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On Soft πGr-Continuous Functions in Soft Topological Spaces

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Abstract: **The purpose of this paper is to study soft πgr-continuous function and soft πgr-irresolute function in soft topological spaces. Also,we introduce the concepts such as soft πgr-closure,soft πgr-interior and exhibit some results related to soft πgr- continuity,soft πgr-open map and soft πgr-closed map . Further, we study the relation between soft πgr-continuous function and other soft continuous functions.**

Key Words: Soft Set , Soft Topology, Soft Continuity , Soft Point, Soft πgr-closure, Soft πgr-interior, Soft Function, Soft πgr-continuity, Soft πgr-irresolute, soft πgr- neighborhood.

Mathematics Subject Classification: 06D72, 54A40.

I. INTRODUCTION.

Molodtsov[16] introduced the concept of soft set as a new mathematical tool.

Furthermore, D.Pei and D.Miao [19] showed that soft sets are a class of special information systems. M.Shabir and M.Naz [20] introduced soft topological spaces. D.N.Georgiou and A.C. Megaritis[6]**,** Soft Set Theory and Topology, Applied General Topology, 14, (2013**)** for the soft set theory, new definitions , examples , new classes of soft sets and properties for mappings between different classes of soft sets are introduced and studied. Levine[13] Introduced g-closed sets in general topology. Kannan [11] introduced soft g-closed sets in soft topological spaces, The concept of π-closed sets in topological spaces was initiated by Zaitsav[23] and the concept of πg-closed set was introduced by Noiri and Dontchev[4]. N.Palaniappan[17] studied and introduced regular closed sets in topological spaces. Jeyanthi.V and Janaki.C [8]introduced πgr-closed sets in topological spaces. Soft semi-open sets and its properties were introduced and studied by Bin Chen[2]. Kharal et al.[12]introduced soft function over classes of soft sets. Cigdem Gunduz Aras et al., [3] in 2013 Studied and discussed the properties of Soft Continuous mappings .Recently, we introduced Soft πgr-closed sets [9]in soft topological spaces. The concept of πgα-closed,πgp-closed,πgs-closed,π*g-closed,πgb-closed sets in topological spaces was introduced by C.Janaki[7], Park. J.H[18], Aslim [1], Ganes M Pandya [5] and Sreeja and Janaki[21].The purpose of this paper is to study soft πgr-continuity on soft topological spaces and investigate its properties.

II. PRELIMINARIES.

Definition 2.1 ([15],[16],[22])
Let U be the initial universe and P(U) denote the power set of U. Let E denote the set of all parameters. Let A be a non-empty subset of E. A pair (F, A) is called a soft set over U, where F is a mapping given by F: A→P(U). In other words, a soft set over U is a parameterized family of subsets of the universe U. For Ɛ∈A, F(Ɛ) may be considered as the set Ɛ-approximate elements of the soft set (F,A).Clearly, a soft set is not a set.

For two soft sets (F,A) and (G,B) over the common universe U, we say that (F,A) is a soft subset of (G,B) if (i)A⊆B and (ii) for all e∈A, F(e) and G(e) are identical approximations. We write (F,A) $\tilde{\subset}$ (G,B). (F,A) is said to be a soft superset of (G,B) , if (G,B) is a soft subset of (F,A). Two soft sets (F,A) and (G,B) over a common universe U are said to be soft equal if (F,A) is a soft subset of (G,B) and (G,B) is a soft subset of (F,A).

Definition:2.2([15],[16],[22])
For a soft set (F,A) over the universe U, the relative complement of (F,A) is denoted by (F,A)′ and is defined by (F,A)′=(F′,A), where F′: A →P(U) is a mapping defined by F′(e)= U– F(e) for all e∈A.

Definition :2.3([15],[16],[22])
A Soft set (F,A) over X is said to be a Null soft set denoted by $\bar{\varphi}$ or φ_A if for all e∈A, F(e)=φ(null set).

Definition:2.4([15],[16],[22])
A soft set (F,A) over X is said to be absolute soft set denoted by $\bar{A}$ or X_A if for all e∈A, F(e) =X. Clearly, we have $X_A' = \varphi_A$ and $\varphi_A' = X_A$.

Definition :2.5 ([15],[16],[22])
The union of two soft sets of (F,A) and (G,B) over the common universe U is soft set (H,C), where C = A∪B and for all e∈C, H(e) = F(e) if e ∈A–B, H(e) = G(e) if e∈ B–A and H(e)=F(e) ∪G(e) if e ∈A∩B. We write (F,A) ∪(G,B)= (H,C).

Definition:2.6([15],[16],[22])
The Intersection (H,C) of two soft sets (F,A) and (G,B) over a common universe U denoted (F,A)∩(G,B) is defined as C= A∩B and H(e) = F(e) ∩G(e) for all e∈C.

