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On Optimal Production scheduling of an EPQ model with Stock dependent Production Rate having Selling Price Dependent Demand and Pareto decay

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Abstract

EPQ models play an important role in production and manufacturing units. Much work has been reported in literature regarding EPQ models with finite rate of production. But in many industries like agricultural products manufacturing units the production is dependent on stock on hand. Hence in this paper we develop and analyze an EPQ model for deteriorating items with stock dependent production rate having selling price dependent demand and Pareto rate of decay. Using the differential equations the instantaneous state of inventory is derived and with suitable cost considerations the optimal quantity, production uptime and production downtime are obtained for two cases of with and without shortages. The sensitivity analysis of the model revealed that the stock dependent production has a significant influence an optimal production schedule and can reduce total cost of production. This model also includes the finite rate of production inventory model with Pareto decay as a particular case.

Key words: EPQ model, Stock dependent production, Pareto decay.

1. Introduction

Much work has been reported in literature regarding Economic Production Quantity (EPQ) models during the last two decades. The EPQ models are also a particular case of inventory models. The major constituent components of the EPQ models are 1) Demand 2) production (Replenishment) and 3) Life time of the commodity. Several EPQ models have been developed and analyzed with various assumptions on demand pattern and life time of the commodity. In general it is customary to consider that the replenishment is either finite or infinite in production inventory models.

. Goel and Aggrawal (1980) , Teng, et al.(2005), Srinivasa Rao and Begum (2007), Maiti, et al. (2009), Srinivasa Rao and Patnaik (2010), Tripathy and Misra (2010), Sana (2011) and others have studied inventory models having selling price dependent demand. In all these papers they considered that the replenishment is infinite/finite and constant rate. Sridevi, et al. (2010) developed and analyzed an inventory model with the assumption that the rate of production is random and follows a weibull distribution. However, in many practical situations arising at production processes the production (replenishment) rate is dependent on the stock on hand. The consideration of production rate being dependent on on-hand inventory can significantly reduced wastage of resources and increase profitability.

Another important consideration for developing the EPQ models for deteriorating items is the life time of the commodity. For items like agricultural products, chemicals etc., the life time of the commodity is random and follows a Pareto distribution. (Srinivasa Rao, et al. (2005), Srinivasa Rao and Begum (2007), Srinivasa Rao and Eswara Rao (2011)). Very little work has been reported in the literature regarding EPQ models for deteriorating items with Pareto decay having stock dependent production rate and selling price dependent demand, even though these models are more useful for deriving the optimal production schedules of many production processes. Hence, in this paper we develop and analyze an economic production quantity model with stock dependent production having selling price dependent demand and Pareto decay. The Pareto distribution is capable of characterizing the life time of the commodities which have a minimum period to start deterioration and the rate of deterioration is inversely proportionate to time.

Using the differential equations the instantaneous state of inventory is derived. With suitable cost considerations the total cost function and profit rate function are derived. By maximizing the profit rate function the optimal production quantity, production up time, production down time are derived. A numerical illustration is also discussed. The sensitivity of the model with respect to the costs and parameters is also discussed.

2. Assumptions and notations of the model

The following assumptions are made for developing the inventory model under study.

i. Life time of the commodity is random and follows a pareto distribution having probability density function of the form

$f(t) = \frac{b\theta^b}{t^{b-1}};\ t \geq \theta,\ \ b, \theta \geq 0$

The instantaneous rate of deterioration is $h(t) = \frac{f(t)}{1-F(t)} = \frac{b}{t}$; b $> 0, t > \theta$.

ii. The demand is a function of selling price and is of the form $\lambda(s) = a - ds$ where, a and d are constant, a > 0, d ≥ 0, s is the unit selling price. If d = 0 then the demand rate will be constant

iii. The rate of production is dependent on stock on hand and is of the form $R(t) = \eta - kI(t)$, such that R (t) ≥ 0. where, I (t) is the stock on hand at time t, $\eta > 0$, $0 \leq k \leq 1$. When k=0, this production rate reduces to constant rate of production.

iv. There is no repair or replacement of deteriorated items.

v. The planning horizon is finite. Each cycle will have length T.

vi. Lead time is zero.

vii. The inventory holding cost per unit time (h), the shortage cost per unit per unit time (π), the unit production cost per unit time (c) and set up cost(A) per cycle are fixed and known.

H total inventory holding cost in a cycle time

I (t) inventory level at any time t

Q production quantity

S_1 maximum inventory level

S_2 maximum shortage level

R (t) rate of production at any time t

S_h total shortage cost in a cycle time

t_1 time point at which production stops (production down time)

t_2 time point at which shortage begins

t_3 time point at which production resumes (production uptime)

3 EPQ model without shortages

3.1 Model formulation

Consider a production system in which the production starts at time t = 0 and inventory level gradually increases with the passage of time due to production and demand during the time interval (0, t_1). At time t_1 the production is stopped and let S_1 be the inventory level at that time. During the time interval (t_1, T) the inventory decreases partly due to demand and partly due to deterioration of items. The cycle continues when inventory reaches zero at time t = T. The schematic diagram representing the model is shown in fig.1.

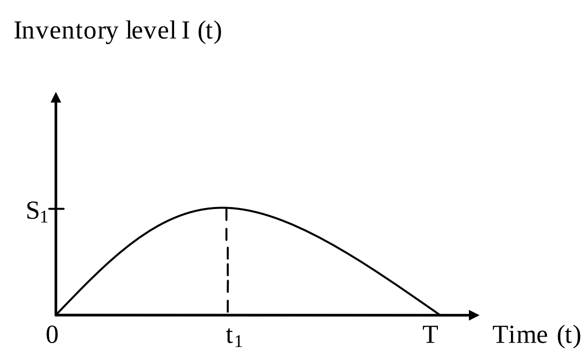

Fig.1.The schematic diagram representing the inventory level of the system without shortages.

The differential equations governing the system in the cycle time (0, T) are;

$$\frac{dI(t)}{dt} = \eta - kI(t) - h(t)\, I(t) - (a - ds), 0 \leq t \leq t_1 \quad (1).$$

$$\frac{dI(t)}{dt} = -h(t)\, I(t) - (a - ds)\ ,\ \ t_1 \leq t \leq T \qquad (2)$$

With the boundary conditions I (0) = 0 and I (T) = 0. Solving the equations (1) and (2), the instantaneous state of inventory at any time t during the interval $(0, t_1)$ is obtained as

$$I(t) = e^{-kt}t^{-b}\{\eta - (a - ds)\}g(t,b,k), 0 < t < t_1 \quad (3)$$

where, $g(t,b,k) = \int_0^t e^{ku} u^b du$ (4)

The instantaneous state of inventory at any time t during the interval (t_1, T) is obtained as

$$I(t) = \frac{(a-ds)}{b+1}\left[t^{-b}T^{b+1} - t\right], \; t_1 \le t \le T \quad (5)$$

The total inventory in the time period $0 \le t \le t_1$ is

$$\int_0^{t_1} I(t)\,dt = \int_0^{t_1} e^{-kt}\,t^{-b}\{\eta - (a - ds)\}g(t,b,k)\,dt \quad (6)$$

where, g (t, b, k) is as defined as in equation (4)

