On the Behaviour of Hyperelastic Materials, a Mooney-Riviln approach

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***Abstract*—This paper emphasizes the superior elastic properties attained, and behavior of the Hyperelastic materials which are basically natural rubber composites with certain polymers. The most elegant aspect is the wide range of properties of these materials finding universal applications in critical and complex environments viz hydrophones and other under water warfare systems. All structural materials used in engineering components have linear elastic properties in the strain range of 10%, in contrast the elastomeric materials possessing high rate of strains even up to 300%. This special category of materials finds their application in shock mounts, Transducer assemblies (Hydrophones), Medical equipment and rubber seals etc. Analysis of these materials need special attention, as mechanical behavior exceeds linear elastic theory. FEA programs involve parameters with Mooney constants in the strain energy potential. In the present investigation two types of rubber material models are studied viz polyurethane (PU) and Neoprene for determining the Mooney constants which are needed for FEA Program.**

Keywords—* *Neoprene;Polyurethane(PU); Strain Energy; tensile testing;

I. INTRODUCTION

Attention of researchers due to ever increasing demands and for modeling Elastomeric materials behavior under mechanical and geometrical boundary conditions gained popularity in recent times. It is common practice to characterize the mechanical behavior of these materials represent the constitutive equation through a strain energy density function. Hyperelastic materials are often considered for various industrial applications, due to their remarkable properties of flexibility, recovery after load release and resistance to high deformation levels. Many attempts have been made to develop more general hyperelastic models to include different aspects of materials behavior. Henky H [1] derived the elastic behavior of hyperelastic materials, large extensions up to 270 % analytically by a simple function. The deformed and unreformed stresses are the functions of two constants which are bulk modulus and logarithmic extension ratio. The deformed state of stress in tension is high when compared to unreformed state of stress, whereas in case of compression, the deformed state of stress is less when compared with unreformed state of stress. Ellen M. Arruda [2] states that, no existing model which accurately represents the behavior of hyperelastic materials in various deformation states and satisfies the criterion of requiring only a small number of physically based parameters or constants. The main condition is any constitutive model constants should be independent of deformation state to provide predictive capability.

In Structural Analysis usually elastic material properties are expressed in terms of strain energy density function. This strain energy function into a separable form related to the principal directions is derived by the Mooney [3] and Rivlin [4]. This advance led to the Ogden model [5] which is largely used today. Strain energy density function is expressible in the form of even powered series of the principal stretches. A variety of strain energy density functions have been extracted from Rivlin's model. Strain energy density is sum of independent functions of the principal stretches for incompressible materials.

The strain invariants and coefficients are required for strain energy density function for rubber like materials as determined by D.W. Haines [6]. The strain-energy function is expressed as a power series of invariants. The material models formulated in invariants of the strain tensor are based on a series approach in different powers of the first and second basic invariant. Because of the incompressibility of the material, the third basic invariant is constant and, hence, does not contribute to the stored energy. Formulations of the strain energy function based on Eigen values were presented by Ogden [5]. These models show a good adaptability to the experimental data resulting from the high degree of non-linearity. The examples of these strain energy density functions have been presented in the references [7–9]. The computations involved in the nonlinear equations of the problem must be set into an appropriate quadratic form for obtaining the optimal efficiency. This can be obtained in general by introducing additional variables and/or differential relations between the variables. In the context of hyperelastic models, strongly nonlinear terms are involved, such as logarithmic or fractional functions.

Two nonlinear materials viz PU and Neoprene have been modeled by using uni-axial tension/compression test in the present investigation, and the material constants are determined through least-squares-fit procedures. In order to estimate the constants that best fits the curve-fit, a program was written in MATLAB optimization solver which inputs experimental stress-strain data and constraints. To minimize the errors of the program and also to know the closeness of the stress-strain curve with the expected constants an RMS function is created and applied for determining the 2-Mooney, 5-Mooney and 9-Mooney constants.

