On Separation Axioms With Respect To Gem Set

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Abstract : **In this paper, we create a new definition of space namely" R^*_space, M^*_space and N^*_space in topological space " and we do a new definition of separation axioms by using the idea of "Gem set''.**
Keyword- New separation axioms, $R^* -$ space , M^*_space and N^*_space , Gem set and topological ideal .

1-INTRODUCTION AND PRELIMINARIES :

The epigram of ideal presented first by K. Kuratowski **[1].** In general topological Hamlett and Jankovi´c **[2, 3, 4,5]** they introduced the application of topological ideal in generalization of most essential properties and the ideal as this form: An ideal I on a topological space (X,T) is a non-empty collection of subsets of X which satisfies the following properties: (1) $A \in I$ and $B \subseteq A$ implies $B \in I$. (2) $A \in I$ and $B \in I$ implies $A \cup B \in I$ and called the (X,T,I) ideal topological space in addition to K. Kuratowski **[1]** used the ideal to define space (X,T,I) and a subset $A \subseteq X$, $A^* (I) =\{ x \in X : U \cap A \notin I$ for every $U \in T(x)\}$ is called the local function of A with respect to I. We simply write A^*instead of $A^*(I)$,F.G. Arenas , J .Dontchev and M.L .Puertas **[6]** introduce some weak separation axioms under the concept ideal . In recent years Al-Swidi and AL-sadaa **[7]** they defined for each element had ideal by this form: let (X,T) be a topological space and x∈ X ,we denote by I_x to an ideal $\{G\subseteq X: x\in G^c\}$, Where X is non-empty set ,again Al-Swidi and AL-Nefee **[8]** they use the idea of ideal I_x *to* defined new set namely "Gem set " which means that A subset B of a topological space (X,T) . Then they defined B^{*x} with respect to space (X,T) as follows : $B^{*x}=\{y\in X: G\cap B \notin I_x$, for every $G\in T(y)\}$ where $T(y)=\{G\in T: y \in G\}$ .A set B^{*x} was called "Gem set" and define a new separation axioms by using Gem set namely it the"I^*-T_i-space "and "I^{**}-T_i- space " ,i=0,1,2. Through out this paper we defined anew separation axioms by benefit of Gem set namely "S_i -space " i=0,1,2,3,4 and studied proprieties and the relationship between "I^*-T_i-space "and T_i-space. I=0,1,2. Also we define anew space called R^*_space , M^*_space and N^*_space and we study the proprieties and relation between them.

Definition1.1 [6].A topological space (X,T) is called

- I^* _$T_\circ$_space if and only if for each pair of distinct point x ,y of X there exist nonempty subset A ,B of X such that $y\notin A^{*x}$or $x\notin B^{*y}$.
- I^*_T_1_space if and only if for each pair of distinct point x, y of X there exist nonempty subset A ,B of X such that $y\notin A^{*x}$and $x\notin B^{*y}$.
- I^* _T_2_space if and only if for each pair of distinct point x, y of X there exist nonempty subset A,B of X such that $y\notin A^{*x}$and $x\notin B^{*y}$with $B^{*y} \cap A^{*x} = \emptyset$.
- I^{**} _$T_\circ$_space if and only if for each pair of distinct point x ,y of X there exist nonempty subset A of X such that $y\notin A^{*x}$or $x\notin A^{*y}$.
- I^{**}_T_1_space if and only if for each pair of distinct point x ,y of X there exist nonempty subset A of X such that $x\notin A^{*y}$and $y\notin A^{*x}$.
- I^{**} _T_2_space if and only if for each pair of distinct point x ,y of X there exist nonempty subset A of X such that $x\notin A^{*y}$and $y\notin A^{*x}$ with $A^{*y} \cap A^{*x} = \emptyset$.

Definition1.2 [6] Let (X,T) be a topological space and A⊆ X We define $Pr^{*x}(A) = A^{*x} \cup A$, for each $x \in X$.

