ON θ Generalized Pre- Open sets in a Topological Space

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Abstract:- **In this paper, a new class of sets called theta generalized pre-open set in a topological space introduced and some of their basic properties are investigated. Several examples are provided to illustrate the behaviour of new sets.**

Keywords: Pre- closed set, Pre – open set, θ gp -Closed, θ gp – open set.

1. INTRODUCTION

N.V.Velico and J.Dontchev etal are introduced the concepts of θ-genaralized closed sets. The concept of generalized closed sets introduced by Levin plays a significant role in topology. After the introduction of generalized closed sets many research papers were published which deal with different types of generalized closed sets. H.Maki et al. defined the concept of gp-closed set in topological spaces and established results related to it. These concepts motivated us to define a new class of sets called the theta generalized pre-closed sets and gp-open sets.

2. θ GENERALIZED PRE -OPEN SETS.

Definitions 2.1: A subset A of a topological space X is called a Theta generalized pre – open (briefly, θ gp – open) set if A^C is θ gp-closed.
Example: Let x={ a, b, c, } and topology τ = { x, ∅, { a} } then θ gp- open set = { x, ∅, {a}, {c}, {b}, {a,b} {a,c} }

Theorem 2.2 A set A⊆ X is θ gp – open iff G ⊆ pint(A) whenever G is θ – closed and G ⊆ A.
Proof: Let A be θ gp – open set and suppose G ⊆ A where G is θ-closed. Then X-A is a θ gp – closed set contained in the θ – open set X-G, pcl (X-A) ⊆ X-G .
Since pcl (X-A) = X – Pint (A) [9], then X – pint (A) ⊆ X-G. That is G⊆pint(A).
Conversely, let G⊆pint(A) be true whenever G ⊆ A and G is θ – closed, then X-pint (A)⊆ X-G. that is pcl (X-A)⊆ X-G. this implies X-A is θ gp – closed and A is θ gp – open in X.

Example 2.3: Let X={ a, b, c,} and topology τ = { x, ∅, {c}, {a, c} } then θ gp- open set = { x, ∅, {a,c}, { a.b}, {c}, {a} } . the converse of the above theorem need not be true.

Theorem 2.4 If A is θ gp – open and B is any set in X such that pint (A) ⊆ B⊆A, then B is θ gp – open in X.
Proof: Follows from the definitions and theorem 3.8 [7]

Theorem 2.5 If A is θ gp – open and B is any set in X such that pint (A)⊆B, then A⋂B is θ gp – open in X.
Proof: Let A be a θ gp – open set of X and pint(A)⊆B, then A∩ pint(A) ⊆ A∩B⊆A.
Since pint(A) ⊆ A, then pint(A) ⊆ A∩B⊆A and from theorem 2.4, A⋂B is θ gp – open in X.

Theorem 2.6 If a set A⊆X is θ gp-closed, then pcl (A) -A is θ gp – open in X .
Proof: suppose that A is θ gp-closed and M is θ – closed such that M ⊆ pcl (A) – A, then by theorem 3.5 [7], M = ∅ and hence M ⊆ pint(pcl (A) -A). Therefore by Theorem 2.2pcl(A) - A is θ gp – open.

Example 2.7 Let X= { a, b, c} and topology τ ={ x, ∅, {a}, {a,b}, }, then θ gp – open x = { x , ∅, {a,c}, {a,b}, {c}, {a}, {b},}.

Definitions 2.8 [3] Let A and B be two non void subsets, of a topological space X. Then A and B are said to be θ separated if A∩ θ cl(B) = θ cl(A)∩ B = ∅.

Theorem 2.9 If A and B are θ separated θ gp – open sets, then A∪B is θ gp open.
Proof: Let F be a θ – closed subset of A∪B. Then F ⋂ θcl (A)⊆ (A∪B)∩ θcl(A)= (A∩ θcl(A))∪(B∩cl(A))=A∪ ∅=A. That is, F ⋂ θcl (A) ⊆A. Therefore Then F ⋂ θcl (A) is a θ – closed set contained in A and A is a θ gp – open, then by Theorem 2.2, F

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∩ θcl (A)⊆ pint(A). similarly F ∩ θcl (B)⊆pint(B). Thus we have F = F∩(A∪B)=(F∩ A) ∪(F∩B) ⊆ (F ∩ θcl (A)) ∪ (F ∩ θcl (B)) ⊆ pint(A)∪pint(B)⊆pint(A∪B). That is F⊆pint (A∪B). Hence by Theorem 2.2, A∪B is θ gp – open.

Related Nbhds, closure and Interior.

Definition 3.0 a subset M of a topological space X is called θ gp -neighbouhood (briefly ,θ gp -nbhd) of a point x∈X, if there exixts a θ gp-open set U such that x∈U⊆M.

The collection of all θ gp -nbhds of a point x∈X is called θ gb -nbhd system of x and is denoted by θ gpN(x).

Theorem 3.1 If A is θ gp -open set ,then it is θ gp -nbhd of each of its points .

Proof: let A be any θ gp -open set of of X, then for each x∈ A, x∈A⊆A. Therefore A is θ gp -nbhd of each of its points.

Theorem 3.2 if A⊆X is a θ gp -closed set and x ∈ A^c, then there exists a θ gp -nbhd F of x such that F∩A=∅.

Proof : Let A⊆X is a θ gp -closed set, then A^c is θ gp -open. Therefore, By theorem 3.0, A^c is θ gp -nbhd of each of its points. Let x ∈ A^c then there exists a θ gp -open set F such that x∈ $F \subseteq A^c$. That is, F∩A=∅.

Theorem 3.3 Let x be a point in a space X, then

(i) θ gpN(x)≠ ∅.

(ii) If A∈ θ gpN(x),then x∈ A.

(iii) If A∈ θ gpN(x) and B ⊇A ,then B∈ θ gpN(x).

(iv) if $A_\lambda \in \theta$ gpN(x) for each $\lambda \in \wedge$ then $\cup A_\lambda \in \theta$ gpN(x).

Proof (i) since X ∈ θ gpN(x), θ gpN(x) ≠ ∅.

(ii) let A∈ θ gpN(x), then there exists a θ gp-open set G such that x∈G⊆A. This implies x∈A.

(iii) Let A∈ θ gpN(x), then there exists a θ gp-open set G such that x∈G⊆A. Since A⊆B, then x∈G⊆B. This shows B∈ θ gpN(x).

(iv) since for each $\lambda \in \wedge$, A_λ is θ gp -nbhd of x, then there exists a θ gp-open set G_λ such that x∈ $G_\lambda \subseteq A_\lambda$. Which implies that x∈ $G_\lambda \subseteq \cup A_\lambda$ and hence $\cup A_\lambda \in \theta$ gpN(x).

Theorem 3.4 Let A be a subset of a X. Then $x \in \theta gpCl(A)$ if and only if $U \cap A \neq \emptyset$, for every $\theta gp - open$ set U containing x.

Proof:

Let $x \in \theta gpCl(A)$. Suppose that there exisits a $\theta gp - open$ set U containing x such that $U \cap A \neq \emptyset$, then $A \subseteq X - U$ and $X - U$ is $\theta gp - closed$. Therefore $\theta gpCl(A) \subseteq X - U$, which implies $x \notin \theta gpCl(A)$, a contradiction.

Conversely, suppose that $x \notin \theta gpCl(A)$. Then there exists a $\theta gp - closed$ set F containing A such that $x \notin F$. Hence F^c is a $\theta gp - open$ set containing x. Therefore $F^c \cap A = \emptyset$, which contradicts the hypothesis.

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