The total inventory in the time period $t_1 \le t \le T$ is

$$\int_{t_1}^{T} I(t)\,dt = \int_{t_1}^{T} \frac{(a-ds)}{b+1}[t^{-b}T^{b+1} - t]\,dt \quad (7)$$

The maximum inventory level $I(t_1) = S_1$ is

$$S_1 = e^{-kt_1}\,t_1^{-b}\{\eta - (a - ds)\}g(t_1,b,k) \quad (8)$$

where, $g(t_1,b,k) = \int_0^{t_1} e^{ku} u^b du$ (9)

The stock loss due to deterioration in the interval (0, T) is given by

$$L(t) = \int_0^t R(t)\,dt - \int_0^t \lambda(s)\,dt - I(t)$$

This implies

$$L(t) = \eta t_1 - k[\eta - (a - ds)]\int_0^{t_1} e^{-kt}\; t^{b} g(t,b,k)\,dt - \int_0^{t_1}(a - ds)dt - \int_{t_1}^{T}(a - ds)\,dt - I(t) \quad (10)$$

where, g (t,b,k) is as defined as in equation (4)

The total production in the cycle time T is

$$Q = \int_0^{t_1} R(t)\,dt = \int_0^{t_1}[\eta - kI(t)]dt.$$

This implies

$$Q = \eta t_1 - k[\eta - (a - ds)]\int_0^{t_1} e^{-kt}\,t^{-b} g(t,b,k)\,dt \quad (11)$$

where ,g(t,b,k) is as defined as in equation (4)

Let TC (t_1, T, s) is the total cost per unit time. Then, $TC(t_1,T,s)$ sum of the set up cost per unit time, purchasing cost per unit time and holding cost per unit time i.e.

$$TC(t_1,T,s) = \frac{A}{T} + \frac{C}{T}Q + \frac{H}{T}$$

The total holding cost in a cycle time T is

$$H = h\left[\int_0^{t_1} I(t)\,dt + \int_{t_1}^{T} I(t)\,dt\right]$$

.By substituting the values for I (t) and Q from the equations (3), (5) and (11) in $TC(t_1,T,s)$ equation one can get

$$TC(t_1,T,s) = \frac{A}{T} + \frac{c}{T}\eta t_1 + \left(\frac{h-ck}{T}\right)\int_0^{t_1}[\eta - (a - ds)]\,e^{-kt}t^{-b}g(t,b,k)\,dt + \frac{h(a-ds)}{2T(1-b^2)}[T^2(b+1) + t_1^2(1-b) - 2t_1^{1-b}T^{1+b}] \quad (12)$$

where ,g (t, b, k) is as defined as in equation (4)

Let TR (t_1, T, s) be the total revenue per unit time.

$$TR(t_1,T,s) = \frac{S}{T}\int_0^{T}\lambda(s)\,dt = \frac{S}{T}\int_0^{T}(a - ds)dt = s(a - ds) \quad (13)$$

Let TP (t_1, T, s) be the profit rate function. Then,

The total profit per unit time = total revenue per unit time – total cost per unit time,

This implies

$$TP(t_1,T,s) = s(a - ds) - TC(t_1,T,s) \quad (14)$$

where ,TC (t_1,T, s) is as defined as in equation (12)

3.2. Optimal Operating Policies of the model

In this section, we obtain the optimal pricing and ordering policies of the inventory model developed in section.3.1. The problem is to find the optimal values of t_1 and s that maximize the profit rate function TP (t_1, T, s) over (0, T). To obtain these values, differentiate TP (t_1, T, s) given in equation (14) with respect to t_1 and s and equate them to zero. The condition for the solutions to be optimal (minimum) is that the determinant of the Hessian matrix is negative definite i.e.

$$D = \begin{vmatrix} \frac{\partial^2 TP(t_1,T,s)}{\partial t_1^2} & \frac{\partial^2 TP(t_1,T,s)}{\partial t_1 \partial s} \\ \frac{\partial^2 TP(t_1,T,s)}{\partial t_1 \partial s} & \frac{\partial^2 TP(t_1,T,s)}{\partial s^2} \end{vmatrix} < 0$$

Differentiating TP (t_1, T, s) with respect to t_1 and equating to zero one can get

$$\frac{\partial TP(t_1,T,s)}{\partial t_1} = 0 \text{ implies}$$

$$\frac{c}{T}\eta + \left(\frac{h-ck}{T}\right)(\eta-(a-ds))e^{-kt_1}t_1^{-b}g(t_1,b,k) + \frac{(a-ds)}{2(1-b^2)}\left[2t_1(1-b)-2(1-b)t_1^{-b}T^{1+b}\right]=0$$

This implies

$$c\eta + (h-ck)(\eta-a+ds)e^{-kt_1}t_1^{-b}g(t_1,b,k) + \frac{h(a-ds)}{(1+b)}\left[t_1 - t_1^{-b}T^{1+b}\right]=0 \quad (15)$$

where , $g(t_1,b,k)$ is as defined as in equation (9)

$$\frac{\partial TP(t_1,T,s)}{\partial S} = 0 \text{ implies}$$

$$T\left(\frac{a}{d}-2s\right) - (h-ck)\int_0^{t_1} e^{-kt}t^{-b}g(t,b,k) + \frac{h}{2(1-b^2)}\left[T^2(b+1)+t_1^2(1-b)-2t_1^{1-b}T^{1+b}\right]=0 \quad (16)$$

where, $g(t,b,k)$ is as defined as in equation (4)

Solving the non-linear equations (15) and (16) simultaneously using numerical methods and verifying the determinant of Hessian matrix to be negative semi definite for concavity one can get the optimal values for t_1 and s. Substituting the optimal values of t_1 and s in the equations (11) and (14) the optimal values of production quantity Q and total profit TP can be obtained.

3.4. Numerical illustration

To expound the model developed, consider the case of deriving an economic production quantity and production down time for an edible oil manufacturing unit. Here, the product is deteriorating type and has random life time and assumed to follow a Pareto distribution. Based on the discussions held with the personnel connected with the production and marketing of the plant and the records, the values of different parameters are considered as T = 12 months, A = Rs. 50, b = 1.2, a = 30, d = 1, h = Rs. 1, c = Rs. 5, k = 0.4 and η = 60.By substituting these values of the parameters and costs in the equations (15) and (16) then solving numerically, the optimal values for production down time t_1, unit selling price s, production quantity Q and total profit TP are obtained and are presented in Table.1.

From Table 1, It is observed that the increase in deterioration parameter b from 1.2 to 1.6 increases the production down time t_1^* from 3.876 to 4.678 months, decreases the unit selling price s* from Rs.17.343 to Rs.17.073, increases the production quantity Q* from 184.141 to 222.324 units and decrease the total profit TP* from Rs. 71.908 to Rs. 61.484. The increase in the parameter a 25 to 45 increase the production down time t_1^*, the unit selling price s*, the production quantity Q* and the total profit TP*. Whereas the increase in the parameter d 0.8 to 1.2 decrease in the production down time t_1^*, the unit selling price s*, the production quantity Q* and the total profit TP*.