Proposition 1.3 [6] Let (X,T)be a topological space x∈ A if and only if x∈ A^{*x} ,for each A⊆ X ,x∈X.

Definition1.4 [6] A mapping f :X→ Y is called I^*-map .if and only if , for every subset A of X ,x∈ X $f((A^{*x}) = f(A)^{*f(x)}$.

Remark 1.5 [5] Let f: $(X,T) \to (Y,\rho)$ be I^*-map .then ,$f(pr^{*x}(A))= pr^{*f(x)}(A))$ for every subset A of X and $x\in X$

Proposition 1.6 Let (X,T) be topological space, and A subset of X ,x∈ X. If x∉ A , Then $A^{*x} = \emptyset$

Proof:- Let $A^{*x} \neq \emptyset$ Then there exist at least one element, say $y \in A^{*x}$ by definition of Gem set then $A\cap G_y \in I_x$.Hence $x\in A \cap G_Y$ so x∈ A which Contradiction ,then $A^{*x} = \emptyset$

Definition1.7 Let (X,T) be a topological space ,for each x∈ X ,anon empty subset A of X ,is called a strong set if and only if (A^{*x} is open set and x∈ A).

Definition 1.8 A topological space (X,T) is said strong space if every sub set of X is strong set .

2-"R^*_space , M^*_space and N^*_space in topological space "

In this section, we offer new definitions of the spaces through Gem set call them R^*_space , M^*_space and N^*_space with study some results and properties.

Remark 2.1 . If a function f:(X, T) → (Y, ρ) is one to one ,then$f^{-1}(I_y) = I_{f^{-1}(y)}$for each y ∈Y.

Remark 2.2 . If a function f:(X, T) → (Y, ρ) is bijection , then $f(I_{f^{-1}(y)}) = I_y$ for each y ∈Y.

Theorem 2.3 . If a function f:(X, T) → (Y, ρ) is continuous and one to one ,then$f^{-1}(B^{*y}) \subseteq (f^{-1}(B))^{*f^{-1}(y)}$ for each y ∈Y.

Proof:- Let a∈ $f^{-1}(B^{*y})$,then there exist d∈ B^{*y} with a=f^{-1}(d). And $H_d \cap B \notin I_y$ for each $H_d \in$ T(d) ,then $f^{-1}(H_d) \cap f^{-1}(B) \notin I_{f^{-1}(y)}$ (by remark 2.1).Hence by continuity of f we get that a ∈ $((f^{-1}(B))^{*f^{-1}(y)}$ .Then $f^{-1}(B^{*y}) \subseteq ((f^{-1}(B))^{*f^{-1}(y)}$.

Theorem 2.4 . If a function f:(X, T) → (Y, ρ) is continuous, open and bijection , then $(f^{-1}(B))^{*f^{-1}(y)} \subseteq f^{-1}(B^{*y})$ for each y∈Y.

Proof :- Let a $\in (f^{-1}(B))^{*f^{-1}(y)}$ then $U_a \cap f^{-1}(B) \notin I_{f^{-1}(y)}$ for each $U_a \in$ T(a) thus $f(U_a) \cap B \notin I_y$ (by Remark 2.2).Therefor by properties of open map ,we get that a∈ ($f^{-1}(B^{*y})$ hence $(f^{-1}(B))^{*f^{-1}(y)} \subseteq f^{-1}(B^{*y})$.

Corollary 2.5 If a function f:(X, T) → (Y, ρ) is continuous, open and bijection . Then $(f^{-1}(B))^{*f^{-1}(y)} = f^{-1}(B^{*y})$ for each y∈Y.

Proof :- By using the theorem 2.3 and 2.4 , we get the prove.

Definition 2.6 . A topological space (X,T) is called R^*_space if and only if for each x∈ Xand a nonempty subset A of X such that x∉ Pr^{*x}(A) then there exist a nonempty subset B, C of X such that x∈ B^{*x}, $Pr^{*x}(A) \subseteq C^{*x}$.

we noted that in definition of R^*_space that $B^{*x}$and C^{*x} are disjoint where if x∉ C $C^{*x} = \emptyset$ this contradiction with $Pr^{*x}(A) \subseteq C^{*x}$.

