On Nαg^*S Closed and Nαg^*S Contra Continuous Functions

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***Abstract*:- The aim of this paper is to give and discuss stronger form of nano continuity called nano contra continuity using Nαg^*s closed sets.**

Keywords: Nano topology, Nαg^*s closed sets, Nαg^*s contra continuity.
2010 AMS subject classification: 54B05,54C05.

1.INTRODUCTION

Ganster and Reily [4] discussed Lc continuous functions. Dontchev [3] had given contra continuous functions. Lellis Thivagar [5] introduced a nano topological space with respect to a subset X of an universe which is defined in terms of lower and upper approximations of X. The elements of a nano topological space are called nano open sets. 'Nano' is a Greek word which means 'very small'. The topology studied here is given the name nano topology as it has at most five elements Certain weak forms of nano sets were studied by various authors. Here we study a new form of closed functions called Nαg^*s closed function and its relation to various other closed functions. Also we analyze a new form of contra continuous function namely, Nαg^*s contra continuous function and its relation to other contra continuous functions.

2. PRELIMINARIES

Definition 2.1: [5] Let $\mathcal{U}$ be a non-empty finite set of objects called the universe and R be an equivalence relation on $\mathcal{U}$ named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair $(\mathcal{U}, R)$ is said to be the approximation space. Let X⊆ $\mathcal{U}$.

(i) The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \underset{x\epsilon\mathcal{U}}{U}\{R(x): R(x) \subseteq X\}$, where R(x) denotes the equivalence class determined by x.

(ii) The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X)$= $\underset{x\epsilon\mathcal{U}}{U}\{R(x): R(x) \cap X \neq \emptyset\}$.

(iii) The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not-X with respect to R and it is denoted by $B_R(X)$. That is $B_R(X)$=$U_R(X) - L_R(X)$.

Definition 2.2:[5] let $\mathcal{U}$ be the universe, R be an equivalence relation on $\mathcal{U}$ and τ_R(X)= $\{\mathcal{U}, \varphi, L_R(X), U_R(X), B_R(X)\}$, where X⊆ $\mathcal{U}$. τ_R(X) satisfies the following axioms.

(i) $\mathcal{U}$ and $\varphi\epsilon\tau_R(X)$.
(ii) The union of the elements of any subcollection of τ_R(X) is in τ_R(X).
(iii) The intersection of the elements of any finite subcollection of τ_RX) is in τ_R(X).

That is, τ_R(X) forms a topology on $\mathcal{U}$ called as the nano topology on $\mathcal{U}$ with respect to X. we call ($\mathcal{U}, \tau_R$(X)) as the nano topological space. The elements of τ_R(X) are called as nano-open sets. A set A is said to be nano closed if its complement is nano-open.

Definition 2.3: [5] If ($\mathcal{U}, \tau_R$(X)) is a nano topological space with respect to X where X⊆ $\mathcal{U}$ and if A⊆ $\mathcal{U}$, then the nano interior of A is defined as the union of all nano-open subsets of A and it is denoted by Nint(A). That is, Nint(A) is the largest nano-open subset of A. The nano closure of A is defined as the intersection of all nano closed sets containing A and it is denoted by Ncl(A). That is, Ncl(A) is the smallest nano closed set containing A.

Definition 2.4: A nano subset A of a nano topological space ($\mathcal{U}, \tau_R$(X)) is called a

(i) Nano pre closed if Ncl Nint(A) ⊆ A
(ii) Nano semi closed if Nint Ncl(A) ⊆ A
(iii) Nano α closed if Ncl Nint Ncl(A) ⊆ A
(iv) Nano semi pre closed if Nint Ncl Nint(A) ⊆ A
(v) Nano regular closed if Ncl Nint(A) =A

For a nano subset A of $(\mathcal{U},\tau_R(X))$ the intersection of all nano pre closed (nano semi closed, nano α closed, nano semi pre closed) sets of $(\mathcal{U},\tau_R(X))$ containing A is called nano pre closure of A (nano semi closure of A, nano α closure of A, nano semi pre closure of A) and is denoted by Npcl(A) (Nscl(A), Nαcl(A), Nspcl(A)).

Definition 2.5: A nano subset A of a nono topological space $(\mathcal{U},\tau_R(X))$ is called a

1) Nano generalized closed (briefly Ng closed) if Ncl(A) $\subseteq U$ whenever A$\subseteq U$ and U is nano open in $\mathcal{U}$.
2) Nano generalized semi closed (briefly Ngs closed) if Nscl(A) $\subseteq U$ whenever A$\subseteq U$ and U is nano open in $\mathcal{U}$.
3) Nano α generalized regular closed (briefly Nαgr closed) if Nαcl(A) $\subseteq U$ whenever A$\subseteq U$ and U is nano regular open in $\mathcal{U}$.
4) Nano α generalized semi closed (briefly Nαgs closed) if Nαcl(A) $\subseteq U$ whenever A$\subseteq U$ and U is nano semi open in $\mathcal{U}$.
5) Nano α generalized closed (briefly Nαg closed) if Nαcl(A) $\subseteq U$ whenever A$\subseteq U$ and U is nano open in $\mathcal{U}$.
6) Nano generalized semi pre closed (briefly Ngsp closed) if Nspcl(A) $\subseteq U$ whenever A$\subseteq U$ and U is nano open in $\mathcal{U}$.
7) Nano generalized pre closed (briefly Ngp closed) if Npcl(A) $\subseteq U$ whenever A$\subseteq U$ and U is nano open in $\mathcal{U}$.
8) Nano g^* pre closed (briefly Ng^*p closed) if Npcl(A) $\subseteq U$ whenever A$\subseteq U$ and U is Ng open in $\mathcal{U}$.
9) Nano generalized pre regular closed (briefly Ngpr closed) if Npcl(A) $\subseteq U$ whenever A$\subseteq U$ and U is nano regular open in $\mathcal{U}$.
10) Nano semi generalized closed (briefly Nsg closed) if Nscl(A) $\subseteq U$ whenever A$\subseteq U$ and U is nano semi open in $\mathcal{U}$.
11) Nano $g^\#\alpha$ closed (briefly N$g^\#\alpha$ closed) if Nαcl(A) $\subseteq U$ whenever A$\subseteq U$ and U is Ng open in $\mathcal{U}$.
12) Nano $g^\# s$ closed (briefly N$g^\# s$ closed) if Nscl(A) $\subseteq U$ whenever A$\subseteq U$ and U is Nαg open in $\mathcal{U}$.

