ISSN: 2278-0181
RACMS-2014 Conference Proceedings

On Λ_μ-Generalized Closed Sets in Generalized Topological Spaces

[1]M. Jeyaraman,
[1] Assistant Professor, Department of Mathematics,
RDM Government Arts College,
Sivagangai.

[2]S. Chinthamani Vasthirani,
[2]Assosiate Professor, Department of Mathematics,
Sree Sevugan Annamalai College,
Devakottai.

***Abstract* – We introduce new classes of sets called $\Lambda_{\mu\text{-}g}$-closed sets and $\Lambda_{\mu\text{-}g}$-open sets in GTS. We also investigate several properties of such sets. It turns out that $\Lambda_{\mu\text{-}g}$-closed sets and $\Lambda_{\mu\text{-}g}$-open sets are weaker forms of closed sets and open sets, and stronger forms of g-μ-closed sets and g-μ-open sets, respectively.**
2010 Mathematics Subject Classification: 57C10, 57C08, 57C05

Keywords and phrases: $\Lambda_{\mu\text{-}g}$-closed set, locally closed sets, g-μ-closed sets.

1. INTRODUCTION

In 2010 E. Ekici and B. Roy introduced the notion of Λ_μ-sets in GTS. A Λ_μ-set is a set A which is equal to its kernel. In 2011 Bishwabhar and Ekici introduced and investigate (Λ, μ)-closed sets by involving Λ_μ-sets and μ-closed sets. The objective of this paper is to introduce new classes of sets called $\Lambda_{\mu\text{-}g}$-closed sets and $\Lambda_{\mu\text{-}g}$-open sets in GTS. It turns out that $\Lambda_{\mu\text{-}g}$-closed sets and $\Lambda_{\mu\text{-}g}$-open sets are weaker forms of closed sets and open sets, and stronger forms of g-μ-closed sets and g-μ-open sets, respectively.

We recall some notion defined in [1, 2]. Let X be a non-empty set and μ be a collection of subsets of X. Then μ is called a generalized topology(briefly GT) on X if $\phi \in \mu$ and $G_i \in \mu$ for $i \in I \neq \phi$ implies $G = \cup_{i\epsilon I} G_i \in \mu$. We say μ is strong if $X \in \mu$, and we call the pair (X, μ) a generalized topological space (briefly GTS) on X. The elements of μ are called μ-open sets and their complements are called μ-closed sets. For $A \subseteq X$, we denote by $c_\mu(A)$ the intersection of all μ-closed sets containing A, i.e., the smallest μ-closed set containing A; and by $i_\mu(A)$ the union of all μ-open sets contained in A, i.e., the largest μ-open set contained in A.

Definition: 1.1[3]

Let (X, μ) be a GTS and $A \subseteq X$. Then the subset $\Lambda_\mu(A)$ is defined by

$$\Lambda_\mu(A) = \begin{cases} \cap\{G : A \subseteq G, G \in \mu\}, & if\ \exists\ G\ \in\ \mu\ such\ that\ A \subseteq G, \\ X, & otherwise. \end{cases}$$

Definition: 1.2[3]

In a GTS (X, μ) a subset A is called a Λ_μ-set if $A = \Lambda_\mu(A)$.

Definition: 1.3[3]

A subset A of a GTS (X, μ) is called (Λ, μ)-closed set if $A = L \cap D$, where L is a Λ_μ-sets and D is a μ-closed set.

The intersection of all (Λ, μ)-closed sets containing a subset A of X is called the (Λ, μ)-closure of A and is denoted by $c_{(\Lambda,\mu)}(A)$. The complement of (Λ, μ)-closed set is called (Λ, μ)-open set. We denote the collection of (Λ, μ)-open sets (resp.(Λ, μ)-closed sets) by Λ_μ-O(X, μ) (resp. Λ_μ-C(X, μ)).

Lemma:1.1 [4]

For subsets A_i ($i \in I$) of a GTS (X, μ), the following properties hold:

(i). If A_i is (Λ, μ)-closed for each $i \in I$, then $\cap_{i\epsilon I} A_i$ is (Λ, μ)-closed.

(ii). If A_i is (Λ, μ)-open for each $i \in I$, then $\cup_{i\epsilon I} A_i$ is (Λ, μ)-open.

2. Λ_μ-GENERALIZED CLOSED SETS

Definition: 2.1

A subset A of a generalized topological space (X, μ) is called Λ_μ-generalized closed, briefly $\Lambda_{\mu\text{-}g}$-closed, if $c_\mu(A) \subseteq U$ whenever $A \subseteq U$ and U is (Λ, μ)-open.

Definition: 2.2

A subset A of a generalized topological space (X, μ) is called Λ_μ-g-closed, if $c_{(\Lambda,\mu)}(A) \subseteq U$ whenever $A \subseteq U$ and U is (Λ, μ)-open.