Definition:2.7 ([2],[11],[15])
Let τ be the collection of soft sets over X, then τ is called a soft topology on X if τ satisfies the following axioms:

1) φ, $\widetilde{X}$ belong to τ.

2) The union of any number of soft sets in τ belongs to τ.
3) The intersection of any two soft sets in τ belongs to τ.

The triplet (X, τ, E) is called a soft topological space over X. For simplicity, we can take the soft topological spaces (X,τ_1,E) as X and (Y,τ_2,E) as Y throughout this work respectively. Let us denote the collection of soft sets over the universe X and Y as SS(X) and SS(Y) respectively.

Definition:2.8 ([2],[11],[15])

Let (X, τ, E) be soft topological space over X. A soft set (F,E) over X is said to be soft closed in X, if its relative complement $(F,E)'$ belongs to τ. The relative complement is a mapping $F':E\rightarrow P(X)$ defined by $F'(e)= X-F(e)$ for all $e\in A$.

Definition:2.9 ([2],[11],[15])

Let (X,τ, E) be a soft topological space over X and the soft interior of (F,E) denoted by s-int(F,E) is the union of all soft open subsets of (F,E). Clearly, (F,E) is the largest soft open set over X which is contained in (F,E).The soft closure of (F,E) denoted by s-cl(F,E) is the intersection of all closed sets containing (F,E). Clearly, (F,E) is smallest soft closed set containing (F,E).

s-int (F,E) $=\cup\{$ (O,E): (O,E) is soft open and $(O,E)\tilde{\subseteq}(F,E)\}$.

s-cl(F,E) $=\cap\{$ (O,E): (O,E) is soft closed and $(F,E)\tilde{\subseteq}(O,E)\}$.

Definition:2.10[3]

Let (X,τ,E) be a soft topological space over X, (G,E) be a soft set over X and $x\in X$. Then (G,E) is said to be a soft neighborhood of x, if there exists a soft open set (F,E) such that $x\in(F,E)\tilde{\subseteq}(G,E)$.

Definition:2.11 [24]

Let (X,τ,E) be a soft topological space over X, (G,E) be a soft set over X and $x\in X$. Then (G,E) is said to be a soft interior point of (G,E), if there exists a soft open set (F,E) such that $x\in(F,E)\tilde{\subseteq}(G,E)$.

Definition:2.12

A soft subset (A,E) of X is called

(i) a soft generalized closed (Soft g-closed)[11] in a soft topological space (X,τ,E) if s-cl(A,E) $\tilde{\subseteq}$ (U,E) whenever $(A,E)\tilde{\subseteq}(U,E)$ and (U,E) is soft open in X.

(ii) a soft semi open [3]if $(A,E)\tilde{\subseteq}$ s-int(s-cl(A,E))

(iii))a soft regular open if (A,E)= s-int(s-cl(A,E)).

(iv) a soft α-open if $(A,E)\tilde{\subseteq}$ s-int(s-cl(s-int(A,E)))

(v) a soft b-open if $(A,E)\tilde{\subseteq}$ s-cl(s-int(A,E)) $\cup$ s-int(s-cl(A,E))

(vi) a soft pre-open set if $(A,E)\tilde{\subseteq}$s-int(s-cl(A,E)).

(vii) a soft clopen if (A,E) is both soft open and soft closed.

The complement of the soft semi open , soft regular open , soft α-open, soft b-open , soft pre-open sets are their respective soft semi closed , soft regular closed , soft α-closed , soft b-closed and soft pre -closed sets.

The finite union of soft regular open sets is called soft π-open set and its complement is soft π-closed set.

The soft regular open set of X is denoted by SRO(X) or SRO(X,τ,E).

Definition:2.13([11])

A soft topological space X is called a soft $T_{1/2}$-space if every soft g-closed set is soft closed in X.

Definition:2.14([9])

The soft regular closure of (A,E) is the intersection of all soft regular closed sets containing (A,E). (i.e)The smallest soft regular closed set containing (A, E) and is denoted by srcl(A,E).The soft regular interior of (A,E) is the union of all soft regular open sets contained in (A,E) and is denoted by srint(A,E).

Similarly , we define soft α-closure, soft pre-closure, soft semi closure and soft b-closure of the soft set (A,E) of a topological space X and are denoted by sαcl(A,E), spcl(A,E), sscl(A,E) and sbcl(A,E) respectively.

Definition:2.15 ([10],[14])

Let (F,E) be a soft set over X. The soft set (F,E) is called a soft point denoted by e_F if for the element $e\in E$, $F(e)\neq\varphi$ and $F(e')=\varphi$ for all $e'\in E-\{e\}$.

Definition:2.16 ([3])

A soft set (G,E) in a soft topological space (X,τ,E) is called a soft neighborhood of the soft point e_F if there exists a soft open set (H,E) such that $e_F\in(H,E)\tilde{\subseteq}(G,E)$.