II. HYPERELASTIC MATERIALS ANALYSIS

Strain Energy Density functions requires a number of mathematical constitutive theories of nonlinear, large elastic deformations. These theories, coupled with the numerical method, can be used very effectively by the user to analyze the elastomer products operating under highly deformed states. According to this theory in large deformation, rubber is assumed isotropic in elastic behavior and very nearly incompressible. The elastic properties of rubber can be explained in terms of a strain energy function based on the strain invariants I_1, I_2 and I_3. These invariants are as the function of extension ratios λ_1,λ_2 and λ_3 have following properties in equations 1,2 and 3.

$$I_1 = \lambda_1^2 + \lambda_2^2 + \lambda_3^2 \quad (1)$$

$$I_2 = \lambda_1^2\lambda_2^2 + \lambda_2^2\lambda_3^2 + \lambda_3^2\lambda_1^2 \quad (2)$$

$$I_3 = \lambda_1^2\lambda_2^2\lambda_3^2 \quad (3)$$

The incompressible materials require the third invariant which is equal to one, and hence eqs. 1,2 and 3 can further be reduced to eqs. 4 and 5. This is because when the material is compressible the third invariant becomes equal to one, and hence the third stretch ratio can be expressed as a function of the first two in equation 7. Next, when a load is only applied in one principal direction as in the case of uni-axial loading, the second stretch ratio (λ_2) is equal and to the third stretch ratio (λ_3). Thus invariants can be expressed as a function of only two stretch ratios and Equation 8 can be expressed as a direct relationship of the first and second stretch ratio [10].

$$I_1 = \lambda_1^2 + \lambda_2^2 + 1/\lambda_1^2\lambda_2^2 \quad (4)$$

$$I_2 = \lambda_1^2\lambda_2^2 + 1/\lambda_1^2 + 1/\lambda_2^2 \quad (5)$$

$$I_3 = 1 = \lambda_1\lambda_2^2 \quad (6)$$

Which implies $\lambda_3^2 = 1/\lambda_1^2\lambda_2^2$ (7)

$\lambda_2^2 = 1/\lambda_1^2\lambda_2^2$ implies $\lambda_2 = 1/\sqrt{\lambda_1}$ (8)

A. *Mooney-Rivlin Approach*

It is Shown that in the generalised Mooney's approach [3] most general strain energy function for a homogenous, isotropic, incompressible and elastic material is in eq. 9

$$W = \sum_{i,j=0}^{N} C_{ij}(I_1 - 3)^i(I_2 - 3)^j + \sum_{i=1}^{N} \frac{1}{D_i}(J_{el} - 1)^{2i} \quad (9)$$

Where W is the strain energy density function, I_1 and I_2 are the measure of distortion in the material, C_{ij} describes the shear behavior of the material, Di introduces the material incompressibility and J_{el} is the elastic volume strain.

The two constant mooney-Rivlin function in eq. 10 obtains if N=1 in Mooney generalized eq. 9

$$W = C_{10}(I_1 - 3) + C_{01}(I_2 - 3) \quad (10)$$

The five constant mooney-Rivlin function in eq. 11 can obtains if N=2 in Mooney generalized eq. 9

$$W = C_{10}(I_1 - 3) + C_{01}(I_2 - 3) + C_{20}(I_1 - 3)^2 + C_{11}(I_1 - 3)(I_2 - 3) + C_{02}(I_2 - 3)^2 \quad (11)$$

To evaluate constants redefining the strain energy density function is needed. The equations 12 to 15 were obtained because $\sigma_i = \frac{\partial W}{\partial \lambda_i}$, where 'i' is the different axis [4] and of σ_3 is equal to zero

$$\sigma_1 - \sigma_3 = \frac{\partial W}{\partial \lambda_1} - \frac{\partial W}{\partial \lambda_3} = \frac{\partial W}{\partial \lambda_1} - 0 = \frac{\partial W}{\partial \lambda_1} \quad (12)$$

$$\sigma_2 - \sigma_3 = \frac{\partial W}{\partial \lambda_2} - \frac{\partial W}{\partial \lambda_3} = \frac{\partial W}{\partial \lambda_2} - 0 = \frac{\partial W}{\partial \lambda_2} \quad (13)$$

$$\sigma_1 = \frac{\partial W}{\partial \lambda_1} = \frac{\partial W}{\partial I_1}\frac{\partial I_1}{\partial \lambda_1} + \frac{\partial W}{\partial I_2}\frac{\partial I_2}{\partial \lambda_1} \quad (14)$$

$$\sigma_2 = \frac{\partial W}{\partial \lambda_2} = \frac{\partial W}{\partial I_1}\frac{\partial I_1}{\partial \lambda_2} + \frac{\partial W}{\partial I_2}\frac{\partial I_2}{\partial \lambda_2} \quad (15)$$

In case of uni-axial test and to get the stress function a derivation is necessary and a multiplication with $\frac{\partial w}{\partial I_i}$ is required. The equations 16 and 17 for obtained for 5-Mooney equations by deriving equation 11 w.r.t to I_1 and I_2.

$$\frac{\partial W}{\partial I_1} = C_{10} + 2C_{20}(I_1 - 3) + C_{11}(I_2 - 3) \quad (16)$$

$$\frac{\partial W}{\partial I_2} = C_{01} + 2C_{02}(I_2 - 3) + C_{11}(I_1 - 3) \quad (17)$$