The increase in unit cost c from Rs. 5 to Rs. 9 has a decreasing effect on t_1^*, Q* and TP* and increasing effect on s* viz. Production down time t_1^* from 3.876 to.2.173 months, production quantity Q* from 184.141to 112.513units and total profit TP* from Rs. 71.908 to Rs.23.562 and unit selling price from Rs. 17.343 to Rs. 19.33 respectively. The increase in holding cost h from Rs. 1 to Rs. 1.8 results increase in optimal values of t_1^*, s* and Q* and decrease in TP* i.e. production down time t_1^* from3.876 to 5.114 months, unit selling price s* from Rs. 17.343 to Rs. 17.591, production quantity Q* from 184.141 to 228.62 units and total profit TP* from Rs.71.908 to 26.078.

The increase in production rate parameter k from 0.4 to 0.8 results an increase in optimal values of t_1^*, Q* and TP* and decreasing in s* i.e. production down time t_1^* from 3.876to 5.053 months, production quantity 184.141to 189.103 units and total profit TP* from Rs. 71.908 to Rs 88.524. and from Rs. 17.343 to Rs. 16.384 Whereas the increase in production rate parameter η from 60 to 80 results a decrease in optimal values of production down time t_1^* from 3.876 to 2.912 months, total profit Rs.71.908 to Rs.47.218, increase in optimal values of unit selling price s*from Rs. 17.343 to Rs. 18.245 and production quantity from 184.141to 190.872 units respectively.

3.5. Sensitivity Analysis

To study the effects of changes in the parameters on the optimal values of production down time and production quantity, sensitivity analysis is performed taking the values of the parameters as b = 1.2, c = Rs. 5, h = Rs. 1, k = 0.4, η = 60, a = 30, d = 1,T = 12 months and A = Rs. 50.

Sensitivity analysis is performed by changing the parameter values by -15%, -10%, -5%, 0%, 5%, 10% and 15%. First changing the value of one parameter at a time while keeping all the rest at fixed values and then changing the values of all the parameters simultaneously, the optimal values of production down time, production quantity, selling price and total

Table 1
OPTIMAL VALUES OF
t_1, s, Q, TP for different values of the parameters for model- without shortages

PARAMETERS								OPTIMAL VALUES			
b	a	D	c	h	k	η	A	t_1^*	s^*	Q^*	TP^*
1.2	30	1.0	5	1.0	0.4	60	50	3.876	17.343	184.141	71.908
1.3								4.102	17.265	194.657	69.069
1.4								4.309	17.195	204.450	66.398
1.5								4.500	17.135	213.310	63.875
1.6								4.678	17.073	222.324	61.484
	25							3.081	15.559	150.366	16.476
	35							4.495	19.424	211.029	142.568
	40							5.001	21.647	234.136	227.433
	45							5.426	23.950	254.872	325.966
		0.8						3.998	21.003	189.365	127.103
		0.9						3.939	18.963	186.834	96.370
		1.1						3.809	16.031	181.281	52.014
		1.2						3.737	14.952	178.215	35.556
			6					3.332	17.839	162.406	57.486
			7					2.875	18.338	143.366	44.763
			8					2.493	18.838	126.836	33.520
			9					2.173	19.335	112.513	23.562
				1.2				4.290	17.412	119.600	59.293
				1.4				4.563	17.474	211.603	47.564
				1.6				4.893	17.532	221.063	36.581
				1.8				5.114	17.591	228.620	26.078
					0.5			4.197	17.163	187.093	76.282
					0.6			4.506	17.02	188.804	80.559
					0.7			4.794	16.912	189.426	84.658
					0.8			5.053	16.834	189.103	88.524
						65		3.599	17.566	186.712	65.125
						70		3.347	17.792	188.609	58.772
						75		3.119	18.018	189.981	52.814
						80		2.912	18.245	190.872	47.218
							40	3.876	17.343	184.141	72.741
							45	3.876	17.343	184.141	72.325
							55	3.876	17.343	184.141	71.491
							60	3.876	17.343	184.141	71.075
Cycle length T = 12 months											

profit are computed. The results are presented in Table 2. The relationships between parameters, costs and the optimal values are shown in Fig.2.

From Table 2, It is observed that variation in the deterioration parameters b has considerable effect on production down time t_1^*, unit selling price s*, optimal production quantity Q* and total profit TP*.Similarly variation in demand parameters a and d has slight effect on production down time t_1^*, unit selling price s*, production quantity Q* and significant effect on total profit TP*.

The decrease in unit cost 'c' results an increase in production down time t_1^*, optimal production quantity Q*, total profit TP* and decrease in unit selling price s*. The increase in production rate parameter k result variation in production down time t_1^*, slight increase in production quantity Q* and total profit TP*.Whereas the increase in production rate parameter η result decrease in production down time t_1^*, total profit TP* and slight increase in production quantity Q*.The increase in holding cost h has significant effect on optimal values of production down time t_1^*, production quantity Q* and total profit TP*. When all the parameters change at a time it has a significant effect on optimal values of production down time t_1^*, unit selling price s*, production quantity Q* and total profit TP*.

4. EPQ Model with Shortages

4.1 Model Formulation

Consider an inventory system for deteriorating items in which the life time of the commodity is random and follows a pareto distribution. Here, it is assumed that shortages are allowed and fully backlogged. In this model the stock level for the item is initially zero. Production starts at time t=0 and continues adding items to stock until the on hand inventory reaches its maximum level S_1 at time $t = t_1$. During the time $(0, t_1)$ stock is depleted by demand and deterioration while production is continuously adding to it. At $t = t_1$ the production is stopped and stock will be depleted by deterioration and demand until it reaches zero at time $t = t_2$. As demand is assumed to occur continuously, at this point shortages begin to accumulate until the backlog reaches its maximum level of S_2 at $t = t_3$. At this point production resumes meeting the current demand and clearing the backlog. Finally shortages will be cleared at time t = T. Then the cycle will be repeated identically. These types of production systems are common in production process dealing agricultural products, where production rate is stock dependent. The schematic diagram representing the inventory system is shown in figure 3

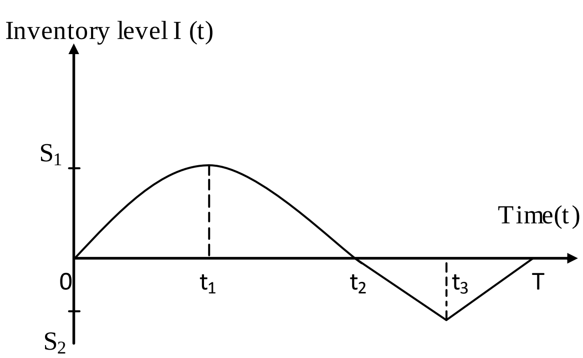

Fig 3; Schematic diagram representing the inventory level of the system for the modelwith shortages