Example2.7. Let X = {x, y, z } , T ={X,Ø}I_x={Ø,{z},{y},{z, y}},I_y = {Ø,{z},{x},{z, x}}andI_z={Ø,{y},{x},{y, x}}

set A ={y},B ={x, y}and C ={z, x} then pr^{*x}(A)={ y},B^{*x} = {x, y, z}and C^{*x}={x,z,y} it follows x∈ B^{*x}, pr^{*x}(A)⊆ C^{*x}.

eroehTm 2.8. Every subspace of R^*_space is R^*_space **.**

Proof :- Let Y⊆ X, y∈ Yand A⊆ Y such that y∉ pr^{*y}(A) ,then there exist a subset H of X such that pr^{*y}(A)= pr^{*y}(H)∩ Y ,so we get that y∉ pr^{*y}(H)∩ Y, but X is R^*_space then there exist a nonempty subsets B,C of X such that pr^{*y}(H)⊆ $B^{*y}$and y∈ C^{*y} .Thus pr^{*y}(H)∩ Y ⊆ B^{*y} ∩ Yand y ∈ C^{*y} ∩ Y .Put $D^{*y} = B^{*y} \cap Y$ and $S^{*y} = C^{*y} \cap Y$ it is follows that $pr^{*y}(A) \subseteq D^{*y}$and y∈ S^{*y}.Hence Y is R^*_subspace.

Theorem 2.9. Let f be bijection open and continuous map from (X,T) space onto R^*- space (Y,ρ). Then (X,T) is R^*-space ,if f is I^*-map.

proof:-Let x∈ X and A⊆ X such that x∉ pr^{*x}(A). Since f is I^*-map. then f(x) ∉ $pr^{*f(x)}(f(A))$,but Y is R^*- space so there exist a nonempty sub sets B,C of Y such that $pr^{*f(x)}(f(A)) \subseteq B^{*f(x)}$ and f(x) ∈ $C^{*f(x)}$.So $f^{-1}(B^{*f(x)})$ and $f^{-1}(C^{*f(x)})$ is a nonempty subset of X such that . $f^{-1}(pr^{*f(x)}(f(A)) \subseteq f^{-1}(B^{*f(x)})$and x ∈ $f^{-1}(C^{*f(x)}) = [f^{-1}(C)]^{*x}$,(by corollary 2.5) $pr^{*x}(A) \subseteq B^{*x}$ and x∈ $[f^{-1}(C)]^{*x}$ .Hence (X ,T) is R^*-space.

Theorem 2.10. Let f be bijection open and continuous map from (X,T) R^*-space onto (Y,ρ). Then (Y,ρ) is R^*-space ,if f is I^*-map.

Definition 2.11.:A topological space (X,T) is called M^*_space if and only if for each x∈ X and a nonempty subset A of X such that x∉ Pr^{*x}(A) then there exist a nonempty B,C of X , such that x∈ Pr^{*x}(C) and $Pr^{*x}(A) \subseteq Pr^{*x}(C)$,With $pr^{*x}(B) \cap Pr^{*x}(C) = \emptyset$.

Example 2.12. Let (X,T) be a topological space such that X ={x, y, z } ,T={X,Ø,{y},{z},{y, z}},I_x ={Ø,{z},{y},{z ,y}} , I_y ={Ø,{z},{x},{z ,x}} and I_z={Ø,{x},{y},{x ,y}} . let x∈ X and a nonempty subset set A={y} of X such that x∉ pr^{*x}(A)={y} then there exist a nonempty subset B ={x}and C ={z, y} of X then,pr^{*x} (B)={x}and pr^{*x}(C)={z,y} it follows x∈ pr^{*x} (B) , pr^{*x}(A)⊆ pr^{*x}(C) with pr^{*x} (B)∩ pr^{*x}(C)={x } ∩ {z, y} = Ø .