The derivatives of the invariants w.r.t streatch ratios are given in equation number 18 and 19

$$\frac{\partial I_1}{\partial \lambda_1} = 2\lambda_1 - \frac{2}{\lambda_1^3\lambda_2^2}, \quad \frac{\partial I_1}{\partial \lambda_2} = 2\lambda_2 - \frac{2}{\lambda_1^2\lambda_2^3} \quad (18)$$

$$\frac{\partial I_2}{\partial \lambda_1} = 2\lambda_1\lambda_2^2 - \frac{2}{\lambda_1^3}, \quad \frac{\partial I_2}{\partial \lambda_2} = 2\lambda_1^2\lambda_2 - \frac{2}{\lambda_2^3} \quad (19)$$

III. EXPERIMENTAL STUDY

Experiments were conducted by using Uni-axial Tensile Test Machine as shown in Fig. 1 having 10KN load cell, it could be used to calculate the Tensile Strength, Shear strength and Adhesive Strength.

Fig 1: Polyurethane Dumbbell type Specimen & Fixing position in M/s INSTRON

The polyurethane material specimen of 75 mm Gauge Length and Area 6.6 mm2 was considered basing on the guidelines given in ASTM standards (ATM D412). The specimen is fixed between the two grips. During the test, the upper grip moves up at a speed of 500mm/min velocity in the upward direction (axial direction).

A. *Uni-axial Tensile test Specimen dimensions.*

Plain rubber sheet is used for the specimens employing a cutting die, manufactured from the standard dimensions of a stock of length 115 mm width 25 mm and thickness 2.5 mm. The sample specimen is shown below fig 2. For each material namely PU and Neoprene four specimens are prepared as shown in Fig. 3 to Fig. 4 and the tests were carried out and Experimental results are given in table I.

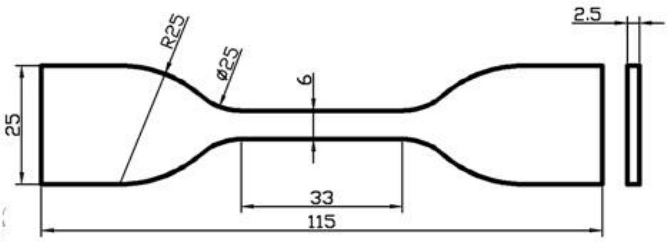

Fig 2: Dimensions of Dumbbell specimen

Fig :3 PU Dumbbell specimen

Fig: 4 NEOPRENE Dumbbell Specimens

TABLE 1: EXPERIMENTAL RESULTS

MODEL	Width (mm)	Thk (mm)	Load Max (N)	Stress Max (MPa)	Strain Max
PU	5.81	1.26	178	24.39	4.743
	5.85	1.27	176	23.386	5.076
	5.80	1.32	175	22.92	4.99
Neoprene	5.68	2.36	168.6	12.57	4.6
	5.6	2.53	163	11.64	3.9
	5.6	2.33	174.6	13.2	4.73

IV. RESULTS AND CONCLUSIONS

A. *Results*

From the practical data of two different test specimens the Mooney constants are extracted as given in table II an table III. In order to estimate the best constant that fits the curve-fit, a program was written, to ensure that the results are as good as possible and the program is created as an optimization solver. The estimated Mooney constants are obtained at minimized RMS value from MATLAB OPTIM tool. The corresponding stress strain diagrams and curve fit for different Mooney constants like 2-Mooney, 5-Mooney and 9-Mooney fits are given for all tested specimens in graphs from Fig. 5 to Fig.12.

TABLE II: EXTRACTED MOONEY CONSTANTS OF PU FROM MATLAB PROGRAM

Model	PU		
constants	2-Mooney	5-Mooney	9-Mooney
C_{10}(MPa)	1.636	-0.452	-0.452
C_{01}(MPa)	0.728	4.853	4.853
C_{20}(MPa)	-	0	0
C_{11}(MPa)	-	0.109	0.119
C_{02}(MPa)	-	0	0
C_{30}(MPa)	-	-	0
C_{21}(MPa)	-	-	-0.003
C_{12}(MPa)	-	-	0.003
C_{03}(MPa)	-	-	0
Bulk Modulus	23640	44010	44010
Error %	104.4	29.15	29.15

