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On ψg* - Closed Sets in Topological Spaces

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Abstract **- In this paper, we introduce a new class of sets called ψg*-closed sets in topological spaces. A subset A of X is said to be ψg*-closed if ψCl(A) ⊆ U whenever A ⊆ U and U is g*-open in X. Also we study some of its basic properties and investigate the relationship with other existing closed sets in topological space. As an application, we introduce four new spaces namely $T_{\psi g^*}$-space and $_gT_{\psi g^*}$-space**

Keywords: **g*-open sets, ψ-closure, ψ-closed sets, ψg*-closed sets.**

1. INTRODUCTION

Levine [8] introduced generalized closed sets (briefly g-closed sets) in topological spaces and studied their basic properties. Veerakumar [17] introduced and studied ψ-closed sets. Veerakumar [16] introduced g*-closed sets in topological spaces and studied their properties. The aim of this paper is to introduce a new class of generalized closed sets called ψg*-closed sets. Applying these sets, we obtain four new spaces namely $T_{\psi g^*}$-space and $_gT_{\psi g^*}$-space.

2. PRELIMINARIES

Throughout this paper (X, τ) (or simply X) represents topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of (X,τ), Cl(A), Int(A) and A^c denote the closure of A, interior of A and the complement of A respectively. We are giving some definitions.

Definition 2.1: A subset A of a topological space (X,τ) is called

1. a semi-open set[9] if A ⊆ Cl(Int(A)).
2. an α-open set[11] if A ⊆ Int(Cl(Int(A))).
3. a regular open set[15] if A = Int(Cl(A)).
4. an semi pre-open set[1] if A ⊆ Cl(Int(Cl(A)).

The complement of a semi–open (resp.α–open, regular-open and semi pre-open) set is called semi-closed (resp.α–closed, regular-closed,semi pre-closed) set.

The intersection of all semi-closed (resp.α-closed, regular-closed and semi pre-closed) sets of X containing A is called the semi-closure (resp.α-closure, regular closure and semi pre-closure) of A and is denoted by sCl(A) (resp.αCl(A), rCl(A) and spCl(A)). The family of all semi-open (resp. α-open, regular-open and semi pre-open) subsets of a space X is denoted by SO(X) (resp. αO(X), rO(X) and spO(X)).

Definition 2.2: A subset A of a topological space (X,τ) is called

1) a generalized closed set (briefly g-closed)[8] if Cl(A) ⊆ U whenever A ⊆ U and U is open in X.
2) a sg-closed set[3] if sCl(A) ⊆ U whenever A ⊆ U and U is semi-open in X.
3) a gs-closed set[2] if sCl(A) ⊆ U whenever A ⊆ U and U is open in X.
4) a αg-closed set[10] if αCl(A) ⊆ U whenever A ⊆ U and U is open in X.
5) a gr*-closed set[7] if rCl(A) ⊆ U whenever A ⊆ U and U is g-open in X.
6) a g*-closed set[16] if Cl(A) ⊆ U whenever A ⊆ U and U is g-open in X.
7) a g**-closed set[12] if Cl(A) ⊆ U whenever A ⊆ U and U is g*-open in X.
8) a g*s-closed set[13] if sCl(A) ⊆ U whenever A ⊆ U and U is gs-open in X.
9) a (gs)*-closed set[6] if Cl(A) ⊆ U whenever A ⊆ U and U is gs-open in X.
10) a gsp-closed set[5] if spCl(A) ⊆ U whenever A ⊆ U and U is open in X.
11) a ψ-closed set[17] if sCl(A) ⊆ U whenever A ⊆ U and U is sg-open in X.
12) a ψg-closed set [14]if ψCl(A) ⊆ U whenever A ⊆ U and U is open in X.

The complement of a g-closed (resp. sg-closed, gs-closed, αg-closed, gr*–closed, g*-closed, g**-closed, g*s-closed, (gs)*-closed, gsp-closed, ψ-closed and ψg-closed) set is called g-open (resp. sg-open, gs-open, αg-open, gr*–open, g*-open, g**-open, g*s-open, (gs)*-open, gsp-open, ψ-open and ψg-open) set.

Definition 2.3: ψCl(A) is defined as the intersection of all ψ-closed sets containing A.

Definition 2.4: A space (X,τ) is called a

i. a $T_{1/2}$ – space[8] if every g-closed set in X is closed.
ii. a $T^*_{1/2}$ – space[16] if every g*-closed set in X is closed.
iii. a $_\alpha T_b$ – space[4] if every αg-closed set in X is closed.

Remark 2.5: r-closed(r-open) → closed(open) → α-closed(α-open) → semi-closed(semi-open) → ψ-closed(ψ-open) → semi pre-closed(semi pre-open)

3. ψg^*-CLOSED SETS

We introduce the following definition.