Theorem 2.13. Every subspace of M^*_space , is M^*_subspace.

Theorem 2.14. Let f be bijection open and continues map from M^*- space (X,T),onto (Y,ρ) space. Then (Y,ρ) is M^*-space if f is I^*-map .

Proof:-Let y∈ Y and B⊆ Y such that y∉ pr^{*y}(B) (By Remark 1.5) then $f^{-1}(y) \notin pr^{*f^{-1}(y)}(f^{-1}(B))$ since X is M^*- space then there exist a nonempty sub sets D,C of X such that $pr^{*f^{-1}(y)}(f^{-1}(B)) \subseteq pr^{*f^{-1}(y)}(D)$and $f^{-1}(y) \in pr^{*f^{-1}(y)}(C)$.with $pr^{*f^{-1}(y)}(D) \cap pr^{*f^{-1}(y)}(C) = \emptyset$. Now $f(pr^{*f^{-1}(y)}(f^{-1}(B)) \subseteq f(pr^{*f^{-1}(y)}(D)), f(f^{-1}(y)) \in f(pr^{*f^{-1}(y)}(C))$ since f is I^*-map ,then $pr^{*y}(B) \subseteq pr^{*y}(f(D))$, y∈ $pr^{*y}(f(C))$ with $f(pr^{*x}(D) \cap pr^{*x}(C)) = pr^{*y}(f(D)) \cap pr^{*y}(f(C)) = f(\emptyset) = \emptyset$ then Y is M^*- space.

Theorem 2.15 Let f be bijection I^*_map and continues map from (X,T) space onto M^*- space (Y,ρ) space. Then (X,T) is M^*- space if f is open map .

proof :- The same of above theorem .

Definition 2 .16 A topological space (X,T) is said to be $N^* -$ space if and only if for each x∈ X and every tow subsets M,L of X then there exist a nonempty subsets B,C of X such that $pr^{*x}(M) \subseteq B^{*x}$and $pr^{*x}(L) \subseteq C^{*x}$.

Remark 2.17. We can not say $B^{*x} \cap C^{*x} = \emptyset$ is disjoint set if $B^{*x} \cap C^{*x} = \emptyset$ then x∉ C then $C^{*x} = \emptyset$ this contradiction with $pr^{*x}(L) \subseteq C^{*x}$,similar x∉ B .

Theorem 2.18 Every subspace of $N^* -$ space is $N^* -$ space.

Theorem 2.19 Let f be is a bijection and continues function of space (X,T) onto N^*- space (Y,ρ) space. Then (X,T) is N^*- space if f is open map.

Proof:-Let L,M pair sub set of X. so f(L),f(M) is disjoint subset of Y since Y is N^*- space then there exist a nonempty sub sets B,C of Y such that $pr^{*y}(f(L)) \subseteq B^{*y}$ and $pr^{*y}(f(M)) \in C^{*y}$. so $f^{-1}(B^{*y})$and $f^{-1}(C^{*y})$is a nonempty subset of X, thus $f^{-1}(pr^{*X}(f(L)) \subseteq f^{-1}(B^{*y})$, $f^{-1}(pr^{*X}(f(M)) \subseteq f^{-1}(C^{*y})$ (by corollary 2.5)

$f^{-1}(\mathrm{pr}^{*y}(f(\mathrm{L})) = \mathrm{pr}^{*x}(\mathrm{L}) \subseteq (f^{-1}(\mathrm{B}))^{*x}$ and $\mathrm{pr}^{*x}(\mathrm{M}) \subseteq (f^{-1}(\mathrm{C}))^{*x)}$.Hence (X ,T) is N^*- space.

3 − "S_i separation axioms"

In this section, we introduce the concept of new definition separation axioms called S_i -spaces and investigate some of their properties and study the relationship between "I^*-T_i-space ","I^{**}-T_i-space and T_i-space.