The complements of the above mentioned nano closed sets are respective nano open sets.

Definition: 2.6 Let $(\mathcal{U},\tau_R(X))$ and $(\gamma,\tau'_R(Y))$ be nano topological spaces. A function f: $(\mathcal{U},\tau_R(X))\longrightarrow(\gamma,\tau'_R(Y))$ s called

1. Ng^*s closed function if the image of every nano closed set of $\mathcal{U}$ is Ng^*s closed in γ.
2. Nαgr closed function if the image of every nano closed set of $\mathcal{U}$ is Nαgr closed in γ.
3. Nαgs closed function if the image of every nano closed set of $\mathcal{U}$ is Nαgs closed in γ.
4. Nαg closed function if the image of every nano closed set of $\mathcal{U}$ is Nαg closed in γ.
5. Ngs closed function if the image of every nano closed set of $\mathcal{U}$ is Ngs closed in γ.
6. Ngsp closed function if the image of every nano closed set of $\mathcal{U}$ is Ngsp closed in γ.
7. Ngp closed function if the image of every nano closed set of $\mathcal{U}$ is Ngp closed in γ.
8. Ngpr closed function if the image of every nano closed set of $\mathcal{U}$ is Ngpr closed in γ.
9. Ng^*p closed function if the image of every nano closed set of $\mathcal{U}$ is Ng^*p closed in γ.
10. Nsg closed function if the image of every nano closed set of $\mathcal{U}$ is Nsg closed in γ.
11. N$g^\#\alpha$ closed function if the image of every nano closed set of $\mathcal{U}$ is N$g^\#\alpha$ closed in γ.
12. N$g^\# s$ closed function if the image of every nano closed set of $\mathcal{U}$ is N$g^\# s$ closed in γ.

Definition: 2.7 Let $(\mathcal{U},\tau_R(X))$ and $(\gamma,\tau'_R(Y))$ be nano topological spaces. A function f: $(\mathcal{U},\tau_R(X))\longrightarrow(\gamma,\tau'_R(Y))$ is called

1. Ng^*s contra continuous function if $f^{-1}(V)$ is Ng^*s closed in $\mathcal{U}$ for every nano open set V of γ.
2. Nαgr contra continuous function if $f^{-1}(V)$ is Nαgr closed in $\mathcal{U}$ for every nano open set V of γ.
3. Nαgs contra continuous function if $f^{-1}(V)$ is Nαgs closed in $\mathcal{U}$ for every nano open set V of γ.
4. Nαg contra continuous function if $f^{-1}(V)$ is Nαg closed in $\mathcal{U}$ for every nano open set V of γ.
5. Ngs contra continuous function if $f^{-1}(V)$ is Ngs closed in $\mathcal{U}$ for every nano open set V of γ.
6. Ngsp contra continuous function if $f^{-1}(V)$ is Ngsp closed in $\mathcal{U}$ for every nano open set V of γ.
7. Ngp contra continuous function if $f^{-1}(V)$ is Ngp closed in $\mathcal{U}$ for every nano open set V of γ.
8. Ngpr contra continuous function if $f^{-1}(V)$ is Ngpr closed in $\mathcal{U}$ for every nano open set V of γ.
9. Ng^*p contra continuous function if $f^{-1}(V)$ is Ng^*p closed in $\mathcal{U}$ for every nano open set V of γ.
10. Nsg contra continuous function if $f^{-1}(V)$ is Nsg closed in $\mathcal{U}$ for every nano open set V of γ.
11. N$g^\#\alpha$ contra continuous function if $f^{-1}(V)$ is N$g^\#\alpha$ closed in $\mathcal{U}$ for every nano open set V of γ.
12. N$g^\# s$ contra continuous function if $f^{-1}(V)$ is N$g^\# s$ closed in $\mathcal{U}$ for every nano open set V of γ.

3. Nαg^*s CLOSED FUNCTION

Definition:3.1 A subset A of a nano topological space $(\mathcal{U},\tau_R(X))$ is said to be Nαg^* semi closed (briefly Nαg^*s closed) set if Nαcl(A) ⊆U, whenever A⊆U and U is Ngs open in $(\mathcal{U},\tau_R(X))$.

Definition:3.2 Let $(\mathcal{U},\tau_R(X))$ and $(\gamma,\tau'_R(Y))$ be nano topological spaces. A function f: $(\mathcal{U},\tau_R(X))\longrightarrow(\gamma,\tau'_R(Y))$ is called Nαg^*s continuous. If $f^{-1}(V)$ is Nαg^*s open in $\mathcal{U}$ for every nano open set V of γ.

Definition:3.3 Let $(\mathcal{U},\tau_R(X))$ and $(\gamma,\tau'_R(Y))$ be nano topological spaces. A function f: $(\mathcal{U},\tau_R(X))\longrightarrow(\gamma,\tau'_R(Y))$ is called Nαg^*s closed if the image of every nano closed set of $\mathcal{U}$ is Nαg^*s closed in γ.

Theorem:3.4 Every nano closed map is Nαg^*s closed but not conversely.