A soft set (G,E) in a soft topological space (X,τ,E) is called a soft neighborhood of the soft set (F,E) if there exists an soft open set (H,E) such that $(F,E)\in(H,E)\tilde{\subseteq}(G,E)$.

The neighborhood system of the soft point e_F denoted by $N_\tau(e_F)$, is the family of all its neighborhoods.

Definition:2.17([3])

Let (X,τ,E) be a soft topological space over X, (G,E) be a soft set over X and $x\in X$. Then (G,E) is said to be a soft interior point of (G,E), if there exists a soft open set (F,E) such that $x\in(F,E)\tilde{\subseteq}(G,E)$.

Definition:2.18 ([10],[14])

Let $SS(X)_A$ and $SS(Y)_B$ be two soft classes. Then u: $X\rightarrow Y$ and $p:A\rightarrow B$ be two functions. Then the function f_{pu}: $SS(X)_E\rightarrow SS(Y)_B$ and its inverse are defined as

(i) Let (F,A) be a soft set in $SS(X)_A$. The image of (F,A) under f_{pu}, written as $f_{pu}((F,A))$ = $(f_{pu}(F),p(A))$is a soft set in $SS(Y)_B$ such that

$$f_{pu}(F)(y)=\begin{cases}\left(\bigcup_{x\in p^{-1}(y)\cap A}u(F(x))\right), & p^{-1}(y)\cap A\neq\phi\\ \phi, \text{ otherwise.}\end{cases}$$

for all $y\in B$.

(ii) Let (G,B) be a soft set in $SS(Y)_B$. Then the inverse image of (G,B) under f_{pu}, written as $f_{pu}^{-1}((G,B))$= $(f_{pu}^{-1}(G),p^{-1}(B))$ is a soft set in $SS(X)_A$ such that

$$f_{pu}^{-1}(G)(x)=\begin{cases}f_{pu}^{-1}(G(p(x))), & p(x)\in B,\\ \phi, \text{ otherwise.}\end{cases}$$

for all $x\in A$.

Definition:2.19([10],[14])

Let (X,τ,A) and (Y,τ*,B) be soft topological spaces and f_{pu} : $SS(X)_A \rightarrow SS(Y)_B$ be a function. Then function f_{pu} is called soft continuous if f_{pu}^{-1}((G,B)) ∊ τ for all (G,B)∊τ*.

Definition:2.20 [10]

Let (X,τ,A) and (Y,τ*,B) be soft topological spaces and $f_{pu}:SS(X)_A \rightarrow SS(Y)_B$ be a function. Then the function f_{pu} is called soft open mapping if f_{pu}((G,A)) ∊ τ* for all (G,A)∊τ. Similarly, a function $f_{pu}:SS(X)_A \rightarrow SS(Y)_B$ is called a soft closed map if for a closed set (F,A) in τ, the image f_{pu}((G,B)) is soft closed in τ*.

Definition:2.21([9])

A soft subset (G,A) of a soft topological space X is called a soft πgr-closed set in X if srcl(G,A) $\tilde{\subseteq}$(X,A)) whenever (G,A) $\tilde{\subseteq}$(X,A), where (X,A) is soft π- open in X. We denote the soft πgr-closed set of X by SπGRC(X) . The complement of soft πgr-closed is soft πgr-open set and is denoted by SπGRO(X).

III. SOFT πGR-CLOSURE AND SOFT πGR-INTERIOR.

Let us introduce the following definitions.

Definition:3.1

Let (X,τ,A) be a soft topological space over X, (G,A) be a soft set over X and x∊X. Then (G,A) is said to be a soft πgr-neighborhood of x, if there exists a soft πgr-open set (F,A) such that x∊(F,A) $\tilde{\subseteq}$ (G,A).The soft πgr-neighborhood of a point x is denoted by sπgr-nbd(x).

Definition:3.2

The Soft πgr-Closure of a soft set (G,A) is defined to be the intersection of all soft πgr-closed sets containing the soft set (G,A) and is denoted by sπgr-cl(G,A).

The Soft πgr-Interior of a soft set (G,A) is defined to be the union of all soft πgr-open sets contained the soft set (G,A) and is denoted by sπgr-int(G,A).

Lemma :3.3

Let (F_1,A) and (F_2,A) be subsets of (X,τ,A) . Then

(i) sπgr-cl (φ_A) = φ_A and sπgr-cl(X) =X.

(ii) If (F_1,A) $\tilde{\subseteq}$ (F_2,A), then sπgr-cl(F_1,A) $\tilde{\subseteq}$ sπgr-cl(F_2,B)

(iii) (F_1,A) $\tilde{\subseteq}$ sπgr-cl(F_1,A)

(iv) sπgr-cl(F_1,A)= sπgr-cl(sπgr-cl(F_1,A))

(v) sπgr-cl((F_1,A)∪(F_2,B))= sπgr-cl(F_1,A) ∪ sπgr-cl(F_2,B)

Proof: Obvious.