Definition 3. 1 :-

1. A topological space (X,T) is said $S_\circ$_space if and only if for each pair of distinct point x ,y of X there exist nonempty subset A of X and A contain at least one of them such that $y \notin A^{*x}$ or $x \notin A^{*y}$.
2. A topological space (X,T) is said S_1 space if and only if for each pair of distinct point x ,y of X there exist nonempty subset A of X and A contain at least one of them such that $x \notin A^{*y}$ and $y \notin A^{*x}$.
3. A topological space (X,T) is said S_2 space if and only if for each pair of distinct point x ,y of X there exist nonempty subset A of X and A contain at least one of them such that $x \notin A^{*y}$ and $y \notin A^{*x}$ with $A^{*y} \cap A^{*x} = \emptyset$.
4. A topological space (X,T) is said S_3_space if it was M^* − space and S_1 space
5. A topological space (X,T) is said S_4_space if it was N^* − space and S_1 space.

Theorem 3.2 For topological space (X,T), then the following properties hold:

1. Every $T_\circ$_space is $S_\circ$_space
2. Every T_1 space is S_1_space.
3. Every T_2 space is S_2 space.
4. Every $S_\circ$_space is I^{**}_$T_\circ$_space.
5. Every S_1 space is I^{**}_T_1_space.
6. Every S_2 space is I^{**} _T_2 space.
7. Every $S_\circ$_space is I^*_$T_\circ$_space.
8. Every S_1_space is I^*_T_1_space.
9. Every S_2_space is I^*_T_2_space.

Proof :- Straight forward .

Remark. 3.3. The converse of theorem need not be true as since from the following examples .

Example 3.4 Let(X,T) be a topological space such that X={x, y,z,h} T={ X,Ø,{z},{y,z}}and, $I_x = \{\emptyset,,\{y\},\{z\},\{h\},\{y,z\},\{y,h\},\{z,h\},\{y,z,h\}\}$ and $I_Y = \{\emptyset,\{x\},\{z\},\{h\},\{x,z\},\{z,h\}\{x,h\},\{x,z,h\}\}$. Let x, y $\in X$ such that $x \neq y$ then there exist a nonempty subset A={x} of X contain at least one element of them we say that $x \in A$ such that $x \notin A^{*y} = \emptyset$. Then (X,T) is $S_\circ$-space but not $T_\circ$-space.

Example 3.5 Let(X,T) be a topological space such that X={x, y, z} T={X,Ø,{x},{x ,z},{x ,y},{y}}and $I_x = \{X,\{y\},\{z\},\{y,z\}\}$, $I_y = \{X,\{x\},\{z\},\{x,z\}\}$ .Let x, y $\in X$ such that $x \neq y$ then there exist a nonempty subset A={x} of X contain at least one element of them we say that $x \in A$ such that $x \notin A^{*y} = \emptyset$.and $y \notin A^{*x} = \{x,z\}$.Hence (X,T) is ($S_1$-space and S_2-space)but not (T_1-space and T_2-space)

Example 3.6 Let X={x, y, z} T={X,Ø,{z}} $I_x = \{\emptyset,\{y\},\{z\},\{y,z\}\}$, $I_y = \{\emptyset,\{x\},\{z\},\{x,z\}\}$, . Let x ,y $\in X \ni x \neq y$ there exist A={z} . A^{*x}=Ø and $A^{*y} = \emptyset$ so $y \notin A^{*x}$ and $x \notin A^{*y}$ .hence (X,T) is (I^{**}_ T_1_space and I^{**}_ T_2_space)but not (S_1_space and S_2_space)since Let x ,y $\in X$such that $x \neq y$ there exist A={x} . A^{*x}={x, y} so $y \in A^{*x}$ Then (X,T) is not(S_1_space and S_2_space).