TABLE III: EXTRACTED MOONEY CONSTANTS OF NEOPRENE FROM MATLAB PROGRAM

Model	**Neoprene**		
constants	2-Mooney	5-Mooney	9-Mooney
C_{10}(MPa)	1.141	-0.141	1.073
C_{01}(MPa)	-1.141	0.968	-0.494
C_{20}(MPa)	-	0.037	0.008
C_{11}(MPa)	-	-0.03	0.68
C_{02}(MPa)	-	-0.007	-0.536
C_{30}(MPa)	-	-	0
C_{21}(MPa)	-	-	-0.194
C_{12}(MPa)	-	-	0.198
C_{03}(MPa)	-	-	-0.002
Bulk Modulus	0	8270	5790
Error %	122.9566	15.04	14.4

B. PU Curve Fit

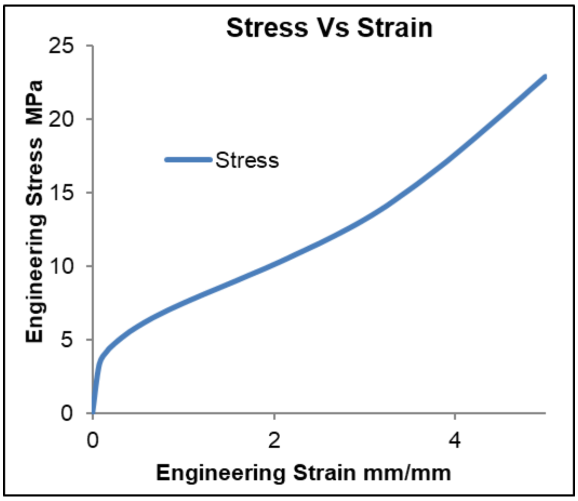

Fig 5: Stress vs. Strain behavior of PU

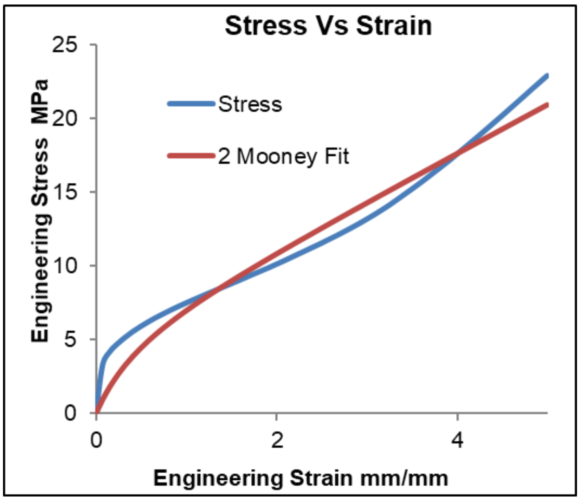

Fig 6: Curve fit for PU Material with 2-Mooney Constants

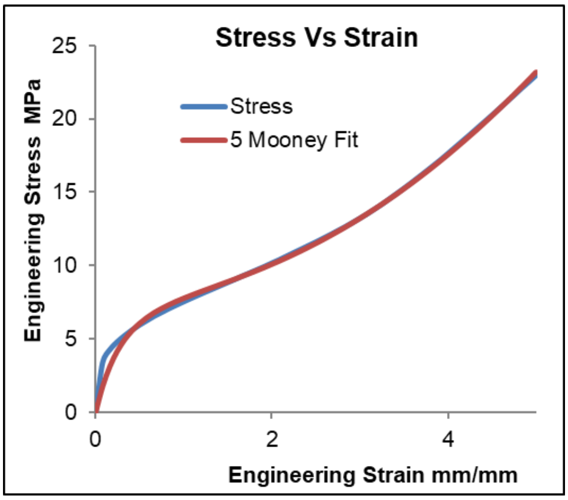

Fig 7: Curve fit for PU Material with 5-Mooney Constants

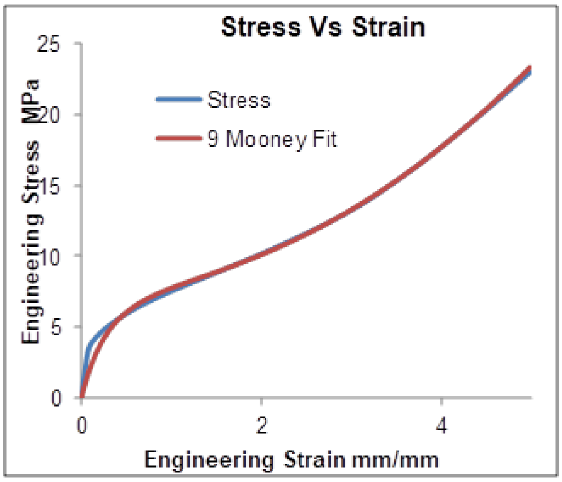

Fig 8: Curve fit for PU Material with 9-Mooney Constants

C. Neoprene Curve Fit

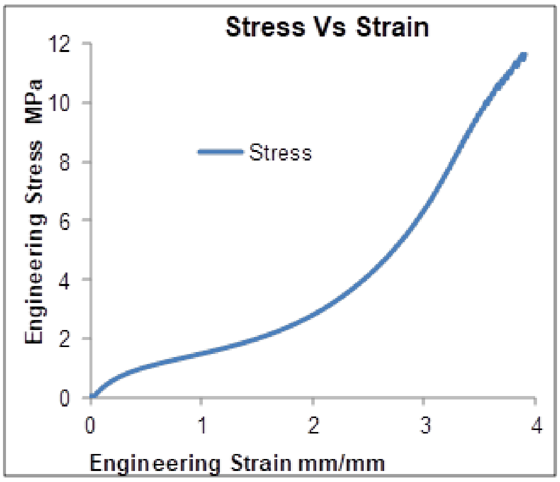