Lemma: 3.4

Let (F,A) and (G,A) be soft subsets of the soft topological space X. Then sπgr-cl ((F,A) ∩ (G,A)) $\tilde{\subseteq}$sπgr-cl((F,A)) ∩ sπgr-cl((G,A))

Proof: Since (F,A)∩(G,A) $\tilde{\subseteq}$ (F,A), (G,A)

⇒sπgr-cl((F,A)∩(G,A)) $\tilde{\subseteq}$ sπgr-cl((F,A)) and sπgr-cl((F,A)∩(G,A)) $\tilde{\subseteq}$sπgr-cl((G,A))

⇒ sπgr-cl((F,A)∩(G,A)) $\tilde{\subseteq}$sπgr-cl(F,A)∩ sπgr-cl(G,A)

Lemma:3.5

Let (F,A) and (G,A) be soft subsets of a soft topological space X. Then

(i)sπgr-int(X) =X, sπgr-int(φ_A) = φ_A

(ii)sπgr-int((F,A)) $\tilde{\subseteq}$ (F,A)

(iii)If (F,A) is any soft πgr-open set contained in (G,A), then ((F,A) $\tilde{\subseteq}$sπgr-int(G,A)

(iv)If (F,A) $\tilde{\subseteq}$ (G,A) , then sπgr-int((F,A)) $\tilde{\subseteq}$ sπgr-int(G,A)

(v)sπgr-int(sπgr-int((F,A)))= sπgr-int((F,A)).

Proof: Straight Forward.

Theorem:3.6

If a subset (F,A) of a soft topological space X is soft πgr-open, then sπgr-int((F,A)) =(F,A).

Proof: Obvious.

Theorem:3.7

If (F,A) and (G,A) are soft subsets of a soft topological space X, then sπgr-int((F,A)) ∪ sπgr-int((G,A)) $\tilde{\subseteq}$sπgr-int ((F,A)∪(G,A))

Proof: We know that (F,A) $\tilde{\subseteq}$ (F,A)∪(G,A) and (G,A) $\tilde{\subseteq}$ (F,A)∪(G,A)

Then sπgr-int((F,A)) $\tilde{\subseteq}$sπgr-int ((F,A)∪(G,A)), sπgr-int((G,A)) $\tilde{\subseteq}$ sπgr-int ((F,A)∪(G,A)

Hence sπgr-int ((F,A)) ∪ sπgr-int((G,A)) $\tilde{\subseteq}$sπgr-int ((F,A)∪(G,A)).

Theorem:3.8

If (F,A) and (G,A) are soft subsets of a space X, then sπgr-int ((F,A)∩(G,A))= sπgr-int ((F,A)∩ sπgr-int ((G,A))

Proof: We know that (F,A)∩(G,A) $\tilde{\subseteq}$ (F,A), (F,A)∩(G,A) $\tilde{\subseteq}$(G,A).Then sπgr-int ((F,A) ∩(G,A)) $\tilde{\subseteq}$ sπgr-int((F,A)) and sπgr-int ((F,A)∩(G,A))$\tilde{\subseteq}$sπgr-int((G,A)). Hence sπgr-int ((F,A)∩(G,A))= sπgr-int ((F,A))∩ sπgr-int ((G,A))---------------(*)

Again , let x∈ sπgr-int ((F,A))∩ sπgr-int ((G,A)). Then x∈ sπgr-int ((F,A)) and x∈ sπgr-int ((G,A)).Hence x is a soft πgr-interior point of each of sets (F,A) and (G,A). It follows that ((F,A) and (G,A) are sπgr-nbds of x, so that their intersection (F,A)∩(G,A) is also a sπgr-nbd of x . Hence x∈ sπgr-int ((F,A)∩(G,A))

Thus, x∈sπgr-int ((F,A))∩ sπgr-int ((G,A)) ⇒ x∈ sπgr-int ((F,A)∩(G,A))

Therefore, sπgr-int ((F,A))∩ sπgr-int ((G,A)) $\tilde{\subseteq}$ sπgr-int ((F,A)∩(G,A))---(**)

From (*) and (**) ,

sπgr-int ((F,A)∩(G,A))= sπgr-int ((F,A))∩ sπgr-int ((G,A)).

Lemma:3.9

If (F,A) be a soft subset of a soft topological space X, then (X– sπgr-int((F,A)) = sπgr-cl(X–(F,A)).

Proof: Let x∈X– (sπgr-int((F,A))). Then x∉ sπgr-int((F,A)). That is every soft πgr-open set (G,A) containing x is such that (G,A)$\tilde{\not\subseteq}$ (F,A).Every soft πgr-open set (G,A) containing x intersects X–(F,A).