The differential equations describing the instantaneous states of I(t) in the interval (0, T) are given by

$$\frac{dI(t)}{dt} = \eta - kI(t) - h(t)I(t) - (a - ds),$$

$$0 \le t \le t_1 \quad (17)$$

Let I (t) denote the inventory level of the system at time t. $(0 \le t \le T)$

$$\frac{dI(t)}{dt} = -h(t)\,I(t) - (a - ds), \ t_1 \le t \le t_2 \quad (18)$$

$$\frac{dI(t)}{dt} = -(a - ds), t_2 \le t \le t_3 \quad (19)$$

$$\frac{dI(t)}{dt} = \eta - (a - ds), t_3 \le t \le T \quad (20)$$

with the boundary conditions I (0) =0, I (t_2) =0 and I (T) =0.Solving the equations (18) to (21) ,the instantaneous state of inventory at any time t, during the interval $(0,t_1)$ is obtained as

$$I(t) = [\eta - (a - ds)]t^{-b}e^{-kt}g(t, b, k), 0 \le t \le t_1 \quad (21)$$

where , g (t,b,k) is as defined as in equation (4)
The instantaneous state of inventory at any time t, during the interval (t_1, t_2) is obtained as

$$I(t) = \frac{(a - ds)}{b + 1}[t_2^{b+1}t^{-b} - t], \quad t_1 \le t \le t_2 \quad (22)$$

The instantaneous state of inventory at any time t, during the interval (t_2, t_3) is obtained as

$$I(t) = (a - ds)\,(t_2 - t), \quad t_2 \le t \le t_3 \quad (23)$$

The instantaneous state of inventory at any time t during the interval (t_3, T) is obtained as

$$I(t) = [\eta - (a - ds)]\,(t - T)\,, \quad t_3 \le t \le T \quad (24)$$

Using the equations (21) and (22) the total volume of inventory for the respective time periods are obtained as follows
The total inventory in the time period $0 \le t \le t_1$ is

$$\int_0^{t_1} I(t)\,dt = \int_0^{t_1} e^{-kt}\,t^{-b}\{\eta - (a - ds)\}g(t, b, k)\,dt \quad (25)$$

where ,g(t,b,k) is as defined as in equation (4)

The total inventory in the time period $t_1 \le t \le t_2$ is

$$\int_{t_1}^{t_2} I(t)\,dt = \int_{t_1}^{t_2} \frac{(a-ds)}{b+1}\left[t_2^{b+1}t^{-b} - t\right]dt \quad (26)$$

Table 2

Sensitivity analysis of the model- without shortages

Variation Parameters	Optimal Policies	Change in parameters (T = 12 Months)						
		-15%	-10%	-5%	0%	5%	10%	15%
b(1.2)	t_1^*	3.414	3.577	3.731	3.876	4.014	4.144	4.269
	s^*	17.508	17.499	17.394	17.343	17.296	17.251	17.209
	Q^*	163.100	170.461	177.473	184.141	190.541	196.637	202.544
	TP^*	77.525	75.571	73.702	71.908	70.183	68.522	66.920
a(30)	t_1^*	3.303	3.508	3.698	3.876	4.044	4.202	4.352
	s^*	15.959	16.395	16.859	17.343	17.845	18.36	18.887
	Q^*	159.798	168.489	176.555	184.141	191.341	198.172	204.095
	TP^*	28.813	42.188	56.563	71.908	88.198	105.415	116.112
d(1)	t_1^*	3.969	3.939	3.908	3.876	3.843	3.809	3.774
	s^*	19.921	18.963	18.109	17.343	16.654	16.031	15.466
	Q^*	188.120	186.834	185.508	184.141	183.994	181.281	179.789
	TP^*	110.815	96.370	83.478	71.908	61.467	52.014	43.411
c(5)	t_1^*	4.347	4.184	4.027	3.876	3.732	3.593	3.460
	s^*	16.977	17.098	17.220	17.343	17.466	17.59	17.714
	Q^*	202.236	196.045	190.011	184.141	178.479	172.952	167.606
	TP^*	83.975	79.826	75.805	71.908	68.131	64.471	60.924
h(1)	t_1^*	3.500	3.632	3.758	3.876	3.988	4.095	4.195
	s^*	17.281	17.303	17.324	17.343	17.361	17.379	17.396
	Q^*	169.558	174.739	179.621	184.141	188.383	192.394	196.106
	TP^*	82.401	78.786	75.293	71.908	68.619	65.415	62.289
k(0.4)	t_1^*	3.685	3.748	3.812	3.876	3.941	4.005	4.069
	s^*	17.468	17.425	17.383	17.343	17.304	17.267	17.231
	Q^*	181.863	182.650	183.422	184.141	184.848	185.47	186.050
	TP^*	69.276	70.152	71.030	71.908	72.786	73.663	74.539
η(60)	t_1^*	4.451	4.248	4.057	3.876	3.706	3.546	3.396
	s^*	16.951	17.080	17.211	17.343	17.477	17.611	17.747
	Q^*	177.715	180.152	182.303	184.141	185.741	187.114	188.301
	TP^*	85.324	80.669	76.200	71.908	67.785	63.821	60.010
A(50)	t_1^*	3.876	3.876	3.876	3.876	3.876	3.876	3.876
	s^*	17.343	17.343	17.343	17.343	17.343	17.343	17.343
	Q^*	184.741	184.741	184.741	184.741	184.741	184.741	184.741
	TP^*	72.533	72.325	72.116	71.908	71.7	71.491	71.283
All parameters	t_1^*	3.44	3.590	3.735	3.876	4.013	4.144	4.271
	s^*	17.534	17.457	17.394	17.343	17.302	17.270	17.246
	Q^*	144.276	157.362	170.646	184.141	197.822	211.588	225.495
	TP^*	90.993	85.331	78.970	71.908	64.146	55.683	46.522