Proof: Let f: $(\mathcal{U},\tau_R(X))\longrightarrow(\gamma,\tau'_R(Y))$ be a nano closed map. If A is nano closed in $\mathcal{U}$, then f(A) is nano closed in γ and hence Nαg^*s closed in γ.

Example:3.5 Let $\mathcal{U}=\{a,b,c\}$, $\mathcal{U}$/R=$\{\{a,b\},\{c\}\}$, X = $\{c\}$, τ_R(X) = $\{\mathcal{U},\varphi,\{c\}\}$
Define f: $(\mathcal{U},\tau_R(X)) \longrightarrow (\mathcal{U},\tau_R(X))$ by f(a)= a, f(b)= a, f(c)= b.
f is Nαg^*s closed but not nano closed function as f $(\{a,b\}) = \{a\}$ is not nano closed.
Theorem:3.6

1. Every Nαg^*s closed function is Ng^*s closed
2. Every Nαg^*s closed function is Nαgr closed
3. Every Nαg^*s closed function is Nαgs closed
4. Every Nαg^*s closed function is Nαg closed
5. Every Nαg^*s closed function is Ngs closed
6. Every Nαg^*s closed function is Ngsp closed
7. Every Nαg^*s closed function is Ngp closed
8. Every Nαg^*s closed function is Ngpr closed
9. Every Nαg^*s closed function is Ng^*p closed
10. Every Nαg^*s closed function is Nsg closed
11. Every Nαg^*s closed function is N$g^{\#}\alpha$ closed
12. Every Nαg^*s closed function is N$g^{\#}s$ closed

Proof: Obvious from [2]
The converse of the above statements need not be true can be seen from the following examples.
Example:3.7 Let $\mathcal{U}=\{a,b,c,d\}$, $\mathcal{U}$/R=$\{\{a\},\{c\},\{b,d\}\}$, X = $\{a,b\}$,
τ_R(X) = $\{\mathcal{U},\varphi,\{a\},\{a,b,d\},\{b,d\}\}$
Define f: $(\mathcal{U},\tau_R(X)) \longrightarrow (\mathcal{U},\tau_R(X))$ by
f(a)= b, f(b)= a, f(c)= c, f(d)= d
f is Ng^*s closed function but not Nαg^*s closed function as f$\{a,c\}$=$\{b,c\}$ is not Nαg^*s closed.
Example: 3.8 Refer Example 3.7
f is Nαgr closed function but not Nαg^*s closed function as f$\{a,c\}$=$\{b,c\}$ is not Nαg^*s closed.

Example: 3.9 Refer Example 3.7
f is Nαgs closed function but not Nαg^*s closed function as f$\{a,c\}$=$\{b,c\}$ is not Nαg^*s closed.
Example: 3.10 Refer Example 3.7
f is Nαg closed function but not Nαg^*s closed function as f$\{a,c\}$=$\{b,c\}$ is not Nαg^*s closed.
Example: 3.11 Refer Example 3.7
f is Ngs closed function but not Nαg^*s closed function as f$\{a,c\}$=$\{b,c\}$ is not Nαg^*s closed.
Example: 3.12 Refer Example 3.7
f is Ngsp closed function but not Nαg^*s closed function as f$\{a,c\}$=$\{b,c\}$ is not Nαg^*s closed.
Example: 3.13 Refer Example 3.7
f is Ngp closed function but not Nαg^*s closed function as f$\{a,c\}$=$\{b,c\}$ is not Nαg^*s closed.
Example: 3.14 Refer Example 3.7
f is Ngpr closed function but not Nαg^*s closed function as f$\{a,c\}$=$\{b,c\}$ is not Nαg^*s closed.
Example: 3.15 Refer Example 3.7
f is Ng^*p closed function but not Nαg^*s closed function as f$\{a,c\}$=$\{b,c\}$ is not Nαg^*s closed.
Example: 3.16 Refer Example 3.7
f is Nsg closed function but not Nαg^*s closed function as f$\{a,c\}$=$\{b,c\}$ is not Nαg^*s closed.
Example: 3.17 Refer Example 3.7
f is N$g^{\#}\alpha$ closed function but not Nαg^*s closed function as f$\{a,c\}$=$\{b,c\}$ is not Nαg^*s closed.
Example: 3.18 Refer Example 3.7
f is N$g^{\#}s$ closed function but not Nαg^*s closed function as f$\{a,c\}$=$\{b,c\}$ is not Nαg^*s closed.
Theorem:3.19 If f: $(\mathcal{U},\tau_R(X)) \longrightarrow (\gamma,\tau'_R(Y))$ is nano closed and g: $(\gamma,\tau'_R(Y)) \longrightarrow (\eta,\tau''_R(Z))$ is Nαg^*s closed, then g o f: $(\mathcal{U},\tau_R(X)) \longrightarrow (\eta,\tau''_R(Z))$ is Nαg^*s closed.
Proof: obvious.
Theorem:3.20 A map f: $(\mathcal{U},\tau_R(X)) \longrightarrow (\gamma,\tau'_R(Y))$ is Nαg^*s closed if and only if for each nano subset S of γ and each nano open set U of $\mathcal{U}$ such that $f^{-1}(S)\subseteq$U, there is Nαg^*s open subset V of γ such that S⊆V and f^{-1}(V) ⊆U.
Proof: Let f be Nαg^*s closed. Let S⊆ V and U be a nano open set of $\mathcal{U}$ such that $f^{-1}(S)\subseteq$U.
$\mathcal{U}-U$ is closed in $\mathcal{U}$. f $(\mathcal{U}-U)$ is Nαg^*s closed in γ.
V= $\gamma-$ f $(\mathcal{U}-U)$ is Nαg^*s open in γ.
f^{-1}(V)= $\mathcal{U}$-f^{-1}(f $(\mathcal{U}-U)$) ⊆ $\mathcal{U}$- $(\mathcal{U}-U)$=U
Conversely, let F be nano closed in $\mathcal{U}$.
f^{-1}(f (F^c)) ⊆ F^c and F^c is nano open in $\mathcal{U}$.