⇒ x∈ sπgr-cl(X–(F,A))

Hence (X– sπgr-int((F,A))) $\tilde{\subseteq}$ sπgr-cl(X–(F,A)).

Conversely, let x∈sπgr-cl(X–(F,A)). Thus every soft πgr-open set (H,A) containing x intersects X-(F,A) (i.e) is every soft πgr-open set (H,A) containing x is such that (H,A) $\tilde{\not\subset}$ (F,A).

$\Rightarrow$ x $\notin$ sπgr-int((F,A)). Thus sπgr-cl(X–(F,A)) $\tilde{\subseteq}$ (X– sπgr-int((F,A))) and hence (X– sπgr-int((F,A))) = sπgr-cl(X–(F,A)).

Remark:3.10

If (F,A) be a soft subset of a soft topological space X, then (X– sπgr-cl((F,A))) = sπgr-int(X–(F,A)).

IV. SOFT πGR-CONTINUOUS FUNCTIONS.

Definition:4.1

Let (X,τ,A) and (Y,τ*,B) be soft topological spaces and $f_{pu}:SS(X)_A \rightarrow SS(Y)_B$ be a function. Then the function f_{pu} is called

1. soft πgr-continuous if f_{pu}^{-1}((G,B)) is soft πgr-closed in (X,τ,A) for every soft closed set (G,B) in (Y,τ*,B).
2. soft πgr-irresolute if f_{pu}^{-1}((G,B)) is soft πgr-closed in (X,τ,A) for every soft πgr- closed set (G,B) in (Y,τ*,B).
3. soft regular continuous if f_{pu}^{-1}((G,B)) is soft regular closed in (X,τ,A) for every soft closed set (G,B) in (Y,τ*,B).
4. soft R- map if for a soft regular closed set (G,B) in τ* , f_{pu}^{-1}((G,B)) is soft regular closed in τ.

Definition:4.2

Let (X,τ,A) and (Y,τ*,B) be soft topological spaces and $f_{pu}:SS(X)_A \rightarrow SS(Y)_B$ be a function. Then

(1) The function f_{pu} is called soft πgr-open if f_{pu}((G,A)) is soft πgr-open in (Y,τ*,B) for every soft open set (G,A) in (X,τ,A).

(2) The function f_{pu} is called soft πgr-closed if f_{pu}((G,A)) is soft πgr-closed in (Y,τ*,B) for every soft closed set (G,A) in (X,τ,A).

Remark:4.3

Soft πgr-continuity and soft continuity are independent concepts.

Example:4.4

(i)Let X={h_1, h_2,h_3}, Y= {a,b}, E={e_1,e_2}. Let F_1, F_2, F_3 are functions defined from E to P(X) as follows:

$F_1(e_1)=\{h_1,h_2\}$, $F_2(e_2=\{h_2\}$

$F_2(e_1)=X$, $F_2(e_2)= \{h_1,h_2\}$

$F_3(e_1) =\{h_3\}$, $F_3(e_2)=\{h_2\}$.

Then τ_1= {φ, X, (F_1,E),(F_2,E),(F_3,E)} is soft topology on X.

Let G_1, G_2 are functions from E to P(Y) and are defined as follows:

$G_1(e_1) =\varphi$, $G_1(e_2) = \{a\}$

$G_2(e_1)= \{a\}$, $G_2(e_2)=Y$.

Then τ_2={φ, Y, (G_1,E), (G_2,E)} be a soft topology on Y. Let f: X→Y be a function defined as $f(h_1)=f(h_2)=\{a\}$ and $f(h_3)=\{b\}$.
Here the inverse image of the soft open sets (G_1,E) and (G_2,E) in Y are soft πgr-open in X but not soft open in X. Hence soft πgr-continuity need not be soft continuity.

(ii) Let X={a,b,c,d}=Y, E={e_1,e_2}. Let F_1,F_2,…, F_6 are functions from E to P(X) and are defined as follows:

$F_1(e_1)=\{c\}$, $F_1(e_2)=\{a\}$,

$F_2(e_1)=\{d\}$, $F_2(e_2)=\{b\}$,

$F_3(e_1)=\{c,d\}$, $F_3(e_2)=\{a,b\}$,

$F_4(e_1)=\{a,d\}$, $F_4(e_2)=\{b,d\}$,

$F_5(e_1)=\{b,c,d\}$, $F_5(e_2)=\{a,b,c\}$,

$F_6(e_1)=\{a,c,d\}$, $F_6(e_2)=\{a,b,d\}$.

Then τ_1 ={φ, $\tilde{X}$,(F_1,E),…,(F_6,E)} is a soft topology and elements in τ are soft open sets.