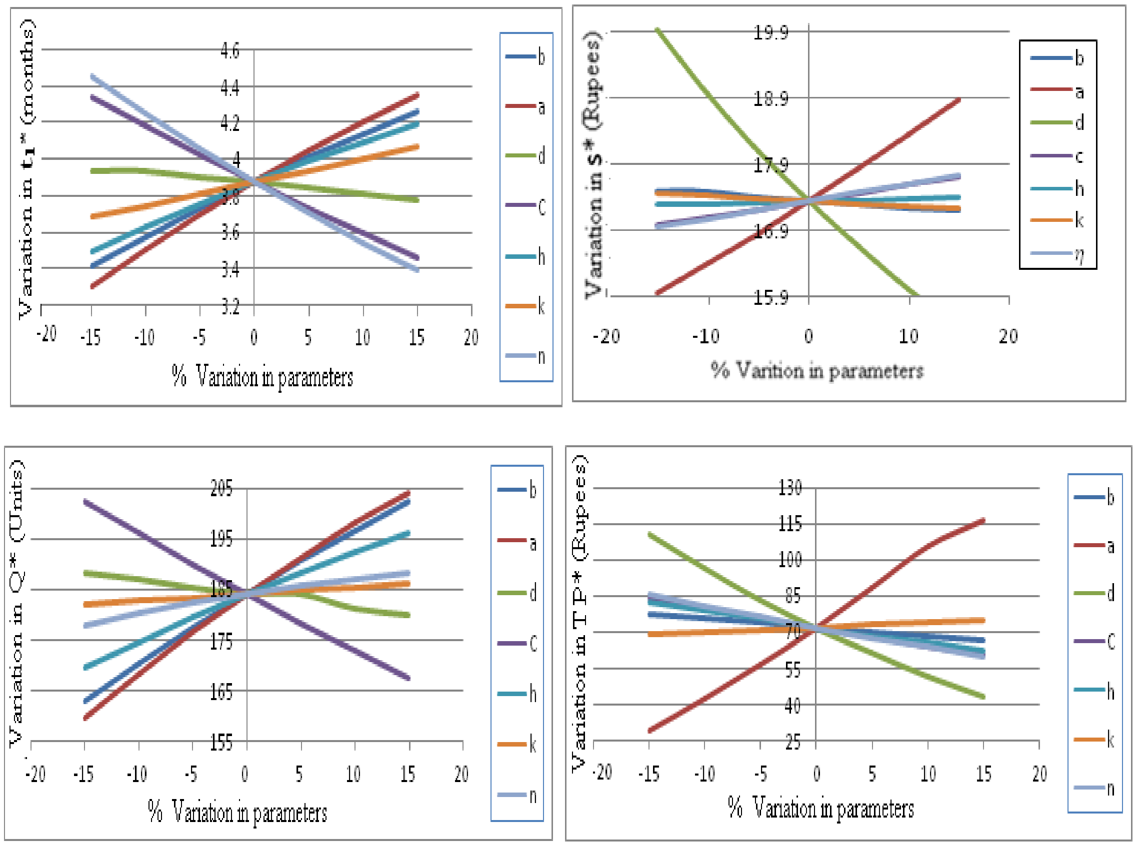

Fig.2: Relationship between optimal values and parameters

Since I (t) is continuous at t_2 equating (22) and (23) one can get

$$t_2 = T + \frac{\eta}{(a-ds)}(t_3 - T) \tag{27}$$

This equation can be used to establish the relationship between t_3 and t_2.
The maximum inventory level I (t_1) = S_1 obtained as

$$S_1 = [\eta - (a - ds)] t_1^{-b} e^{-kt_1} g(t_1, b, k) \tag{28}$$

where, g(t_1,b,k) is as defined as in equation (.9).
Similarly the maximum shortage level

I (t_3) = S_2 obtained as

$$S_2 = (a - ds)(t_2 - t_3) \tag{29}$$

Backlogged demand at time t is

$$B(t) = \int_{t_2}^{t_3} \lambda(s)\, dt = \int_{t_2}^{t_3} (a - ds)\, dt = (a - ds)(t_3 - t_2) \tag{30}$$

The stock loss due to deterioration in the interval (0, T) is

$$L(t) = \int_0^t R(t)\, dt - \int_0^t \lambda(s)\, dt - I(t)$$

This implies

$$L(t) = \eta[t_1 + T - t_3] - k\left[\eta - (a - ds)\right] \int_0^{t_1} t^{-b} e^{-kt} g(t, b, k)\, dt - \int_0^{T} (a - ds)\, dt - I(t)$$

This implies

$$L(t) = \eta[t_1 + T - t_3] - k[\eta - (a - ds)]\int_0^{t_1} t^{-b}e^{-kt}g(t,b,k)dt - \int_0^{t_1}(a-ds)\,dt - \int_{t_1}^{t_2}(a-ds)\,dt - \int_{t_2}^{t_3}(a-ds)dt - \int_{t_3}^{T}(a-ds)dt - I(t) \quad (31)$$

The total production in the cycle time T is

$$Q = \int_0^{t_1} R(t)\,dt + \int_{t_3}^{T} R(t)dt = \int_0^{t_1}\eta dt + \int_{t_3}^{T}\eta dt - k\int_0^{t_1} I(t)\,dt$$
$$= \eta(t_1 + T - t_3) - k[\eta - (a - ds)]\int_0^{t_1} t^{-b}e^{-kt}g(t,b,k)dt \quad (32)$$

where, g(t,b,k) is as defined as in equation (4)
The total cost per unit time TC (t_1, t_3,T, s) is the sum of the setup cost per unit time, purchasing cost per unit time, holding cost per unit time and the shortage cost per unit time i.e.

$$TC(t_1,t_3,T,s) = \frac{A}{T} + \frac{C}{T}Q + \frac{H}{T} + \frac{S_h}{T}$$

The total holding cost in a cycle time is

$$H = h\left[\int_0^{t_1} I(t)\,dt + \int_{t_1}^{t_2} I(t)\,dt\right]$$

The total shortage cost in a cycle time is

$$S_h = \pi\left[\int_{t_2}^{t_3} -I(t)\,dt + \int_{t_3}^{T} -I(t)\,dt\right]$$

Therefore

$$TC(t_1,t_3,T,s) = \frac{A}{T} + \frac{C}{T}Q + \frac{h}{T}\left[\int_0^{t_1} I(t)\,dt + \int_{t_1}^{t_2} I(t)\,dt\right] + \frac{\pi}{T}\left[\int_{t_2}^{t_3} -I(t)\,dt + \int_{t_3}^{T} -I(t)\,dt\right]$$

By substituting the values of I(t) and Q from the equations (21) to (24) and (32) in TC(t_1,t_3,T,s) equation, one can get

$$TC(t_1,t_3,T,s) = \frac{A}{T} + \frac{c}{T}\left\{\eta(t_1 + T - t_3) - k\int_0^{t_1}\left(\eta - (a - ds)\right)t^{-b}e^{-kt}g(t,b,k)dt\right\} + \frac{h}{T}\left\{\int_0^{t_1}\left(\eta - (a - ds)\right)t^{-b}e^{-kt}[g(t,b,k)]\,dt + \int_{t_1}^{t_2}\frac{(a-ds)}{b+1}\{t_2^{b+1}t^{-b} - t\}dt\right\} - \frac{\pi}{T}\left\{\int_{t_2}^{t_3}(a-ds)(t_2 - t)\,dt + \int_{t_3}^{T}\left(\eta - (a - ds)\right)(t - T)dt\right\}$$

On integrating and simplifying the above equation one can get

$$TC(t_1,t_3,T,s) = \frac{A}{T} + \frac{C\eta}{T}(t_1 + T - t_3) + \left(\frac{h - ck}{T}\right)[\eta - (a - ds)]\int_0^{t_1} t^{-b}e^{-kt}g(t,b,k)dt$$
$$+ \frac{h(a-ds)}{2T(1-b^2)}\left\{\left[\left[T + \frac{\eta}{(a-ds)}(t_3 - T)\right]^2(1 + b) + t_1^2(1-b) - 2t_1^{1-b}\left[T + \frac{\eta}{(a-ds)}(t_3 - T)\right]\right]^{1+b}\right\}$$
$$+ \frac{\pi\eta}{2T}(T - t_3)^2\left[\frac{\eta}{(a-ds)} - 1\right] \quad (33)$$

where, g(t,b,k) is as defined as in equation (4)

Let TR (t_1, t_3, T, s) be the total revenue per unit time.

$$TR(t_1,t_3,T,s) = \frac{S}{T}\int_0^{T}\lambda(s)dt = \frac{S}{T}\int_0^{T}(a - ds)\,dt = s(a - ds) \quad (34)$$

