c) = (\hat{\mu}\,int\,(A))^c$

(ii) Follows by taking complements in (i)

(iii) Follows by replacing A by A^c in (i)

Proposition 4.10 :

For a subset A of a topological space X, the following conditions are equivalent :

(i) $\hat{\mu}\,o\,(X, \tau)$ is closed under any union,

(ii) A is $\hat{\mu}$-closed if and only if $\hat{\mu}\,cl(A) = A$.

(iii) A is $\hat{\mu}$-open if and only if $\hat{\mu}\,int(A) = A$.

Proof :

(i) $\Rightarrow$ (ii) : Let A be an $\hat{\mu}$-closed set. Then by definition of $\hat{\mu}$-closure, we get $\hat{\mu}\,cl(A) = A$. Conversely, assume that $\hat{\mu}\,cl(A) = A$. For each $x \in A^c$, $x \notin \hat{\mu}\,cl(A)$, by Proposition 4.5 there exist an $\hat{\mu}$-openset G_x such that $G_x \cap A = \phi$ and hence $x \in G_x \subset A^c$.

Therefore, we obtain $A^c = \bigcup_{x \in A^c} G_x$. By (i) A^c is $\hat{\mu}$-open and hence A is $\hat{\mu}$-closed.

(ii) $\Rightarrow$ (iii) = Follows by (ii) and proposition 4.9.

(iii) $\Rightarrow$ (i) = Let $\{U_\alpha / \alpha \in \wedge\}$ be a family of $\hat{\mu}$-open sets of X. Put $U = \bigcup_{\alpha} U_\alpha$. For each $x \in U$, there exist $\alpha(x) \in \wedge$ such that $x \in U_{\alpha(x)} \subset U$. Since $U_{\alpha(x)}$ is $\hat{\mu}$-open, $x \in \hat{\mu}\,int(U)$ and so $U = \hat{\mu}\,int(U)$. By (iii), U is $\hat{\mu}$-open. Thus $\hat{\mu}\ o\,(X, \tau)$ is closed under any union.

Proposition 4.11:

In a topological space X, assume that $\hat{\mu}\,o\,(X, \tau)$ is closed under any union. Then $\hat{\mu}\,cl(A)$ is an $\hat{\mu}$-closed set for every subset A of X.

Proof :

Since $\hat{\mu}\,cl(A) = \hat{\mu}\,cl\,(\hat{\mu}\,cl(A))$ and by Proposition 4.10, we get $\hat{\mu}\,cl(A)$ is an $\hat{\mu}$-closed set.

Theorem 4.12 :

Let $f : X \to Y$ be a map. Assume that $\hat{\mu}\,o(X, \tau)$ is closed under any union. Then the following are equivalent :

(i) The map f is $\hat{\mu}$-continuous;

(ii) The inverse of each open set is $\hat{\mu}$-open;

(iii) For each point x in X and each open set V in Y with $f(x) \in V$, there is an $\hat{\mu}$-open set U in X such that $x \in U$, $f(U) \subset V$;

(iv) For each subset A of X, $f(\hat{\mu}\,cl(A)) \subset cl(f(A))$;

(v) For each subset B of Y, $\hat{\mu}\,cl(f^{-1}(B) \subset f^{-1}(cl(B))$;

(vi) For each subset B of Y, $f^{-1}(int(B)) \subset \hat{\mu}\ int(f^{-1}(B))$.

Proof :

(i) ⇔ (ii) By theorem 3.26

(i) ⇔ (iii) : Suppose that (iii) holds and let V be an open set in Y and let $x \in f^{-1}(V)$. Then $f(x) \in V$ and thus there exist an $\hat{\mu}$-open set U_x such that $x \in U_x$ and $f(U_x) \subset V$. Now $x \in U_x \subset f^{-1}(V)$ and $f^{-1}(V) = \bigcup_{x \in f^{-1}(V)} U_x$. By assumption $f^{-1}(V)$ is $\hat{\mu}$-open in X and therefore f is $\hat{\mu}$-continuous.

Conversely, suppose that (i) holds and let $f(x) \in V$. Then $x \in f^{-1}(V)$ which is $\hat{\mu}$-open in X, since f is $\hat{\mu}$-continuous. Let $U = f^{-1}(V)$. Then $x \in U$ and $f(U) \subset V$.