Let G_1, G_2 ,G_3 are functions from E to P(Y) and are defined as follows:

$G_1(e_1) =\{a\}$, $G_1(e_2) = \{d\}$,

$G_2(e_1)= \{b\}$, $G_2(e_2)=\{c\}$,

$G_3(e_1)=\{a,b\}$,$G_3(e_2)=\{c,d\}$,

$G_4(e_1)=\{b,c,d\}$,$G_4(e_2)=\{a,b,c\}$.

Then τ_2={φ, Y, (G_1,E), (G_2,E),(G_3,E),(G_4,E)} be a soft topology on Y. Let f: X→Y be an identity map. Here the inverse image the soft open set (G_4,E)={{b,c,d},{a,b,c}} in Y is not soft πgr-open in X but soft open in X. Hence soft continuous function need not be soft πgr-continuous.

Theorem:4.5

Every soft regular continuous function is soft πgr-continuous but not conversely.

Proof: Straight forward.

Example :4.6

In example 4.4(i), The inverse image of the soft open sets (G_1,E) and (G_2,E) in Y are soft πgr-open in X but not soft regular open in X. Hence soft πgr-continuity need not be soft regular continuous.

Theorem:4.7
Every soft πgr-continuous function is soft rg-continuous, soft πg-continuous, soft π^*g-continuous, soft $\pi g\alpha$-continuous, soft πgp-continuous, soft πgs-continuous and soft πgb-continuous .
Proof: Follows from the definitions.
Remark: 4.8
The converse of the above need not be true and is shown in the following example.
Example:4.9
In example 4.4(ii), the inverse image the soft open set $(G_4,E)=\{\{b,c,d\},\{a,b,c\}\}$ in Y is soft not πgr-open in X but soft rg-open, soft $\pi g\alpha$-open, soft πgs-open, soft πg-open, soft π^*g-open, soft πgp-open and soft πgb -open in X. Hence soft rg-continuous, soft $\pi g\alpha$-continuous, soft πgs-continuous, soft πg-continuous, soft π^*g-continuous, soft πgp-continuous and soft πgb-continuous but not soft πgr-continuous.
Remark:4.10
Soft πgr-continuity and soft πgr-irresolute are independent concepts.
Example:4.11
(i)Let $X=\{a,b,c,d\}=Y$, $E=\{e_1,e_2\}$. Let $F_1,F_2,\ldots, F_6$ are functions from E to P(X) and are defined as follows:

$F_1(e_1)=\{c\}$, $F_1(e_2)=\{a\}$,

$F_2(e_1)=\{d\}$, $F_2(e_2)=\{b\}$,

$F_3(e_1)=\{c,d\}$, $F_3(e_2)=\{a,b\}$,

$F_4(e_1)=\{a,d\}$, $F_4(e_2)=\{b,d\}$,

$F_5(e_1)=\{b,c,d\}$, $F_5(e_2)=\{a,b,c\}$,

$F_6(e_1)=\{a,c,d\}$, $F_6(e_2)=\{a,b,d\}$.

Then $\tau_1=\{\varphi, \tilde{X}, (F_1,E),\ldots,(F_6,E)\}$ is a soft topology and elements in τ are soft open sets.

Let G_1, G_2, G_3 are functions from E to P(Y) and are defined as follows:

$G_1(e_1)=\{a\}$, $G_1(e_2)=\{d\}$,

$G_2(e_1)=\{b,c,d\}$, $G_2(e_2)=\{a,b,c\}$.

Then $\tau_2=\{\varphi, Y, (G_1,E), (G_2,E)\}$ be a soft topology on Y. Let f: $X\rightarrow Y$ be an identity map. Here the inverse image of the soft open set in Y is soft πgr-open in X, but the inverse image of soft πgr-open sets in Y are not soft πgr-open in X. Hence soft πgr-continuous function need not be soft πgr-irresolute.
(ii) Let $X=\{a,b,c\}=Y$, $E=\{e_1,e_2\}$. Let F_1, F_2, F_3 are functions from E to P(X) and are defined as follows:

$F_1(e_1)=\{a,c\}$, $F_1(e_2)=\{\varphi\}$,

$F_2(e_1)=\{b\}$, $F_2(e_2)=\{a\}$,

$F_3(e_1)=X$, $F_3(e_2)=\{a\}$

Then $\tau_1=\{\varphi, \tilde{X}, (F_1,E),(F_2,E),(F_3,E)\}$ is a soft topology on X and elements in τ are soft open sets of X.