Example 3.7 Let X={x, y,z} T={X,Ø,{x},{x, y}}and $I_y = \{\emptyset,\{x\},\{z\},\{x,z\}\}$,$I_x = \{\emptyset,\{z\},\{y\},\{z,y\}\}$. Let x, y $\in X \ni x \neq y$ there exist A={z} and B={y}, $A^{*x} = \emptyset$ and ${B}^{*y} = \{y,z\}$, $x \notin B^{*y}$and $y \notin A^{*x}$ ..Hence (X,T) is I^*_ T_1_space but not S_1_space since Let x, y $\in X \ni x \neq y$,A = {x}then y $\in A^{*x} = \{x,y,z\}$then (X,T) is (I^*_ T_1_spaceand I^*_ T_2_space) but not (S_1-space and S_2-space).

Preposition 3.8 Let (X,T) be topological space if X strong space then we have.

1. Every $S_\circ$_space is $T_\circ$_space.
2. Every S_1_space is T_1_space.
3. Every S_2_space is T_2_space.
4. Every I^{**}_$T_\circ$_space is $S_\circ$_space
5. Every I^{**}_T_1_space is S_1_space.
6. Every I^{**}_T_2_space is S_2_space.
7. Every I^* - $T_\circ$_space is $S_\circ$_space.
8. Every I^*_$T_\circ$_space is $T_\circ$_space.
9. Every I^*_T_1_space is T_1_space.
10. Every I^*_T_2_space is T_2_space.
11. Every I^{**}_$T_\circ$_space is $T_\circ$_space.
12. Every I^{**}_T_1_space is T_1_space.
13. Every I^{**}_T_2_space is T_2_space.

Proof:- (1) Let x ,y $\in X$ such that $x \neq y$.Since (X,T) is $S_\circ$_space then there exist a nonempty subset A of X contain at least one element of them we say that $x \in A$ such that $x \notin A^{*y}$or $y \notin A^{*x}$.Assume $y \notin A^{*x}$ since X is strong space then A is strong set so A^{*x} is open set and then $x \in A^{*x}$.Hence (X ,T)is $T_\circ$_space.

Proof:- (2) Let x ,y $\in X$ such that $x \neq y$.Since (X,T) is S_1_spacethen there exist a nonempty subset A of X contain at least one element of them we say that $x \in A$ such that $x \notin A^{*y}$and $y \notin A^{*x}$since X is strong space then A^{*x} is open set with $x \in A$ and A^{*y} is open set with $y \in A$ (by proposition 1.6)) $x \in A^{*x}$ and $y \in A^{*y}$,hence (X,T) is T_1_space.

Proof:- (3) The same way of proof (2)

Proof:-(4) Let x ,y $\in X$ such that $x \neq y$.Since (X,T) is I^{**} - $T_\circ$_space thenThere exist a nonempty subset A of X such that $x \notin A^{*y}$or $y \notin A^{*x}$ .Since X is strong space then A is strong therefor $x \in A$ hence (X,T) is $S_\circ$_space.

Proof:-(5) Let x ,y $\in X$ such that $x \neq y$.And let (X,T) is I^{**} - T_1_space thenThere exist a nonempty subset A of X such that x $\notin A^{*y}$ and y$\notin A^{*x}$ .Since X is strong space then A is strong therefor x$\in A$ hence (X,T) is S_1_space .

Proof:-(6) The same way of proof (5)

Proof:-(7) Let x ,y $\in X$ such that x$\neq y$.And let (X,T) is I^* - $T_\circ\_space$ then

There exist two a nonempty subset B, A of X such that x $\notin A^{*y}$ or y$\notin A^{*x}$ assume x $\notin A^{*y}$ since X is strong space then A is strong set .So Then x$\in A$ .Hence (X,T) is $S_\circ\_space$.

Proof :-(8) Let x ,y $\in X$ such that x$\neq y$.and let (X,T) is I^* - $T_\circ\_space$ then

There exist two a nonempty subset B, A of X such that x $\notin B^{*y}$ or y$\notin A^{*x}$,since X is strong space then A ,B are strong sets .So A^{*x} , B^{*y} are open sets and x$\in A$,y$\in B$.(by proposition 1.6) then x$\in A^{*x}$ and y$\in B^{*y}$ and we have x $\notin B^{*y}$ or y$\notin A^{*x}$ .Hence (X,T) is $S_\circ\_space$.