Definition: 2.3

A subset A of a generalized topological space (X, μ) is called gΛ_μ-closed, if $c_{(\Lambda,\mu)}(A) \subseteq U$ whenever $A \subseteq U$ and U is μ-open.

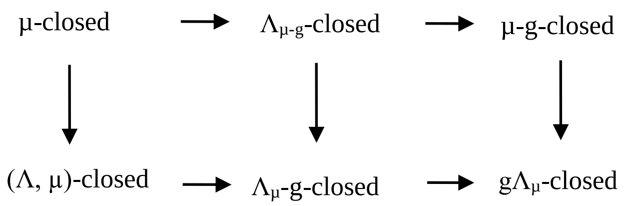
Remark: 2.4

From above definitions, we have the following diagram

(i). $\Lambda_{\mu\text{-}g}$-closed sets and (Λ, μ)-closed sets are independent concepts.

(ii). Λ_μ-g-closed sets and μ-g-closed sets are independent concepts.

(iii). (Λ, μ)-closed sets and μ-g-closed sets are also independent concepts



Example: 2.5;

Let X = {a, b, c,} and μ = {φ,{a},{a, b},{b, c},X}. Thus Λ_μ-O(X, μ) = {φ,{a},{c},{a, b},{b, c},{a, c},X}. Take A = {a, c}. Then we obtained that A is a μ-g-closed set but it is not $\Lambda_{\mu\text{-}g}$-closed.

Let X = {a, b, c,} and μ = {φ,{a},{a, b},{b, c},X}. Then A = {b} is a (Λ, μ)-closed set but it is not μ-g-closed.

Let X = {a, b, c,} and μ = {φ, {a}, X}. Then A = {a, b} is $\Lambda_{\mu\text{-}g}$-closed set but it is not (Λ, μ)-closed

Remark 2.6

The union of two $\Lambda_{\mu\text{-}g}$-closed sets need not be $\Lambda_{\mu\text{-}g}$-closed as can be verified from the following example

Example: 2.7

Let X = {a, b, c} and μ = {φ,{a},{a, b},{b, c},X}. Now put A = {a} and B = {c} are two $\Lambda_{\mu\text{-}g}$-closed sets. But A ∪ B = {a, c} is not a $\Lambda_{\mu\text{-}g}$-closed set.

Remark: 2.8

The intersection of two $\Lambda_{\mu\text{-}g}$-closed sets need not be $\Lambda_{\mu\text{-}g}$-closed as can be verified from the following lemma

Example: 2.9

Let X = {a, b, c} and μ = {φ, {a}}. Now put A = {a} and B = {b} are two $\Lambda_{\mu\text{-}g}$-closed. But $A \cap B$ = φ is not a $\Lambda_{\mu\text{-}g}$-closed set.

Theorem: 2.10

A subset A of GTS (X, μ) is $\Lambda_{\mu\text{-}g}$-closed, then $c_\mu(A) \setminus A$ contains no non empty μ-closed subset of (X, μ).

Proof.

Let F be a μ-closed subset contained in $c_\mu(A) \setminus A$. Clearly $A \subseteq F^c$ where A is $\Lambda_{\mu\text{-}g}$-closed and F^c is an μ-open set of X. Thus $c_\mu(A) \subseteq F^c$ or $F \subseteq (c_\mu(A))^c$. Then $F \subseteq (c_\mu(A))^c \cap (c_\mu(A) \setminus A) \subseteq (c_\mu(A))^c \cap c_\mu(A) = \varphi$. This shows that F = φ.

The converse of the above theorem is not true in general as it is shown in the following example.

Example: 2.11

Let X = {a, b, c,} and μ = {φ,{a},{a, b},{b, c},X}. If A = {a, c}, then $c_\mu(A) \setminus A$ = {b} does not contain a non-empty μ-closed set. But A is not $\Lambda_{\mu\text{-}g}$-closed in (X, μ).

Corollary: 2.12

In a T_1 space, every $\Lambda_{\mu\text{-}g}$-closed set is μ-closed.

Proof.

Let A be a $\Lambda_{\mu\text{-}g}$-closed set in a T_1 space (X, μ). Let $x \in c_\mu(A) \setminus A$. Since (X, μ) is T_1, {x} is a μ- closed set in (X, μ). By theorem 2.10, there exists no non empty μ-closed set in $c_\mu(A) \setminus A$ = φ. Therefore $c_\mu(A)$ = A. Hence A is μ-closed.

Theorem: 2.13

A set A is $\Lambda_{\mu\text{-}g}$-closed if and only if $c_\mu(A) \setminus A$ contains no non empty (Λ, μ)-closed sets.

Proof.