Let G_1, G_2, G_3, G_4, G_5 and G_6 are functions from E to P(Y) and are defined as follows:

$G_1(e_1)=\{a\}$, $G_1(e_2)=\{b\}$,

$G_2(e_1)=\{b,c\}$, $G_2(e_2)=\{a\}$,

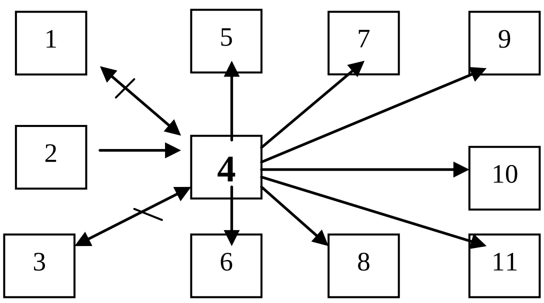
$G_3(e_1)=X$, $G_3(e_2)=\{a,b\}$,

$G_4(e_1)=\{a,b\}$, $G_4(e_2)=\{b,c\}$,

$G_5(e_1)=\{b\}$, $G_5(e_2)=\varphi$,

$G_6(e_1)=\{a,b\}$, $G_6(e_2)=\{b\}$.

Then $\tau_2=\{\varphi, Y, (G_1,E), (G_2,E),(G_3,E),(G_4,E),(G_5,E),(G_6,E)\}$ be a soft topology on Y. Let f: $X\rightarrow Y$ be an identity map. Here the inverse image of the soft πgr-open sets in Y are soft πgr-open in X, but the inverse image of soft open set $(G_4,E)=\{\{a,b\},\{b,c\}\}$ is not soft πgr-open in X. Hence soft πgr-irresoluteness need not be soft πgr-continuous.
Remark:4.12
The above discussions are represented diagrammatically as follows:



1-Soft continuous
2-soft regular continuous
3-soft πgr-irresolute
4-soft πgr-continuous
5-soft πg-continuous
6-soft rg-continuous
7-soft π^*g-continuous
8-soft πgp-continuous
9-soft $\pi g\alpha$-continuous
10-soft πgb-continuous
11-soft πgs-continuous.

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Theorem:4.13

A soft function f_{pu}: $SS(X)_A \rightarrow SS(Y)_B$ is soft πgr-continuous when f_{pu}(sπgr-cl(F,A)) $\tilde{\subseteq}$ s-cl(f_{pu}((F,A))) for every soft set (F,A) of a soft topological space (X,τ,A).

Proof: Let f_{pu}: $SS(X)_A \rightarrow SS(Y)_B$ be a soft πgr-continuous function. Now, s-cl(f_{pu}((F,A))) is a soft closed set of (Y,τ*,B). By the soft πgr-continuity of f_{pu}, f_{pu}^{-1} (s-cl(f_{pu}((F,A))) is soft πgr-closed and (F,A) $\tilde{\subseteq}$ $\boldsymbol{f_{pu}^{-1}}$ (s-cl(f_{pu}((F,A))).But sπgr-cl (F,A) is the smallest soft πgr-closed set containing (F,A), hence sπgr-cl(F,A) $\tilde{\subseteq}$ f_{pu}^{-1} (s-cl(f_{pu}((F,A))). The above implies f_{pu}(sπgr-cl(F,A)) $\tilde{\subseteq}$ s-cl(f_{pu}((F,A))).

Theorem:4.14

A soft function f_{pu}: $SS(X)_A \rightarrow SS(Y)_B$ is soft πgr-continuous when f_{pu}^{-1}(s-int(G,B)) $\tilde{\subseteq}$ sπgr-int(f_{pu}^{-1} ((G,B)) for every soft set (G,B) of a soft topological space (Y,τ*,B).

Proof : Let f_{pu}: $SS(X)_A \rightarrow SS(Y)_B$ be a soft πgr-continuous function. Now, s-int(f_{pu}((G,B))) is a soft open set of (Y,τ*,B), so by soft πgr-continuity of f_{pu} , f_{pu}^{-1}(s-int(f_{pu}((G,B))) is soft πgr-open in (X,τ,A) and f_{pu}^{-1}(s-int((G,B)))) $\tilde{\subseteq}$ (G,B). As sπgr-int(G,B) is the largest soft πgr-open set contained in (G,B), f_{pu}^{-1} (s-int(G,B)) $\tilde{\subseteq}$ sπgr-int(f_{pu}^{-1} ((G,B)) for every soft set (G,B) of a soft topological space (Y,τ*,B).

Theorem:4.15

A soft function f_{pu}: $SS(X)_A \rightarrow SS(Y)_B$ is soft πgr-closed if sπgr-cl(f_{pu}((F,A))) $\tilde{\subseteq}$ f_{pu}(s-cl(F,A)) for every soft set (F,A) of a soft topological space (X,τ,A).

Proof: Suppose that f is soft πgr-closed and (F,A) is any soft set of (X,τ,A). We have, f_{pu}((F,A)) $\tilde{\subseteq}$ f_{pu}(s-cl(F,A)). Now,sπgr-cl(f_{pu}((F,A))) $\tilde{\subseteq}$ sπgr-cl(f_{pu}(s-cl(F,A))).