Fig 9: Stress vs. Strain behavior of Neoprene

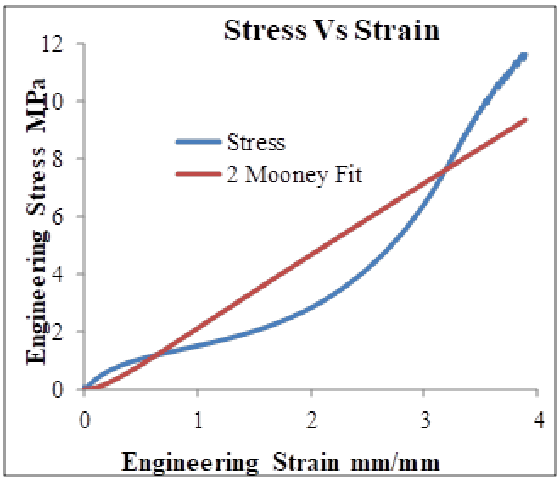

Fig 10: Curve fit for Neoprene Material with 2-Mooney Constants

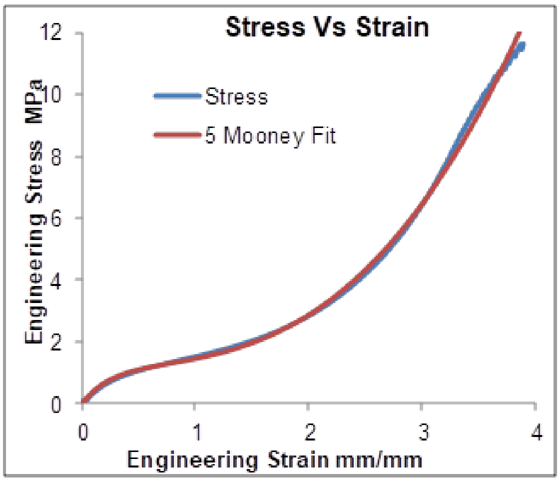

Fig 11: Curve fit for Neoprene Material with 5-Mooney Constants

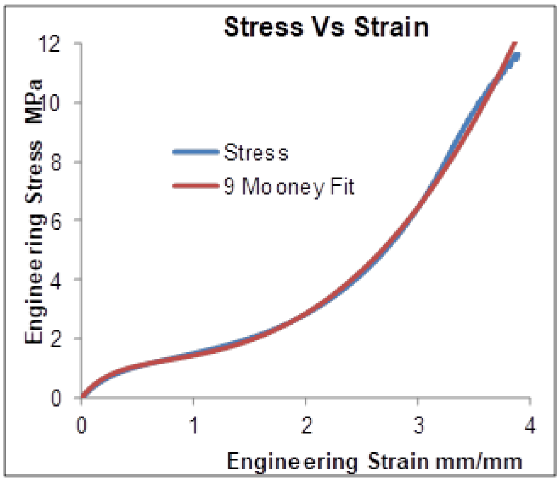

Fig 12: Curve fit for Neoprene Material with 9-Mooney Constants

V. CONCLUSIONS

- PU material showed that they have high elongations up to 500% and has a 104% error for 2-Mooney fit.
- The observed error in PU material reduced to 29% for 5-Mooney and 9-Mooney curve fit.
- Further it is seen that Neoprene material has high elongations up to 470% and has 123% error for 2-Mooney fit.
- Reduction in error for Neoprene is limited only to 15% for 5-Moomey and 9-Mooney curve fit.

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