Also let TP (t_1, t_3, T, s) be the profit rate function. Then,

Total profit per unit time = Total Revenue per unit time – Total cost per unit time.
This implies

$$TP(t_1, t_3, T, s) = s(a - ds) - TC(t_1, t_3, T, s) \quad (35)$$

where ,TC (t_1, t_3, T,s) is as given in equation (33)

4.2 Optimal operating policies of the model

In this section, the optimal policies of the inventory system developed in section 4.1 are derived. To find the optimal values of production down time (t_1) and production up time (t_3) and optimal selling price (s) ,one has to maximize the total profit TP (t_1, t_3,T,s) in equation (35) with respect to t_1, t_3 and s and equate the resulting equations to zero. The condition for the solutions to be optimal (minimum) is that the determinant of the Hessian matrix is negative definite i.e.

$$D = \begin{vmatrix} \frac{\partial^2 TP(t_1,t_3,T,s)}{\partial t_1^2} & \frac{\partial^2 TP(t_1,t_3,T,s)}{\partial t_1 \partial t_3} & \frac{\partial^2 TP(t_1,t_3,T,s)}{\partial t_1 \partial s} \\ \frac{\partial^2 TP(t_1,t_3,T,s)}{\partial t_1 \partial t_3} & \frac{\partial^2 TP(t_1,t_3,T,s)}{\partial t_3^2} & \frac{\partial^2 TP(t_1,t_3,T,s)}{\partial t_3 \partial s} \\ \frac{\partial^2 TP(t_1,t_3,T,s)}{\partial t_1 \partial s} & \frac{\partial^2 TP(t_1,t_3,T,s)}{\partial t_3 \partial s} & \frac{\partial^2 TP(t_1,t_3,T,s)}{\partial s^2} \end{vmatrix} < 0$$

The necessary conditions which maximize TP (t_1, t_3, T, s) is

$$\frac{\partial TP(t_1,t_3,T,s)}{\partial t_1} = 0, \frac{\partial TP(t_1,t_3,T,s)}{\partial t_3} = 0 \text{ and } \frac{\partial TP(t_1,t_3,T,s)}{\partial s} = 0$$

$\frac{\partial TP(t_1,t_3,T,s)}{\partial t_1} = 0$ implies

$$\frac{c\eta}{T} + \left(\frac{h - ck}{T}\right)\left[[\eta - (a - ds)]e^{-kt_1}t_1^{-b}g(t_1, b, k)\right] + \frac{h(a - ds)}{(1+b)T}\left\{t_1 - t_1^{-b}\left[\frac{\eta}{(a - ds)}(t_3 - T) + T\right]^{1+b}\right\} = 0 \quad (36)$$

where, $g(t_1,b,k)$ is as defined as in equation (9)

$\frac{\partial TP(t_1,t_3,T,s)}{\partial t_3} = 0$ implies

$$-c + \frac{h}{(1-b)}\left\{\left[\frac{\eta}{(a - ds)}(t_3 - T) + T\right] - t_1^{1-b}\left[\frac{\eta}{(a - ds)}(t_3 - T) + T\right]^b\right\} + \frac{\pi(T - t_3)(a - ds - \eta)}{(a - ds)} = 0 \quad (37)$$

$\frac{\partial TP(t_1,t_3,T,s)}{\partial t_3} = 0$ implies

$$T\left(\frac{a}{d} - 2s\right) - (h - ck)\int_0^{t_1} t^{-b}e^{-kt}g(t,b,k)dt - \frac{h}{2(1-b^2)}\left\{\left[\frac{\eta}{(a - ds)}(t_3 - T) + T\right]^2(1+b) - 2t_1^{1-b}\left[\frac{\eta}{(a - ds)}(t_3 - T) + T\right]^{1+b} + t_1^2(1-b)\right\} - \frac{\pi\eta^2(T - t_3)^2}{2(a - ds)^2} = 0$$

$\frac{\partial TP(t_1,t_3,T,s)}{\partial s} = 0$ implies

$$\frac{h\eta(t_3 - T)}{(1-b)(a - ds)}\left\{\left[\frac{\eta}{(a - ds)}(t_3 - T) + T\right] - t_1^{-b}\left[\frac{\eta}{(a - ds)}(t_3 - T) + T\right]^b\right\} +$$

$$\frac{h}{2(1-b^2)}\left\{\left[\frac{\eta}{(a-ds)}(t_3 - T) + T\right]^2(1+b) -\right.$$

2t11− bηa−dst3−T+T1+b+t121−b−

$$\frac{\pi\eta^2(T - t_3)^2}{2(a - ds)^2} = 0 \quad (38)$$

where , g(t,b,k) is as defined as in equation (4)

Solving the non-linear equations (36) to (38) by using MathCAD one can obtain the optimal production down and up times t_1^* , t_3^* and selling price s^*.Substituting t_3^* in equation (27) t_2^* is obtained. The optimal production quantity Q* is obtained by substituting t_1^* and t_3^* in equation (32).

4.3 NUMERICAL ILLUSTRATION

To expound the model developed, consider the case of deriving and economic production quantity, production down time, production up time and selling price for an edible oil plant. Here the product is of a deteriorating type and has a random life time which is assumed to follow pareto distribution. Form the records and discussions held with the production and marketing personnel the values of various parameters are considered. For different values of the parameters and costs, the optimal values of production down time, production up time, selling price, optimal production quantity and total profit are computed and presented in Table3.

From Table 3, it is observed that the when b increases from 1.2 to 1.6 units the production down time t_1^* is decreasing, production quantity Q* is increasing and the total profit TP* is decreasing i.e. t_1^* decreases from 1.989 to 1.860 months, Q* increases from 162.212 to 173.697 units and total profit TP* decreases from Rs. 114.092 to Rs.112.809. There is a decrease in production up time t_3^* from 11.038 to 10.870 months and slight increase in selling price s^* from Rs. 13.275 to Rs. 13.330.

When the demand parameter 'a' increases 25 to 29 then the optimal production down time t_1^* is increases, production up time t_3^* is decreasing, optimal values of selling price, production quantity and total profit are increasing i.e. t_1^* from 1.989 to 2.001 months, t_3^* from 11.038 to 10.765 months, s^* from Rs. 13.275 to Rs.15.166, Q* from 162.212 to 179.802 units and TP* from Rs. 114.092 to Rs. 164.702. Similarly when the demand parameter d increases 0.8 to 1.2 results, increase production up time t_3^* from 11.038 to 11.059 months, decrease in production down time t_1^* from 1.989 to 1.976 months, selling price s^* from Rs. 13.275 to Rs. 11.205, production quantity Q* from 162.212 to 160.299 units and total profit TP* from Rs. 114.092 to Rs. 88.241.

The increase in holding cost h from Rs. 0.2 to Rs. 0.6 results decrease in production down time t_1^* from 1.996 to 1.979 months, production up time, t_3^* from 11.280 to 10.733 months, increase in selling price s^* from Rs. 13.170 to Rs. 13.457, production quantity Q* from 148.048 to 179.995 units and decrease in total profit TP* from Rs. 117.352 to Rs.109.026. The increase in unit cost c from Rs. 1 to Rs. 5 results slight increase in production down time t_1^* from 1.986 to 2.005 months, production up time, t_3^* from 10.680 to 11.730 months, selling price s^* from Rs. 12.858 to Rs. 13.854, decrease in production quantity Q* from 183.681 to 121.249 units and total profit TP* from Rs. 127.722 to Rs. 77.587.