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 10 Issue 10, October-2021

By assumption, there exists a Nαg^*s open subset V of γ such that f $(F^c) \subseteq V$ and
f^{-1}(V) $\subseteq F^c$.
This implies F$\subseteq (f^{-1}(\text{V}))^c$
Hence $V^c \subseteq (f(F^c))^c = f(F) \subseteq f(f^{-1}(\text{V}))^c \subseteq V^c$
So, f(F)=V^c, which is Nαg^*s closed.
Definition:3.21 Let $\mathcal{U}$ and γ be a nano topological space. A map f: $(\mathcal{U}, \tau_R(\text{X})) \longrightarrow (\gamma, \tau'_R(\text{Y}))$ is called Nαg^*s open map if the image of every nano open set of $\mathcal{U}$ is Nαg^*s open in γ.
Theorem:3.22 For any bijection map f: $(\mathcal{U}, \tau_R(\text{X})) \longrightarrow (\gamma, \tau'_R(\text{Y}))$, the following are equivalent:

1. f^{-1}: $(\gamma, \tau'_R(\text{Y})) \longrightarrow (\mathcal{U}, \tau_R(\text{X}))$ is Nαg^*s continuous map.
2. f is Nαg^*s open map.
3. f is Nαg^*s closed map.

Proof: (1)$\Longrightarrow$(2)
Let U be nano open in $\mathcal{U}$. $(f^{-1})^{-1}$(U) is Nαg^*s open in γ. That is f(U) is Nαg^*s open in γ.

(2)$\Longrightarrow$(3)
Let F be a nano closed set of $\mathcal{U}$. Then $F^c$is open in $\mathcal{U}$.
By assumption, f (F^c) is Nαg^*s open in γ.
f (F^c) = f $(F)^c$ is Nαg^*s closed open in γ. f(F) is Nαg^*s closed in γ.
(3)$\Longrightarrow$(1)
Let F be nano closed in $\mathcal{U}$. f(F) is Nαg^*s closed in γ. f(F)= $(f^{-1})^{-1}$(F) is Nαg^*s closed in γ.
Hence f^{-1} is Nαg^*s continuous map.

4. Nαg^*s CONTRA CONTINOUS FUNCTIONS

Definition:4.1 A function f: $(\mathcal{U}, \tau_R(\text{X})) \longrightarrow (\gamma, \tau'_R(\text{Y}))$ is called Nαg^*s contra continuous function if f^{-1}(V) is Nαg^*s closed in $\mathcal{U}$ for every nano open set V of γ.
Example:4.2 Refer Example 3.5
Define f: $(\mathcal{U}, \tau_R(\text{X})) \longrightarrow (\mathcal{U}, \tau_R(\text{X}))$ by f(a)= c, f(b)= a, f(c)= b
f is Nαg^*s contra continuous function.
Definition:4.3 A function f: $(\mathcal{U}, \tau_R(\text{X})) \longrightarrow (\gamma, \tau'_R(\text{Y}))$ is called almost Nαg^*s contra continuous if f^{-1}(V) is Nαg^*s closed in $\mathcal{U}$ for every nano regular open set V of γ.
Theorem:4.4 Every Nαg^*s contra continuous function is almost Nαg^*s contra continuous.
Proof: The proof is obvious as every nano regular open set is nano open.
The converse of the above theorem need not be true can be seen from the following example.
Example:4.5 Refer Example 3.5
Let f be the identity function. f is almost Nαg^*s contra continuous but not Nαg^*s contra continuous an $f^{-1}(\{c\}) = \{c\}$ is not Nαg^*s closed.
Theorem:4.6

1. Every Nαg^*s contra continuous function is Ng^*s contra continuous.
2. Every Nαg^*s contra continuous function is Nαgr contra continuous.
3. Every Nαg^*s contra continuous function is Nαgs contra continuous.
4. Every Nαg^*s contra continuous function is Nαg contra continuous.
5. Every Nαg^*s contra continuous function is Ngs contra continuous.
6. Every Nαg^*s contra continuous function is Ngsp contra continuous.
7. Every Nαg^*s contra continuous function is Ngp contra continuous.
8. Every Nαg^*s contra continuous function is Ngpr contra continuous.
9. Every Nαg^*s contra continuous function is Ng^*p contra continuous.
10. Every Nαg^*s contra continuous function is Nsg contra continuous.
11. Every Nαg^*s contra continuous function is N$g^\#\alpha$ contra continuous.
12. Every Nαg^*s contra continuous function is N$g^\# s$ contra continuous.

Proof: Obviuous from[2]
The converse of the above statements need not be true can be seen from the following examples.
Example: 4.7 Refer Example 3.7
Define f: $(\mathcal{U}, \tau_R(\text{X})) \longrightarrow (\mathcal{U}, \tau_R(\text{X}))$ by f(a)= a, f(b)= b, f(c)= d, f(d)= c.
f is Ng^*s contra continuous but not Nαg^*s contra continuous as $f^{-1}(\{a\}) = \{a\}$ is not Nαg^*s closed in $\mathcal{U}$.
Example: 4.8 Refer Example 3.7
Define f: $(\mathcal{U}, \tau_R(\text{X})) \longrightarrow (\mathcal{U}, \tau_R(\text{X}))$ by f(a)= d, f(b)= b, f(c)= a, f(d)= c.
f is Nαgr contra continuous but not Nαg^*s contra continuous as $f^{-1}(\{b, d\}) = \{a, b\}$ is not Nαg^*s closed in $\mathcal{U}$.