(iv) ⇔ (i) Suppose that (i) holds and A be a subset of X. Since $A \subset f^{-1}(f(A))$. We have $A \subset f^{-1}(cl(f(A)))$. Since cl(f(A)) is a closed set in Y, by assumption $f^{-1}(cl(f(A)))$ is an $\hat{\mu}$-closed set containing A. Consequently, $\hat{\mu}\,cl(A) \subset f^{-1}(cl(f(A)))$.

Thus $f(\hat{\mu}\,cl(A)) \subset f(f^{-1}cl(f(A))) \subset cl(f(A))$.

Conversely, suppose that (iv) holds for any subset A of X. Let F be a closed subset of Y. Then by assumption, $f(\hat{\mu}\,cl(f^{-1}(F))) \subset cl(f(f^{-1}(F))) \subset cl(F) = F$. Thus $\hat{\mu}\,cl(f^{-1}(F)) \subset f^{-1}(F)$ and so $f^{-1}(F)$ is $\hat{\mu}$-closed.

(iv) ⇔ (v) : Suppose that (iv) holds and B be any subset of Y. Then replacing A by $f^{-1}(B)$ in (iv) we get $f(\hat{\mu}\,cl(f^{-1}(B))) \subset cl(f(f^{-1}(B))) \subset cl(B)$. Thus $\hat{\mu}\,cl(f^{-1}(B)) \subset f^{-1}(cl(B))$. Conversely, suppose that (v) holds. Let B = f(A) where A is a subset of X. Then we have $\hat{\mu}\,cl(A) \subset \hat{\mu}\,cl(f^{-1}(B)) \subset f^{-1}(cl(f(A)))$ and so $f(\hat{\mu}\,cl(A)) \subset cl(f(A))$.

(v) ⇔ (vi) : Let B be any subset of Y. Then by (v) we have $\hat{\mu}\,cl(f^{-1}(B^c)) \subset f^{-1}(cl(B^c))$ and hence $(\hat{\mu}\,int\ f^{-1}(B))^c \subset (f^{-1}\ int\ (B))^c$. Therefore we obtain $f^{-1}(int(B)) \subset \hat{\mu}\,int(f^{-1}(B))$.

(vi) $\Leftrightarrow$ (i) : Suppose that (vi) holds. Let F be any closed subset of Y. We have $f^{-1}(F^c) = f^{-1}(int(F^c)) \subset \hat{\mu} int(f^{-1}(F^c)) = (\hat{\mu} cl(f^{-1}(F)))^c$ and hence $\hat{\mu} cl(f^{-1}(F)) \subset f^{-1}(F)$. By proposition 4.10 $f^{-1}(F)$ is $\hat{\mu}$-closed in X. Hence f is $\hat{\mu}$-continuous.

Proposition 4.13 :

Let f : (X, τ) → (Y, σ) be a $\hat{\mu}$-continuous map. If (X, τ) is a $T\hat{\mu}$ space, then f is continuous.

Proof :

The proof follows from definition.

Proposition 4.14 :

Let f : (X, τ) → (Y, σ) be a $\hat{\mu}$-continuous map. If (X, τ) is a $\alpha T\hat{\mu}$ space, then f is α-continuous.

Proof :

The proof follows from definition.

Proposition 4.15 :

Let f : (X, τ) → (Y, σ) is $\hat{\mu}$-continuous map. If (X, τ) is a $sT\hat{\mu}$ space, then f is semicontinuous.

Proof :

The proof follows from definition.

Proposition 4.15 :

If f : (X, τ) → (Y, σ) is $\hat{\mu}$-continuous map. If (X, τ) is a $pT\hat{\mu}$ space, then f is precontinuous.

Proof :

The proof follows from definition.

Proposition 4.16:

If f : (X, τ) → (Y, σ) is $\hat{\mu}$-continuous map. If (X, τ) is a $spT\hat{\mu}$ space, then f is semiprecontinuous.

Proof :

The proof follows from definition.

Proposition 4.17:

If f : (X, τ) → (Y, σ) is $\hat{\mu}$-continuous map. If (X, τ) is a $\mu T\hat{\mu}$ space, then f is μ-continuous.

Proof :

The proof follows from definition.

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