Proof :- (9) Let x ,y $\in X$ such that x$\neq y$.And let (X,T) is I^* - T_1_space then

There exist two a nonempty subset B, A of X such that x $\notin B^{*y}$ and y$\notin A^{*x}$ since X is strong space then A ,B are strong sets .so A^{*x} , B^{*y} are open sets and x$\in A$,y$\in B$.(by proposition 1.6) then x$\in A^{*x}$ and y$\in B^{*y}$ and we have x $\notin B^{*y}$ and y$\notin A^{*x}$ .Hence (X,T) is S_1_space.

Proof :- (10) The same way of proof (9)

Proof :- (11) Let x ,y $\in X$ such that x$\neq y$.Since (X,T) is I^{**} - $T_\circ\_space$ thenThere exist a nonempty subset A of X such that x $\notin A^{*y}$ or y$\notin A^{*x}$ .Since X is strong space then A is strong therefor A^{*x} *is open set and* x$\in A$ (by proposition 1.6) x$\in A^{*x}$ .Hence (X,T) is $T_\circ\_space$.

Proof :- (12) Let x, y $\in X \ni x \neq y$ and let X is I^{**} - T_1_space then there exist a subset A of *such that* $x \notin A^{*y}$ and $y \notin A^{*x}$ .since $x \notin A^{*y}$ then $x \notin (A^{*y})^{*x}$ since$(A^{*y})^{*x} = \emptyset$.

Put $A^{*y} = B$ then $B^{*x} = \emptyset$ then y$\notin B^{*x}$ and we have $x \notin A^{*y}$.Since X is strong set then A^{*y} and B^{*x} .Hence (X,T) $I^{**}_T_1_$space.

Proof :- (13) The same way the proof (12).

Corollary 3.9 Let (X,T) be topological space if X is strong space and M^* - space then we have .

1. Every S_2_space is S_3_space.
2. Every I^{**}-T_2_space is S_3_space.

Theorem 3.10 Let (X,T) is S_3-space then $I^{**}_T_2_$space

Proof:- By (Let (X,T) is S_3-space then S_2-space) then X is S_2-space and by S_2-space is $I^{**}_T_2_$space then (X,T) is $I^{**}_T_2_$space.

Remark 3.11 Every S_2 -space is a S_1 $-$space.

Remark 3.12 If X is door space then every T_3 –space is S_3 –space

Theorem 3.13 Let f be is bijection and I^* -map from S_i_space (X,T) onto (Y,ρ) space then (Y,ρ) is S_i_space if f is open and continuous for each i=0,1,2.

Proof :-Assume i=2,let $y_1, y_2 \in Y$ such that $y_1 \neq y_2$ since f is one to one then$f^{-1}(y_1) \neq f^{-1}(y_2)$ because of X is S_1-space then there exist a nonempty subset A of X such that A contain at least one element we say that $f^{-1}(y_1) \in A$ and $f^{-1}(y_1) \notin A^{*f^{-1}(y_2)}$ and $f^{-1}(y_2) \notin A^{*f^{-1}(y_1))}$ with $A^{*f^{-1}(y_2)} \cap A^{*f^{-1}(y_1))} = \emptyset$ so $f(f^{-1}(y_1)) \in A = y_1 \in f(A)$ since f is I^* -map then we get that $y_1 \notin f(A)^{*y_2}$ and $=y_2 \notin f(A)^{*y_1}$ with $f(A)^{*y_2} \cap f(A)^{*y_1}) = f(\emptyset) = \emptyset$. Then Y is S_1-space.