Definition 3.1: A subset A of a topological space (X,τ) is called a ψg^*-closed set if $\psi Cl(A) \subseteq U$ whenever $A \subseteq U$ and U is g*-open in X. The family of all ψg^*-closed sets of X are denoted by ψg^*-C(X).

Definition 3.2: The complement of a ψg^*-closed set is called ψg^*-open set. The family of all ψg^*-open sets of X are denoted by ψg^*-O(X).

Example 3.3: Let X = {a,b,c} and τ = {X,ϕ,{a},{b},{a,b},{a,c}} then {X,ϕ,{b},{c},{a,c},{b,c}} are ψg^*-closed sets and {X,ϕ,{b},{a},{a,c},{a,b}} are ψg^*-open sets in X.

Proposition 3.4: Every closed set is ψg^*-closed set.
Proof: Let A be any closed set in X and U be any g*-open set in X such that $A \subseteq U$. Since A is closed, Cl(A) = A for every subset A of X. By Remark 2.5, every closed set is ψ-closed, $\psi Cl(A) \subseteq Cl(A) = A \subseteq U$. Therefore, $\psi Cl(A) \subseteq U$ where U is g* open. Hence, A is ψg^*-closed set.

The following example shows that the converse of the above proposition need not be true.

Example 3.5: Let X = {a,b,c} and τ = {X, ϕ,{a}}. ψg^*-C(X) = {X,ϕ,{b},{c},{a,b},{b,c},{a,c}}. Here, {b},{c},{a,b},{a,c} are ψg^*-closed sets but not closed sets in X.

Proposition 3.6: Every semi-closed set is ψg^*-closed set.
Proof: Let A be any semi-closed set in X such that $A \subseteq U$ where U is g*-open. Since A is semi-closed, sCl(A) = A. By Remark 2.5, every semi-closed set is ψ-closed, $\psi Cl(A) \subseteq sCl(A) = A \subseteq U$. Therefore, $\psi Cl(A) \subseteq U$ where U is g* open. Hence, A is ψg^*-closed set.

The converse of the above proposition need not be true as shown in the following example.

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Example 3.7: Let X = {a,b,c} and τ = {X, ϕ,{a},{a,b}}. s-C(X) = {X,ϕ,{b},{c},{b,c}} and ψg^*-C(X) = {X, ϕ,{b},{c},{b,c},{a,c}}. Here, {a,c} is ψg^*-closed set but not semi-closed set in X.

Proposition 3.8: Every α-closed set is ψg^*-closed set.
Proof: The proof follows from the result that every α-closed set is semi-closed and by proposition 3.6.

The converse of the above proposition need not be true as shown in the following example.

Example 3.9: Let X = {a,b,c} and τ = {X,ϕ,{b},{a,c}}. α-C(X) = {X,ϕ,{b},{a,c}} and ψg^*-C(X) = {X,ϕ,{a},{b},{c},{a,b},{b,c},{a,c}}. Here, {a},{c},{a,b},{b,c} are ψg^*-closed sets but not α-closed sets in X.

Proposition 3.10: Every regular closed set is ψg^*-closed set.
Proof: The proof follows from the result that every regular closed set is closed and by proposition 3.4.

The reverse implication does not hold as shown in the following example.

Example 3.11: Let X = {a,b,c} and τ = {X, ϕ,{b}}. r-C(X) = {X,ϕ} and ψg^*-C(X) = {X,ϕ,{a},{c},{a,b},{b,c},{a,c}}. Here, {a},{c},{a,b},{b,c},{a,c} are ψg^*-closed sets but not regular closed sets in X.

Proposition 3.12: Every g-closed set is ψg^*-closed set.
Proof: Let A be any g-closed set in X. Let U be any open set in X such that $A \subseteq U$. Since "Every open set is g*-open set", we have $\psi Cl(A) \subseteq Cl(A) \subseteq U$. Therefore, $\psi Cl(A) \subseteq U$ where U is g*-open. Hence, A is ψg^*-closed set.

The reverse implication does not hold as shown in the following example.

Example 3.13: Let X = {a,b,c,d} and τ = {X,ϕ,{d},{a,b},{a,b,d}}. g-C(X) = {X,ϕ,{c},{a,c},{b,c},{c,d},{a,b,c},{b,c,d},{a,c,d}} and ψg^*-C(X) = {X,ϕ,{c},{d},{a,b},{a,c},{b,c},{c,d},{a,b,c},{b,c,d},{a,c,d}}. Here, {d},{a,b} are ψg^*--closed sets but not g-closed sets.

Proposition 3.14: Every αg-closed set is ψg^*-closed set.
Proof: Let A be any αg-closed set in X. Let U be any open set in X such that $A \subseteq U$. Since "Every open set is g*-open set", we have $\psi Cl(A) \subseteq \alpha Cl(A) \subseteq U$. Therefore, $\psi Cl(A) \subseteq U$ where U is g*-open. Hence, A is ψg^*-closed set.

The reverse implication does not hold as shown in the following example.