Example: 4.9 Refer Example 3.7
Define f: $(\mathcal{U},\tau_R(X)) \longrightarrow (\mathcal{U},\tau_R(X))$ by f(a)= a, f(b)= a, f(c)= a, f(d)= c.
f is Nαgs contra continuous but not Nαg^*s contra continuous as $f^{-1}(\{a\}) = \{a,b,c\}$ is not Nαg^*s closed in $\mathcal{U}$.
Example: 4.10 Refer previous Example
f is Nαg contra continuous but not Nαg^*s contra continuous as $f^{-1}(\{a\}) = \{a,b,c\}$ is not Nαg^*s closed in $\mathcal{U}$.

Example: 4.11 Refer Example 3.7
Define f: $(\mathcal{U},\tau_R(X)) \longrightarrow (\mathcal{U},\tau_R(X))$ by f(a)= a, f(b)= b, f(c)= d, f(d)= d.
f is Ngs contra continuous but not Nαg^*s contra continuous as $f^{-1}(\{a\}) = \{a\}$ is not Nαg^*s closed in $\mathcal{U}$.
Example: 4.12 Refer Example 4.9
f is Ngsp contra continuous function but not Nαg^*s contra continuous function.
Example: 4.13 Refer Example 4.8
f is Ngp contra continuous function but not Nαg^*s contra continuous function.
Example: 4.14 Refer Example 4.8
f is N$g$pr contra continuous function but not Nαg^*s contra continuous function.
Example: 4.15 Refer Example 4.8
f is Ng^*p contra continuous function but not Nαg^*s contra continuous function.
Example: 4.16 Refer Example 4.11
f is Nsg contra continuous function but not Nαg^*s contra continuous function.
Example: 4.17 Refer Example 4.9
f is N$g^\#\alpha$ contra continuous function but not Nαg^*s contra continuous function.
Example: 4.18 Refer Example 4.9
f is N$g^\# s$ contra continuous function but not Nαg^*s contra continuous function.
Theorem:4.19 Let arbitrary union of Nαg^*s open sets be Nαg^*s open. Then the following statements are equivalent for a map f: $(\mathcal{U},\tau_R(X)) \longrightarrow (\gamma,\tau_R'(Y))$

1. f is Nαg^*s contra continuous.
2. For every nano closed set F of γ, f^{-1}(F) is Nαg^*s open in $\mathcal{U}$.
3. For each x$\in \mathcal{U}$ and each nano closed set F of γ containing f(x), there exists Nαg^*s open set U containing x such that f(U) $\subseteq F$.
4. For each x$\in \mathcal{U}$ and each nano open set V of γ not containing f(x), there exists Nαg^*s closed set K not containing x such that f^{-1}(V) $\subseteq K$.

Proof: (1)⟹(2)
Let F be nano closed in γ. Then $\gamma - F$ is nano open in γ. By (1) $f^{-1}(\gamma - \text{F}) = \mathcal{U} - f^{-1}(\text{F})$ is Nαg^*s closed in $\mathcal{U}$. So, f^{-1}(F) is Nαg^*s open. Hence (2) holds.
(2)⟹(1)
Let G be nano open in γ. Then $\gamma - G$ is nano closed in γ. By (2) $f^{-1}(\gamma - \text{G}) = \mathcal{U} - f^{-1}(\text{G})$ is Nαg^*s open in $\mathcal{U}$. So, f^{-1}(G) is closed in $\mathcal{U}$. Hence (1) holds.
(2)⟹(3)
Let F be nano closed in γ containing f(x). Hence x$\in f^{-1}$(F). By (2), f^{-1}(F) is Nαg^*s open in $\mathcal{U}$. Let U=f^{-1}(F). This implies U is Nαg^*s open in $\mathcal{U}$ containing x and f(U)=F $\subseteq F$. So (3) holds.
(3)⟹(2)
Let F be a nano closed set of γ containing f(x). So, x$\in f^{-1}$(F). By (3), there exists Nαg^*s open set $U_x$of $\mathcal{U}$ containing x such that f $(U_x) \subseteq F$. So, $f^{-1}(\text{F}) = U\{U_x : x \in f^{-1}(\text{F})\}$. This is a union of N$\alpha g^*$s open sets, hence it is Nαg^*s open. Hence (2) holds.
(3)⟹(4)
Let V be nano open in γ not containing f(x). Then $\gamma - V$ is closed in γ containing f(x). By (3), there exists Nαg^*s open set U in $\mathcal{U}$ containing x such that f(U) $\subseteq \gamma - \text{V}$. This implies U$\subseteq f^{-1}(\gamma - \text{V}) = \mathcal{U} - f^{-1}(V)$. Hence$f^{-1}(V) \subseteq \mathcal{U}$-U. Let K= $\mathcal{U}$-U, K is Nαg^*s closed in $\mathcal{U}$ not containing x such that $f^{-1}(V) \subseteq K$. Hence (4) holds.
(4)⟹(3)
Let F be nano closed in γ containing f(x). Then $\gamma - F$ is nano open in γ not containing f(x). From (4), there exists Nαg^*s closed set K in $\mathcal{U}$ not containing x such that $f^{-1}(\gamma - \text{F}) \subseteq K$.
This implies $\mathcal{U} - f^{-1}(F) \subseteq K$. Hence $\mathcal{U} - K \subseteq f^{-1}(F)$. That is, f $(\mathcal{U} - K) \subseteq F$. Let U= $\mathcal{U} - K$. Then U is Nαg^*s open in $\mathcal{U}$ containing x such that f(U) $\subseteq F$. So (3) holds.

Theorem:4.20 The following are equivalent for a map f: $(\mathcal{U},\tau_R(X)) \longrightarrow (\gamma,\tau_R'(Y))$

1. f is almost Nαg^*s contra continuous.
2. $f^{-1}(Nint(Ncl(G)))$ is Nαg^*s closed in $\mathcal{U}$ for every nano open set G of γ.
3. $f^{-1}(Ncl(Nint(F)))$ is Nαg^*s open in $\mathcal{U}$ for every nano closed set F of γ.