Since f_{pu}(s-cl(F,A)) is soft πgr-closed in (Y,τ*,B), sπgr-cl(f_{pu}(s-cl(F,A)))=f_{pu}(s-cl(F,A)) for every soft set (F,A) of (X,τ,A).Hence sπgr-cl(f_{pu}((F,A))) $\tilde{\subseteq}$ f_{pu}(s-cl(F,A)) for every soft set (F,A) of a soft topological space (X,τ,A).

Theorem:4.16

A soft function f_{pu}: $SS(X)_A \rightarrow SS(Y)_B$ is soft πgr-open if f_{pu}(s-int((F,A))) $\tilde{\subseteq}$ sπgr-int(f_{pu}((F,A))) for every soft set (F,A) of a soft topological space (X,τ,A).

Proof: Let f_{pu}: $SS(X)_A \rightarrow SS(Y)_B$ is soft πgr-open map and (F,A) be any soft set of (X,τ,A).Then s-int (F,A) is soft open in (X,τ,A). Since f is soft πgr-open map, f_{pu}(s-int(F,A)) is soft πgr-open in (Y,τ*,B). We have f_{pu}(s-int (F,A)) $\tilde{\subseteq}$ f_{pu}((F,A)) for every soft set (F,A) of a soft topological space (X,τ,A). Also, f_{pu}(s-int(F,A)) = sπgr-int(f_{pu}(s-int(F,A))) Hence f_{pu}(s-int(F,A)) $\tilde{\subseteq}$ sπgr-int(f_{pu}((F,A))).

Theorem:4.17

Let (X,τ,A) be a soft πgr-$T_{1/2}$-space f_{pu}: $SS(X)_A \rightarrow SS(Y)_B$ be a soft function. Then f_{pu} is soft πgr-continuous iff f_{pu} is soft regular continuous.

Proof: Let f_{pu} be a soft πgr-continuous function. Then f_{pu}^{-1}((G,B)) is soft πgr-closed in (X,τ,A) for every soft closed set (G,B) of (Y,τ*,B). Since (X,τ,A) is a soft πgr-$T_{1/2}$-space, every soft πgr-closed set is soft regular closed . Hence f_{pu}^{-1}((G,B)) is soft regular closed in (X,τ,A) for every soft closed set (G,B) in (Y,τ*,B) and hence f_{pu} is soft regular continuous.

Let f_{pu} be a soft regular continuous function in (X,τ,A) . Then f_{pu}^{-1}((G,B)) is soft regular closed in (X,τ,A) for every soft closed set (G,B) in (Y,τ*,B). Since every soft regular closed set is soft πgr-closed. Then f_{pu}^{-1}((G,B)) is soft πgr-closed in (X,τ,A) for every soft closed set (G,B) of (Y,τ*,B) and hence f_{pu} is soft πgr-continuous.

Theorem:4.18

A soft function f_{pu}: $SS(X)_A \rightarrow SS(Y)_B$ is soft πgr-irresolute, then

(i) f_{pu}(sπgr-cl(F,A)) $\tilde{\subseteq}$ sπgr-cl(f_{pu}((F,A))) for every soft set (F,A) of (X,τ,A).

(ii) sπgr-cl(f_{pu}^{-1}(G,B))) $\tilde{\subseteq}$ f_{pu}^{-1}(sπgr-cl(G,B)) for every soft set (G,B) of (Y,τ*,B).

Proof :(i) For every soft set (F,A) of (X,τ,A), sπgr-cl(f_{pu}((F,A))) is soft πgr-closed in (Y, τ*,B). By hypothesis , f_{pu}^{-1}(sπgr-cl(f_{pu}((F,A)))) is soft πgr-closed in (X,τ,A).Also , (F,A) = f_{pu}^{-1}(f_{pu}((F,A))) $\tilde{\subseteq}$ f_{pu}^{-1}(sπgr-cl(f_{pu}(F,A)))). By the definition of soft πgr-closure, we have sπgr-cl(F,A) $\tilde{\subseteq}$ f_{pu}^{-1}(sπgr-cl(F,A)). Hence, we get f_{pu}(sπgr-cl(F,A)) $\tilde{\subseteq}$ sπgr-cl(f_{pu}((F,A))).

(ii)Since sπgr-cl(G,B) is soft πgr-closed in Y and so by hypothesis, f_{pu}^{-1}(sπgr-cl(G,B)) is soft πgr-closed in (X,τ,A). Since f_{pu}^{-1}((G,B)) $\tilde{\subseteq}$ f_{pu}^{-1}(sπgr-cl(G,B)), it follows that sπgr-cl(f_{pu}^{-1}((G,B))) $\tilde{\subseteq}$ f_{pu}^{-1}(sπgr-cl(G,B)).

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