The increase in shortage cost π from Rs. 0.2 to Rs. 0.6 has effect on all optimal values of t_1^* from 1.990 to 1.899 months, t_3^* from 11.034 to 11.081 months, selling price s^* from Rs.13.316 to Rs.13.227, production quantity Q* from 162.486 to 155.468 units and total profit TP* from Rs. 115.048 to Rs. 111.855.The increase in production rate parameter 'k' 0.3 to 0.7 results decrease in production down time t_1^* from 1.989 to 1.988 months, production up time, t_3^* from11.075 to 10.945 months, selling price s^* from Rs. 13.308 to Rs. 13.188, production quantity Q* from 163.272 to 159.272 units and total profit TP* increase from Rs. 113.514 to Rs. 115.446.Similarly the increase in production rate parameter 'η' 50 to 70 results increase in production down time t_1^* from 1.986 to 1.990 months, production up time, t_3^* from10.719 to 11.259 months, selling price s^* from Rs. 13.160 to Rs. 13.376, production quantity Q* from 151.646 to 173.199 units and total profit TP* decrease from Rs. 117.071 to Rs. 110.952.

4.4 SENSITIVITY ANALYSIS

To study the effect of changes in the parameters and costs on the optimal values of production down time, production up time, unit selling price and production quantity, sensitivity analysis is performed taking the values A = Rs. 50, c =Rs. 2, h = Rs. 0.3, T = 12 months, π = Rs. 0.3, a = 25, d = 1, k = 0.4, b = 1.2 and η = 60.

Sensitivity analysis is performed by changing the parameters by -15%, -10%, -5%, 0%, 5%, 10% and 15%. First changing the value of one parameter at a time while keeping all the rest at fixed values and then changing the values of all the parameters simultaneously, the optimal values t_1,t_3,s,Q and TP are computed and the results are presented in Table 4. The relationships between parameters, costs and the optimal values are shown in figure4.

From Table 4, it is observed that the deteriorating parameter b has less effect on production down time, production up time, unit selling price and significant effect on production quantity and total profit. Decrease in unit cost c results decrease in production down time, production up time, selling price, increase in production quantity Q* and total profit TP*. The increase in production rate parameter η has less effect on production down time, production up time, unit selling price, moderate effect on production quantity Q* and total profit TP* respectively.Increase in holding cost h results significant variation in production quantity Q* and decrease in total profit TP*. The increase in shortage cost results less effect on production quantity Q* and total profit TP*.

Table .3
OPTIMAL VALUES OF
t_1, t_3, s, Q and TP for different values of the parameters and costs for the model- with shortages

PARAMETERS(T = 12 Months)									OPTIMAL POLICIES				
b	a	d	c	h	k	η	π	A	t_1^*	t_3^*	s^*	Q^*	TP^*
1.2	25	1.0	2	0.3	0.4	60	0.2	50	1.989	11.038	13.275	162.212	114.092
1.3									1.988	10.994	13.276	165.385	113.677
1.4									1.987	10.955	13.277	168.212	113.282
1.5									1.986	10.918	13.280	170.878	112.905
1.6									1.860	10.870	13.330	173.697	112.809
	26								1.990	10.973	13.749	166.32	126.021
	27								1.995	10.906	14.320	170.704	138.411
	28								1.996	10.833	14.692	175.325	151.320
	29								2.001	10.765	15.166	179.802	164.702
		0.8							2.009	11.018	16.383	164.387	152.932
		0.9							1.997	11.028	14.656	163.207	131.352
		1.1							1.983	11.049	12.145	161.249	99.981
		1.2							1.976	11.059	11.205	160.299	88.241
			1						1.986	10.680	12.858	183.681	127.722
			3						1.991	11.317	13.562	145.476	101.125
			4						1.999	11.537	13.740	132.589	88.945
			5						2.005	11.730	13.854	121.249	77.587
				0.2					1.996	11.280	13.170	148.048	117.352
				0.4					1.991	10.899	13.350	170.621	111.893
				0.5					1.975	10.803	13.412	175.624	110.341
				0.6					1.979	10.733	13.457	179.995	109.026
					0.3				1.989	11.075	13.308	163.272	113.514
					0.5				1.988	11.005	13.243	160.98	114.617
					0.6				1.988	10.974	13.215	160.154	115.051
					0.7				1.988	10.945	13.188	159.275	115.446
						50			1.986	10.719	13.160	151.646	117.071
						55			1.987	10.895	13.220	156.792	115.605
						65			1.990	11.159	13.328	167.598	112.541
						70			1.990	11.259	13.376	173.199	110.952
							0.2		1.990	11.034	13.316	162.486	115.048
							0.4		1.986	11.044	13.238	161.725	113.170
							0.5		1.988	11.054	13.204	161.228	112.266
							0.6		1.899	11.081	13.227	155.468	111.855
								40	1.989	11.038	13.275	162.212	114.926
								45	1.989	11.038	13.275	162.212	114.509
								55	1.989	11.038	13.275	162.212	113.676
								60	1.989	11.038	13.275	162.212	113.259

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Table 4; sensitivity analysis of the model- with shortages