Proof: (1)⟹(2)
Let G be nano open in γ. Then Nint (Ncl(G)) is nano regular open in γ. Hence $f^{-1}(Nint(Ncl(G)))$ is Nαg^*s closed in $\mathcal{U}$.

(2)⟹(1) obvious
(1)⟹(3)
Let F be nano closed in γ. Then $Ncl(Nint(F))$ is nano regular closed in γ. Hence $f^{-1}(Ncl(Nint(F)))$ is Nαg^*s open in $\mathcal{U}$.
(3)⟹(1) obvious
Definition:4.21 A map f: $(\mathcal{U},\tau_R(X)) \longrightarrow (\gamma,\tau'_R(Y))$ is said to be nano R-map if $f^{-1}(V)$ is nano regular open in $\mathcal{U}$ for each nano regular open set V of γ.
Definition:4.22 A map f: $(\mathcal{U},\tau_R(X)) \longrightarrow (\gamma,\tau'_R(Y))$ is said to be nano perfectly continuous if $f^{-1}(V)$ is nano clopen in $\mathcal{U}$ for each nano open set V of γ.
Theorem:4.23 For two mappings f: $(\mathcal{U},\tau_R(X)) \longrightarrow (\gamma,\tau'_R(Y))$ and g: $(\gamma,\tau'_R(Y)) \longrightarrow (\eta,\tau''_R(Z))$, then for g o f: $\mathcal{U} \longrightarrow \eta$, the following properties hold:

1. If f is almost Nαg^*s contra continuous and g is a nano R map then g o f is almost Nαg^*s contra continuous.
2. If f is almost Nαg^*s contra continuous and g is nano perfectly continuous, then g o f is almost Nαg^*s contra continuous and almost Nαg^*s continuous.

Proof: (1) obvious
(2) Let V be nano regular open in η. $g^{-1}(V)$ is nano clopen in γ and hence nano regular open and nano regular closed. $f^{-1}(g^{-1}(V)) = (g\ o\ f)^{-1}(V)$ is Nαg^*s open and Nαg^*s closed in $\mathcal{U}$ as f is almost Nαg^*s contra continuous. So, g o f is almost contra Nαg^*s continuous and almost Nαg^*s continuous.
Definition:4.24 A nano topological space $(\mathcal{U},\tau_R(X))$ is called $T_{N\alpha g^*s}$ space if every Nαg^*s open set is nano open.
Theorem:4.25 Let f: $(\mathcal{U},\tau_R(X)) \longrightarrow (\gamma,\tau'_R(Y))$ be a Nαg^*s contra continuous map and g: $(\gamma,\tau'_R(Y)) \longrightarrow (\eta,\tau''_R(Z))$ be Nαg^*s continuous. If γ is a $T_{N\alpha g^*s}$ space, then g o f: $\mathcal{U} \longrightarrow \eta$ is an almost Nαg^*s contra continuous map.
Proof: Let V be nano regular open in η . $g^{-1}(V)$ is Nαg^*s open in γ. As γ is $T_{N\alpha g^*s}$ space, $T_{N\alpha g^*s}$ space, $g^{-1}(V)$ is nano open in γ. $f^{-1}(g^{-1}(V)) = (g\ o\ f)^{-1}(V)$ is Nαg^*s closed in $\mathcal{U}$. Hence g o f is almost Nαg^*s contra continuous map.
Definition:4.26 A map f: $(\mathcal{U},\tau_R(X)) \longrightarrow (\gamma,\tau'_R(Y))$ is called strongly Nαg^*s open (strongly Nαg^*s closed) if (V) is Nαg^*s open (Nαg^*s closed) in γ for every Nαg^*s open (Nαg^*s closed) set V of $\mathcal{U}$.
Theorem:4.27 If f: $(\mathcal{U},\tau_R(X)) \longrightarrow (\gamma,\tau'_R(Y))$ is surjective strongly Nαg^*s nano open (strongly Nαg^*s closed) map and g: $(\gamma,\tau'_R(Y)) \longrightarrow (\eta,\tau''_R(Z))$ is a map such that g o f: $\mathcal{U} \longrightarrow \eta$ is an almost Nαg^*s nano contra continuous, then g is almost Nαg^*s contra continuous.
Proof: Let V be any regular closed (nano regular open) set in η. As g o f is almost Nαg^*s contra continuous, $(g\ o\ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is Nαg^*s open (Nαg^*s closed) in $\mathcal{U}$. Since f is surjective and strongly Nαg^*s open (nano strongly Nαg^*s closed), f $(f^{-1}(g^{-1}(V)))$= $g^{-1}(V)$ is Nαg^*s open (Nαg^*s closed) in γ. Hence g is almost Nαg^*s contra continuous.
Definition:4.28 A map f: $(\mathcal{U},\tau_R(X)) \longrightarrow (\gamma,\tau'_R(Y))$ is called weakly Nαg^*s continuous if for each x∈ $\mathcal{U}$ and each nano open set V of γ containing f(x), there exists Nαg^*s open set U of $\mathcal{U}$such that f(U) ⊆ $Ncl(V)$.
Theorem:4.29 If a map f: $(\mathcal{U},\tau_R(X)) \longrightarrow (\gamma,\tau'_R(Y))$ is Nαg^*s contra continuous, then f is weekly Nαg^*s continuous map.
Proof: Let x∈ $\mathcal{U}$ and V be an open set in γ containing f(x). Ncl(V) is closed in γ containing f(x). Since f is Nαg^*s contra continuous, f^{-1}(Ncl(V)) is Nαg^*s open in $\mathcal{U}$. Let U=f^{-1}(Ncl(V)). f(U)=f(f^{-1}(Ncl(V))) ⊆Ncl(V). So, f is weakly Nαg^*s continuous.
Definition:4.30 A space $\mathcal{U}$ is called locally Nαg^*s indiscrete if every Nαg^*s open set is nano closed in $\mathcal{U}$.
Theorem:4.31 If a map f: $(\mathcal{U},\tau_R(X)) \longrightarrow (\gamma,\tau'_R(Y))$ is almost Nαg^*s contra continuous and $\mathcal{U}$ is locally Nαg^*s indiscrete, then f is almost nano continuous.
Proof: Let V be nano regular open in γ. As f is almost Nαg^*s contra continuous. $f^{-1}(V)$ is Nαg^*s closed in $\mathcal{U}$. Since $\mathcal{U}$ is locally Nαg^*s indiscrete space, $f^{-1}(V)$ is nano open in γ. Hence f is almost nano continuous.