Necessity: Suppose that A is $\Lambda_{\mu\text{-}g}$-closed. Let S be a (Λ, μ)-closed subset of $c_\mu(A) \setminus A$. Then $A \subseteq S^c$. Since A is $\Lambda_{\mu\text{-}g}$-closed, we have $c_\mu(A) \subseteq S^c$ consequently $S \subseteq (c_\mu(A))^c$. Hence $S \subseteq c_\mu(A) \cap (c_\mu(A))^c = \varphi$. Therefore S is empty.

Sufficiency: Suppose that $c_\mu(A) \setminus A$ contains no nonempty (Λ, μ)-closed sets. Let $A \subseteq G$ and G be (Λ, μ)-open. If $c_\mu(A) \not\subseteq G$, then $c_\mu(A) \cap G^C$ is a nonempty (Λ, μ)-closed subset of $c_\mu(A) \setminus A$. Therefore, A is $\Lambda_{\mu\text{-}g}$-closed.

Theorem: 2.14

If A is a $\Lambda_{\mu\text{-}g}$-closed set of (X, μ) and $A \subseteq B \subseteq c_\mu(A)$, then B is a $\Lambda_{\mu\text{-}g}$-closed set in (X, μ).

Proof.

Let $A \subseteq B$ and $c_\mu(A) \subseteq c_\mu(B)$. Hence $c_\mu(B) \setminus B \subseteq c_\mu(A) \setminus A$. But by the theorem 2.13, $c_\mu(A) \setminus A$ contains no non empty (Λ, μ)-closed subsets of X and hence neither $c_\mu(B) \setminus B$. Again by theorem 2.13, B is a $\Lambda_{\mu\text{-}g}$-closed set.

Theorem: 2.15

If A is (Λ, μ)-open and $\Lambda_{\mu\text{-}g}$-closed set in (X, μ), then A is μ-closed in (X, μ).

Proof.

Since A is (Λ, μ)-open and $\Lambda_{\mu\text{-}g}$-closed, $c_\mu(A) \subseteq A$ and hence A is μ-closed.

Theorem: 2.16

For each $\{x\} \in X$, either {x} is (Λ, μ)-closed or $\{x\}^c$ is $\Lambda_{\mu\text{-}g}$-closed in (X, μ).

Proof.

Suppose {x} is not (Λ, μ)-closed in(X, μ) then$\{x\}^c$ is not (Λ, μ)-open and the only (Λ, μ)-open set containing$\{x\}^c$ is the space X itself. Therefore $c_\mu(\{x\}^c) \subseteq X$ and so $\{x\}^c$ is $\Lambda_{\mu\text{-}g}$-closed in (X, μ).

Definition: 2.17

A subset S of X is said to be locally-μ- closed if $S = U \cap F$, where U is μ-open F is μ-closed in (X, μ).

Theorem: 2.18.

Let A be locally-μ- closed subset of (X, μ). For the set A the following properties are equivalent:

(i). A is μ-closed;

(ii). A is $\Lambda_{\mu\text{-}g}$-closed;

(iii). A is μ-g-closed.

Proof.

By Remark 2.4, it suffices to prove that (iii) implies (i). By $A \cup (c_\mu(A))^c$ is μ-open in (X, μ) since A is locally-μ-closed. Now $A \cup (c_\mu(A))^c$ is an μ-open set of (X, μ) such that $A \subseteq A \cup (c_\mu(A))^c$. Since A is μ-g-closed, then $c_\mu(A) \subseteq A \cup (c_\mu(A))^c$. But $c_\mu(A) \cap (c_\mu(A))^c = \varphi$. Thus we have $c_\mu(A) \subseteq A$ and hence A is μ-closed.

Definition: 2.19

A subset A in (X, μ) is said to be $\Lambda_{\mu\text{-}g}$-open in (X, μ) if and only if A^c is $\Lambda_{\mu\text{-}g}$-closed.

Theorem: 2.20

A set A is said to be $\Lambda_{\mu\text{-}g}$-open in (X, μ) if and only if $F \subseteq i_\mu(A)$ whenever F is (Λ, μ)-closed in (X, μ) and $F \subseteq A$.

Proof.

Suppose that $F \subseteq i_\mu(A)$ whenever F is Λ, μ)-closed and $F \subseteq A$. Let $A^c \subseteq G$, where G is(Λ, μ)-open. Hence $G^c \subseteq A$. By assumption $G^c \subseteq i_\mu(A)$ which implies that $(i_\mu(A))^c \subseteq G$, so $c_\mu(A^c) \subseteq G$. Hence A^c is $\Lambda_{\mu\text{-}g}$-closed. i.e., A is $\Lambda_{\mu\text{-}g}$-open

Conversely, let A be $\Lambda_{\mu\text{-}g}$-open. Then A^c is $\Lambda_{\mu\text{-}g}$-closed. Also let F be a (Λ, μ)-closed set contained in A. Then F^c is (Λ, μ)-open. Therefore whenever $A^c \subseteq F^c$, $c_\mu(A^c) \subseteq F^c$. This implies that $F \subseteq (c_\mu(A^c))^c = i_\mu(A)$. Thus $F \subseteq i_\mu(A)$.