Variation Parameters	Optimal Policies	Change in parameters(T = 12 Months)						
		-15%	-10%`	-5%	0%	+5%	+10%	+15%
b	t_1^*	1.990	1.989	1.989	1.989	1.989	1.988	1.987
	t_3^*	11.133	11.100	11.071	11.038	11.012	10.986	10.963
	s*	13.277	13.276	13.276	13.275	13.274	13.274	13.274
	Q*	155.38	157.728	159.86	162.212	164.126	165.976	167.629
	TP*	114.893	114.623	114.356	114.092	113.839	113.595	113.361
a	t_1^*	1.984	1.984	1.985	1.989	1.992	1.994	1.996
	t_3^*	11.272	11.202	11.122	11.038	10.954	10.861	10.782
	s*	11.450	12.097	12.687	13.275	13.864	14.561	15.048
	Q*	147.352	151.736	156.785	162.212	167.594	173.438	178.507
	TP*	73.738	86.426	99.885	114.092	129.069	144.79	161.327
d	t_1^*	1.996	1.996	1.993	1.989	1.985	1.983	1.980
	t_3^*	11.021	11.028	11.033	11.038	11.044	11.049	11.054
	s*	15.468	14.656	13.928	13.275	12.683	12.145	11.654
	Q*	163.595	163.161	162.710	162.212	161.654	161.249	160.798
	TP*	141.534	131.356	122.265	114.092	106.703	99.981	93.852
c	t_1^*	1.988	1.988	1.989	1.989	1.989	1.989	1.99
	t_3^*	10.941	10.975	11.007	11.038	11.069	11.099	11.128
	s*	13.165	13.203	13.239	13.275	13.308	13.341	13.372
	Q*	168.020	165.968	164.083	162.212	160.342	158.532	156.828
	TP*	118.126	116.777	115.429	114.092	112.763	111.441	110.12
h	t_1^*	1.991	1.989	1.989	1.989	1.988	1.987	1.987
	t_3^*	11.130	11.096	11.069	11.038	11.013	10.989	10.967
	s*	13.234	13.247	13.265	13.275	13.287	13.300	13.310
	Q*	156.797	158.741	160.355	162.212	163.662	165.02	166.369
	TP*	115.375	114.924	114.497	114.092	113.718	113.363	113.023
k	t_1^*	1.989	1.989	1.989	1.989	1.989	1.989	1.989
	t_3^*	11.060	11.053	11.045	11.038	11.031	11.024	11.018
	s*	13.294	13.288	13.281	13.275	13.268	13.262	13.256
	Q*	162.829	162.593	162.428	162.212	162.006	161.810	161.562
	TP*	113.755	113.871	113.983	114.092	114.199	114.303	114.405
η	t_1^*	1.987	1.988	1.988	1.989	1.989	1.99	1.992
	t_3^*	10.759	10.87	10.956	11.038	11.119	11.180	11.246
	s*	13.173	13.216	13.242	13.275	13.315	13.337	13.348
	Q*	152.602	155.4	158.922	162.212	165.027	168.732	171.786
	TP*	116.782	115.909	115.005	114.092	113.169	112.226	111.252
π	t_1^*	1.989	1.989	1.989	1.989	1.989	1.989	1.988
	t_3^*	11.036	11.037	11.038	11.038	11.039	11.04	11.041
	s*	13.293	13.287	13.28	13.275	13.269	13.263	13.257
	Q*	162.327	162.268	162.211	162.212	162.154	162.096	161.991
	TP*	114.522	114.378	114.235	114.092	113.950	113.809	113.673
All Parameters	t_1^*	1.999	1.991	1.989	1.989	1.988	1.986	1.982
	t_3^*	11.139	11.102	11.073	11.038	11.01	10.983	10.98
	s*	13.165	13.203	13.243	13.275	13.31	13.342	13.352
	Q*	133.922	143.16	152.381	162.212	171.752	181.332	189.304
	TP*	101.411	105.866	110.092	114.092	117.887	121.473	124.883

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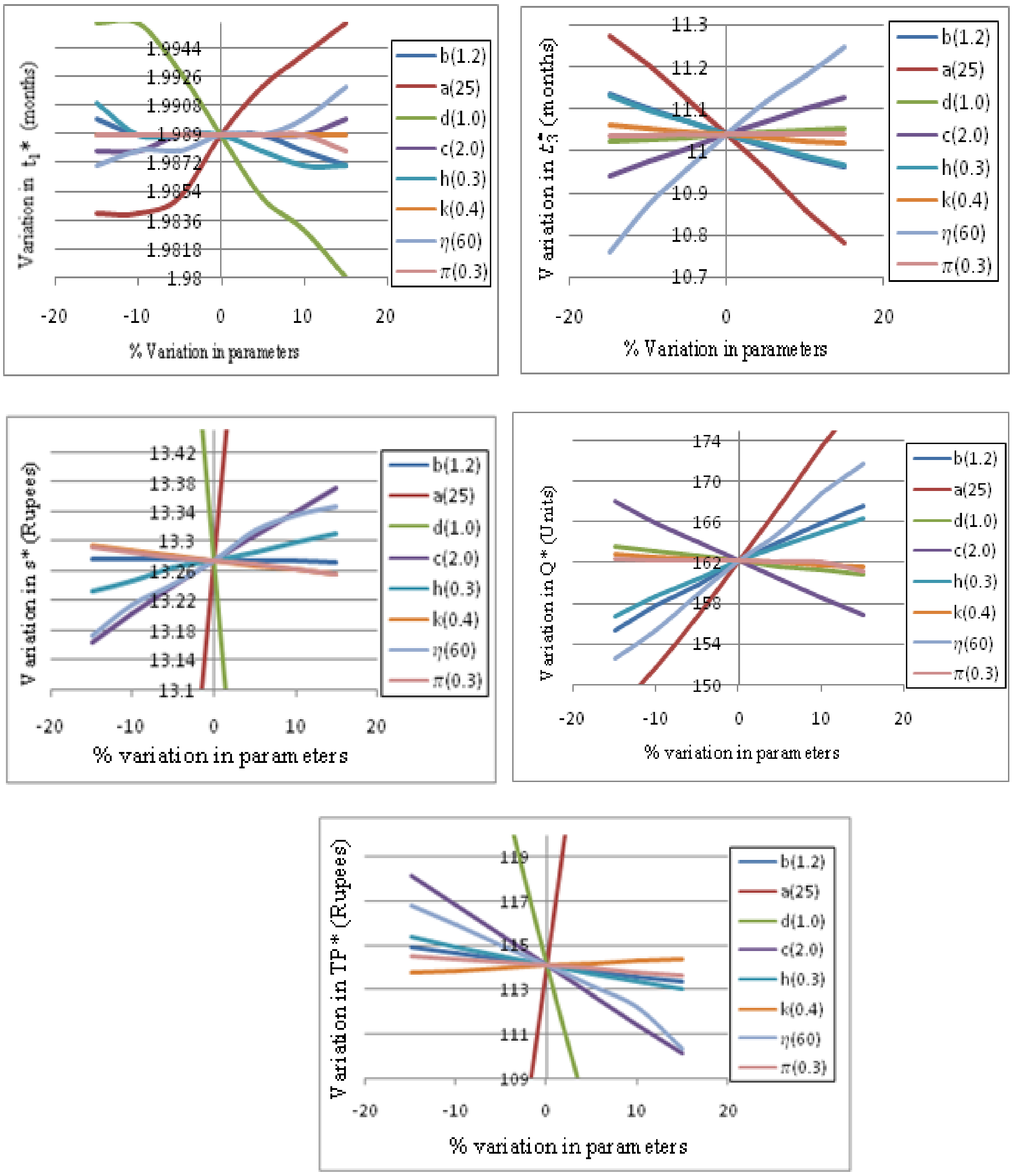

Fig 4. Relationship between optimal values and parameters

A comparative study of with and without shortages revealed that allowing shortages has significant influence in optimal production schedule and total profit. This model includes some of the earlier inventory models for deteriorating items with Pareto decay as particular cases for specific values of the parameters. When k = 0 this model includes EPQ model for deteriorating items with Pareto decay and selling price dependent demand and finite rate of replenishment. When b = 0 this model becomes EPQ model with stock dependent production and selling price dependent demand. When d=0 this model includes EPQ model for deteriorating items with pareto decay and constant demand.

5. Conclusions

In this paper, production level inventory models for deteriorating items with selling price dependent demand and Pareto deterioration for both without and with shortages are developed and analyzed. By maximizing the total profit function the optimal values of the production quantity, production down time, production uptime and unit selling price are derived. The sensitivity model with respect to the parameters and costs revealed that the change in production rate parameters and deteriorating parameters have significant influence on optimal production schedule. By suitably estimating the parameters and costs the production manager can optimally derive the production schedule and reduce waste and variation of resources. This model is having potential applications in manufacturing and production industries like edible oil mills, sugar factories, etc., where the deterioration of the commodity is random and follows Pareto distribution and having selling price dependent demand.

6. References

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