5. DIFFERENT Nαg^*s FUNCTIONS

Definition:5.1 A function f: $(\mathcal{U},\tau_R(X)) \longrightarrow (\gamma,\tau'_R(Y))$ is said to be Nαg^*s continuous if and only if the inverse image of every nano open set in γ is Nαg^*s open in $\mathcal{U}$.
Definition:5.2 A function f: $(\mathcal{U},\tau_R(X)) \longrightarrow (\gamma,\tau'_R(Y))$ is said to be Nαg^*s nano contra continuous if and only if the inverse image of every closed set in γ is Nαg^*s open in $\mathcal{U}$.
Theorem:5.3 Let f: $(\mathcal{U},\tau_R(X)) \longrightarrow (\gamma,\tau'_R(Y))$ be Nαg^*s contra continuous. Thus $f^{-1}(B)$ ⊆Nαg^*s (Nint ($f^{-1}(NclB)$)), for every B ⊆ γ.
Proof: f is Nαg^*s contra continuous. Ncl(B) is nano closed in γ. f^{-1}(Ncl(B)) is Nαg^*s open in $\mathcal{U}$. So, Nαg^*s (Nint(f^{-1}(Ncl(B)))= f^{-1}(Ncl(B))
f^{-1}(B) ⊆ f^{-1}(Ncl(B)) = Nαg^*s (Nint(f^{-1}(Ncl(B)))
Theorem:5.4 For a function f: $(\mathcal{U},\tau_R(X)) \longrightarrow (\gamma,\tau'_R(Y))$, the following conditions are equivalent.

1. f is Nαg^*s contra continuous.
2. For each x∈ $\mathcal{U}$ and each nano closed set V in γ with f(x) ∈ V, there exists an Nαg^*s open set U in $\mathcal{U}$ such that x∈ U and f(U)⊆ V, if arbitrary union of Nαg^*s open sets Nαg^*s open.
3. The inverse image of each nano open set in γ is Nαg^*s closed in $\mathcal{U}$.

Proof: (1)⟹(2)

Let f be Nαg^*s contra continuous.
Let x∈ $\mathcal{U}$ and V be a nano closed in γ containing f(x).
So, x∈ $f^{-1}(V)$, which is Nαg^*s open in $\mathcal{U}$.
Let $f^{-1}(V) = U$.
Hence x∈ U.
f(U)=f$f^{-1}(V) \subseteq V$.
(2)⟹(1)
Let V be nano closed in γ. Let x∈ $f^{-1}(V)$. So, f(x) ∈V. Hence, there exists Nαg^*s open set U_x containing x such that f $(U_x) \subseteq V$. That is, x∈ $U_x \subseteq f^{-1}(V)$. Therefore $f^{-1}(V)$ is Nαg^*s open in X.
(3)⟹(1) is obvious

Definition:5.5 A subset S of $\mathcal{U}$ is called locally nano Nαg^*s closed if S=U∩ F, where U is Nαg^*s open and F is Nαg^*s closed.

Definition: 5.6 f: $(\mathcal{U}, \tau_R(X)) \longrightarrow (\gamma, \tau'_R(Y))$ is said to be Lc- Nαg^*s continuous if and only if the pre image of every nano open set of γ is locally Nαg^*s closed.

Theorem:5.7 Every Nαg^*s continuous function is Lc- Nαg^*s continuous.
Proof: Let f: $(\mathcal{U}, \tau_R(X)) \longrightarrow (\gamma, \tau'_R(Y))$ be Nαg^*s continuous. Let V be nano open in γ. $f^{-1}(V) = f^{-1}(V) \cap$ Nαg^*s (Ncl $(f^{-1}(V))$), which is locally Nαg^*s closed in $\mathcal{U}$. Hence f is Lc- Nαg^*s continuous.

Definition:5.8 Let $(\mathcal{U}, \tau_R(X))$ be a nano topological space. A subset A⊆ $\mathcal{U}$ is said to be Nαg^*s clopen if and only if it is Nαg^*s closed and Nαg^*s open.

Definition: 5.9 A function f: $(\mathcal{U}, \tau_R(X)) \longrightarrow (\gamma, \tau'_R(Y))$ is said to be strongly Nαg^*s continuous if and only if the inverse of every subset of γ is Nαg^*s clopen.

Theorem:5.10 Every strongly nano Nαg^*s continuous function is Nαg^*s contra continuous.
Proof: Let f: $(\mathcal{U}, \tau_R(X)) \longrightarrow (\gamma, \tau'_R(Y))$ be strongly Nαg^*s continuous. Let V be nano closed in γ. Then $f^{-1}(V)$ is Nαg^*s is Nαg^*s clopen and hence $f^{-1}(V)$ is Nαg^*s open. So f is Nαg^*s contra continuous.
The converse of the above theorem need not be true can be seen from the following example.