Theorem: 2.21

A set A is said to be $\Lambda_{\mu\text{-}g}$-open in (X, μ) if and only if G = X whenever G is (Λ, μ)-open and $i_\mu(A) \cup A^c \subseteq G$.

Proof.

Let A be $\Lambda_{\mu\text{-}g}$-open, G be (Λ, μ)-open and $i_\mu(A) \cup A^c \subseteq G$. This gives $G^c \subseteq (i_\mu(A))^c \cap (A^c)^c = (i_\mu(A))^c \setminus A^c = c_\mu(A^c) \setminus A^c$. Since A^c is $\Lambda_{\mu\text{-}g}$-closed and G^c is (Λ, μ)-closed, by theorem 2.13, it follows that $G^c = \varphi$. Therefore X = G. Conversely, suppose that F is (Λ, μ)-closed and $F \subseteq A$. Then $i_\mu(A) \cup A^c \subseteq i_\mu(A) \cup F^c$. It follows that $i_\mu(A) \cup F^c = X$ and hence $F \subseteq i_\mu(A)$. Therefore A is $\Lambda_{\mu\text{-}g}$-open.

Theorem: 2.22

If $i_\mu(A) \subseteq B \subseteq A$ and A is $\Lambda_{\mu\text{-}g}$-open in (X, μ), then B is $\Lambda_{\mu\text{-}g}$-open in (X, μ).

Proof.

Suppose $i_\mu(A) \subseteq B \subseteq A$ and A is $\Lambda_{\mu\text{-}g}$-open in (X, μ). Then $A^c \subseteq B^c \subseteq c_\mu(A^c)$ and A^c is $\Lambda_{\mu\text{-}g}$- closed.

Theorem: 2.23

A set A is $\Lambda_{\mu\text{-}g}$-closed in (X, μ) if and only if $c_\mu(A) \setminus A$ is $\Lambda_{\mu\text{-}g}$-open in (X, μ).

Proof.

Necessity: Suppose that A is $\Lambda_{\mu\text{-}g}$-closed in (X, μ). Let $F \subseteq c_\mu(A) \setminus A$, where F is (Λ, μ)-closed. By theorem 2.10, $F = \varphi$. Therefore $F \subseteq i_\mu(c_\mu(A) \setminus A)$ and by theorem 2.20, $c_\mu(A) \setminus A$ is $\Lambda_{\mu\text{-}g}$-open in (X, μ).

Sufficiency: Let $A \subseteq G$, where G is (Λ, μ)-open set. Then $c_\mu(A) \cap G^c \subseteq c_\mu(A) \cap A^c = c_\mu(A) \setminus A$. Since $c_\mu(A) \cap G^c$ is (Λ, μ)-closed and $c_\mu(A) \setminus A$ is $\Lambda_{\mu\text{-}g}$-open in by theorem 2.20, we have $c_\mu(A) \cap G^c \subseteq i_\mu(c_\mu(A) \setminus A) = \varphi$. Hence A is $\Lambda_{\mu\text{-}g}$-closed in (X, μ).

Theorem: 2.24

For the subset $A \subseteq X$ the following properties are equivalent:

(i). A is $\Lambda_{\mu\text{-}g}$-closed;

(ii). $c_\mu(A) \setminus A$ contains no nonempty is (Λ, μ)-closed set;

(iii). $c_\mu(A) \setminus A$ is $\Lambda_{\mu\text{-}g}$-open

Proof.

This follows from the theorem 2.10 and 2.23

REFERENES

[1] A.Császár, *Generalized open sets*, Acta Math. Hungar., 75(1-2)(1997), 65-87.

[2] A.Császár, *Generalized open sets in generalized topologies*, Acta Math. Hungar., 106 (2005), 53-66.

[3] B. Roy and Erdal Ekici, On *(Λ, μ)-closed sets in generalized topological spaces*, Method of Functional Analysis and Topology, 17(2) (2011), 174-179.

[4] E. Ekici and B. Roy, *New generalized topologies on generalized topological spaces due to Császár*, Acta Math. Hungar., 132(1-2) (2011), 117-124.

[5] B. Roy, *On a type of generalized open sets*, Applied General topology,12 (2) (2011), 163-173.

[6] B. Roy, On generalized of R0 and R1 spaces, Acta Math. Hungar., 127(2010), 291-300.

[7] M. Caldas, S. Jafari and T. Noiri, On Λ- generalized closed sets in topological spaces, Acta. Math. Hungar., 118(4) (2008), 337 – 343.