Theorem 3.14 Let f be is open and bijection from (X,T) onto S_i_space (Y,ρ) then (X,T) is S_i_space if f is $I^* - map$ for each i=0,1,2

Proof:- Assume i=2 ,let $x_1, x_2 \in X$ such that $x_1 \neq x_2$ since f is one to one then$f(x_1) \neq f(x_2)$ but Y is S_1-space then there exist a nonempty subset B of Y such that B contain at least one element we say that $f(x_1) \in B$ with $f(x_1) \notin B^{*f(x_2)}$and $f(x_2) \notin B^{*f(x_1)}$such that $B^{*f(x_2)} \cap B^{*f(x_1)} = \emptyset$ so $x_1 \in f^{-1}(B)$and f^{-1} $(f(x_1)) \notin f^{-1}(B^{*fx_2)})$=$x_1 \notin f^{-1}(B)^{*x_2}$, $x_2 \notin f^{-1}(B)^{x_1}$ with $f^{-1}(B)^{f(x_1)}) \cap f^{-1}(B)^{*f(x_2)}) = f^{-1}(B)^{x_1} \cap f^{-1}(B)^{*x_2} = \emptyset$.Then X is S_2-space.

Proposition 3.15 The product space of S_2-space is S_2-space.

Proof :- Let X and Y are S_2-space to prove X$\times Y$ is S_2-space .Let (x_1,y_1), (x_2,y_2) be two distinct point of X$\times Y$ either $x_1 \neq x_2$ or $y_1 \neq y_2$ take $x_1 \neq x_2$.since X is S_2-space then there exist a nonempty sub set A of X such that contain at least one element we say that $x_1 \in A$ and $x_1 \notin A^{*x_2}$ and $x_2 \notin A^{*x_1}$ with $A^{*x_2} \cap A^{*x_1} = \emptyset$. it follows $A^{*x_1} \times Y$ and $A^{*x_2} \times Y$ is a nonempty subset of X$\times Y$ then $(x_1,y_1)$$\notin A^{*x_2} \times Y$ and $(x_2,y_2)$$\notin A^{*x_1} \times Y$ with $(A^{*x_1} \times Y)$$\cap$ $(A^{*x_2} \times Y)$ =$(A^{*x_1} \cap A^{*x_2}) \times Y = \emptyset \times Y = \emptyset$. Hence X$\times Y$ is S_2-space.

Corollary 3.16 The product space X=$\times\{X_\lambda:\lambda \in \Lambda\}$ If X_λ is S_2-space then X is S_2-space.

Proof :- Let X_λ is S_2-space to prove X =$\times\{X_\lambda:\lambda \in \Lambda\}$ is S_2-space let x=$\{x_\lambda: :\lambda \in \Lambda\}$ and y=$\{y_\lambda: :\lambda \in \Lambda\}$ be two distinct point of X then $x_\mu \neq y_\mu$ for some $\mu \in \Lambda$ where $x_\mu, y_\mu \in X$ .since X_μ is S_2-space then there exist A_μ a nonempty sets in X_μ and contain at least one element we say that $x_\mu \in A_\mu$such that $x_\mu \notin {A_\mu}^{*y_\mu}$ and $y_\mu \notin {A_\mu}^{*x_\mu}$ with ${A_\mu}^{*y_\mu} \cap {A_\mu}^{*x_\mu} = \emptyset$ since $\pi_\mu(x) = x_\mu$ and $\pi_\mu(y) = y_\mu$ it follows $\pi_\mu(x) = x_\mu \notin {A_\mu}^{*y_\mu}$ and $\pi_\mu(y) = y_\mu \notin {A_\mu}^{*y_\mu}$ then x$\notin {\pi_\mu}^{-1}({A_\mu}^{*y_\mu})$ and y$\notin {\pi_\mu}^{-1}({A_\mu}^{*x_\mu})$ with ${\pi_\mu}^{-1}\big({A_\mu}^{*y_\mu} \cap {A_\mu}^{*x_\mu}\big) = {\pi_\mu}^{-1}(\emptyset) = \emptyset$,${\pi_\mu}^{-1}({A_\mu}^{*y_\mu})$ and ${\pi_\mu}^{-1}({A_\mu}^{*x_\mu})$ are a nonempty sets in X .Hence X is S_2-space.

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