Example:5.11 Let $\mathcal{U}$= $\{a, b, c\}$, $\mathcal{U}$/R=$\{\{a, b\}, \{c\}\}$, X = $\{c\}$,
τ_R(X) = $\{ \mathcal{U}, \varphi, \{c\}\}$
Define f: $(\mathcal{U}, \tau_R(X)) \longrightarrow (\mathcal{U}, \tau_R(X))$ by
f(a)= c, f(b)= a, f(c)= b
f is Nαg^*s contra continuous but not Nαg^*s strongly continuous as $f^{-1}\{a, b\}$=$\{b, c\}$ is not Nαg^*s clopen.

Theorem:5.12 Every perfectly Nαg^*s continuous function is Nαg^*s contra continuous.
Proof: Let f: $(\mathcal{U}, \tau_R(X)) \longrightarrow (\gamma, \tau'_R(Y))$ be perfectly Nαg^*s continuous. Let V be nano open in γ. $f^{-1}(V)$ is Nαg^*s clopen which is Nαg^*s closed. Hence f is Nαg^*s contra continuous. The converse of the above theorem need not be true can be seen from the following example.

Example:5.13 Refer Example:5.11
f is Nαg^*s contra continuous but not Nαg^*s perfectly continuous as $f^{-1}\{a\}$=$\{b\}$ is not Nαg^*s clopen.

Definition: 5.14 A nano topological space $(\mathcal{U}, \tau_R(X))$ is called strongly S- Nαg^*s closed if and only if every Nαg^*s closed cover of $\mathcal{U}$ has a finite sub cover.

Theorem:5.15 Nαg^*s nano contra continuous images of strongly S- Nαg^*s closed spaces are nano compact.
Proof: Let f: $(\mathcal{U}, \tau_R(X)) \longrightarrow (\gamma, \tau'_R(Y))$ be Nαg^*s contra continuous and onto. Let $\mathcal{U}$ be strongly S- Nαg^*s closed. Let $(V_i)_{i \in I}$ be an nano open cover of γ. Then $(f^{-1}(V_i)_{i \in J})$ is a Nαg^*s closed cover of $\mathcal{U}$. Since f is Nαg^*s contra continuous, then for some finite J⊆ I, we have $\mathcal{U} = \underset{i \in J}{U} f^{-1}(V_i)$. As f is onto $\gamma = \underset{i \in J}{U} V_i$. That is, γ is nano compact.

Definition: 5.16 A map f: $(\mathcal{U}, \tau_R(X)) \longrightarrow (\gamma, \tau'_R(Y))$ is called Nαg^*s irresolute if and only if the inverse image of every Nαg^*s closed set in γ is Nαg^*s closed in $\mathcal{U}$.

Remark:5.17 f is Nαg^*s irresolute ⇔ the inverse image of every Nαg^*s open set under f is Nαg^*s open.

Theorem:5.18 Let f: $(\mathcal{U}, \tau_R(X)) \longrightarrow (\gamma, \tau'_R(Y))$, g: $(\gamma, \tau'_R(Y)) \longrightarrow (\eta, \tau''_R(Z))$ be functions. Then

1. If f is Nαg^*s continuous and g is nano continuous, then g o f is Nαg^*s continuous.
2. If f and g are Nαg^*s irresolute, g o f is Nαg^*s irresolute.
3. If f is Nαg^*s irresolute and g is Nαg^*s continuous, then g o f is Nαg^*s continuous.

Proof: obvious

Definition: 5.19 f: $(\mathcal{U}, \tau_R(X)) \longrightarrow (\gamma, \tau'_R(Y))$ is said to be almost Nαg^*s continuous if and only if the inverse image of every nano regular open set is Nαg^*s open in $\mathcal{U}$.

Theorem:5.20 f: $(\mathcal{U}, \tau_R(X)) \longrightarrow (\gamma, \tau'_R(Y))$ be a function. f is Nαg^*s continuous⟹ f is almost Nαg^*s continuous.
Proof: Let f be Nαg^*s continuous. Let U be nano regular open in γ. As nano regular open sets are nano open, U is nano open in γ. So, $f^{-1}(U)$ is Nαg^*s open in $\mathcal{U}$. Hence f is almost Nαg^*s continuous.

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 10 Issue 10, October-2021

Definition: 5.21 A nano topological space $(\mathcal{U}, \tau_R(X))$ is said to be Nαg^*s Hausdorff if and only if x and y are distinct points of $\mathcal{U}$, there exists disjoint Nαg^*s open sets U and V containing x and y respectively.

Theorem:5.22 Let $(\mathcal{U}, \tau_R(X))$ be a topological space and let $(\gamma, \tau'_R(Y))$ be a nano Hausdorff space. If f: $(\mathcal{U}, \tau_R(X)) \longrightarrow (\gamma, \tau'_R(Y))$ is injective and Nαg^*s continuous, then $\mathcal{U}$ is a Nαg^*s Hausdorff space.

Proof: Let x and y be distinct points of $\mathcal{U}$. Since f is injective f(x) and f(y) are distinct points of γ. As γ is nano Hausdorff, there exists disjoint nano open sets U and V containing f(x) and f(y) respectively.

Since f is Nαg^*s continuous and U and V are disjoint, $f^{-1}(U)$ and, $f^{-1}(V)$ are disjoint Nαg^*s open sets. So $\mathcal{U}$ is Nαg^*s Hausdorff.

Following the same lines, we can prove the following.

Theorem:5.23 Let $(\mathcal{U}, \tau_R(X))$ be a nano topological space and let γ be Nαg^*s Hausdorff. If f: $(\mathcal{U}, \tau_R(X)) \longrightarrow (\gamma, \tau'_R(Y))$ is injective and Nαg^*s irresolute, then $\mathcal{U}$ is Nαg^*s Hausdorff.

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