Example 3.15: Let X = {a,b,c} and τ = {X,ϕ,{a},{b},{a,b}}. αg-C(X) = {X,ϕ,{c},{b,c},{a,c}} and ψg^*-C(X) = {X,ϕ,{a},{b},{c},{b,c},{a,c}}. Here, {a},{b} are ψg^*--closed sets but not αg-closed sets.

Proposition 3.16: Every gr*-closed set is ψg^*-closed set.
Proof: Let A be any gr*-closed set in X. Let U be any g*-open set in X such that $A \subseteq U$. Since "Every g*-open set is g-open set" and A is gr*-closed, $rCl(A) \subseteq U$. For every subset A of X, $\psi Cl(A) \subseteq rCl(A)$ and so $\psi Cl(A) \subseteq U$ where U is g*-open. Hence, A is ψg^*-closed set.

The reverse implication does not hold as shown in the following example.

Example 3.17: Let X = {a,b,c} and τ = {X,ϕ,{b},{a,b},{b,c}}. gr*-C(X) = {X,ϕ,{a,c}} and ψg^*-C(X) = {X,ϕ,{a}, {c},{a,c}}. Here, {a},{c} are ψg^*-closed sets but not gr*-closed sets.

Proposition 3.18: Every g*-closed set is ψg*-closed set.
Proof: Let A be any g*-closed set in X. Let U be any g*-open set in X such that A ⊆ U. Since "Every g*-open set is g-open set" and A is g*-closed, we have ψCl(A) ⊆ U where U is g*-open. Hence, A is ψg*-closed set.

The following example shows that the converse of the above proposition need not be true.

Example 3.19: Let X = {a,b,c} and τ = {X, ϕ,{a}}. g*-C(X) = {X,ϕ,{b,c}} and ψg*-C(X) = {X,ϕ,{b},{c},{a,b},{b,c},{a,c}}. Here, {b},{c}, {a,b},{a,c} are ψg*-closed sets but not g*closed sets in X.

Proposition 3.20: Every g**-closed set is ψg*-closed set.
Proof: Let A be any g**-closed set in X. Let U be any g*-open set in X such that A ⊆ U. Since A is g**-closed, Cl(A) ⊆ U. For every subset A of X, ψCl(A) ⊆ Cl(A) and so ψCl(A) ⊆ U where U is g*-open. Hence, A is ψg*-closed set.

The following example shows that the converse of the above proposition need not be true.

Example 3.21: Let X = {a,b,c} and τ = {X, ϕ,{a},{b},{a,b}}. g**-C(X) = {X,ϕ,{c},{b,c},{a,c}} and ψg*-C(X) = {X,ϕ,{a},{b},{c},{b,c},{a,c}}. Here, {a},{b} are ψg*-closed sets but not g*closed sets in X.

Proposition 3.22: Every g*s-closed set is ψg*-closed set.
Proof: Let A be any g*s-closed set in X. Let U be any g*-open set in X such that A ⊆ U. Since "Every g*-open set is gs-open set" and A is g*s-closed, sCl(A) ⊆ U. For every subset A of X, ψCl(A) ⊆ sCl(A) and so ψCl(A) ⊆ U where U is g*-open. Hence, A is ψg*-closed set.

The converse of the above proposition need not be true as shown in the following example.

Example 3.23: Let X = {a,b,c} and τ = {X, ϕ,{a}}. g*s-C(X) = {X, ϕ,{b},{c},{b,c}} and ψg*-C(X) = {X, ϕ,{b},{c},{a,b},{b,c},{a,c}}. Here, {a,b},{a,c} are ψg*-closed sets but not g*s-closed sets in X.

Proposition 3.24: Every (gs)*-closed set is ψg*-closed set.
Proof: Let A be any (gs)*-closed set in X. Let U be any g*-open set in X such that A ⊆ U. Since "Every g*-open set is gs-open set" and A is (gs)*-closed, Cl(A) ⊆ U. For every subset A of X, ψCl(A) ⊆ Cl(A) and so ψCl(A) ⊆ U where U is g*-open. Hence, A is ψg*-closed set.

The converse of the above proposition need not be true as shown in the following example.

Example 3.25: Let X = {a,b,c} and τ = {X, ϕ,{a}}. (gs)*-C(X) = {X, ϕ,{b,c}} and ψg*-C(X) = {X, ϕ,{b},{c},{a,b},{b,c},{a,c}}. Here, {b},{c},{a,b},{a,c} are ψg*-closed sets but not (gs)*-closed sets in X.

Proposition 3.26: Every ψg*-closed set is gsp-closed set.
Proof: Let A be any ψg*-closed set in X. Let U be any open set such that A ⊆ U. Since, "Every open set is g*-open set", ψCl(A) ⊆ U. For every subset A of X, spCl(A) ⊆ ψCl(A) and so spCl(A) ⊆ U where U is open. Hence, A is gsp-closed set.

The reverse implication does not hold as shown in the following example.

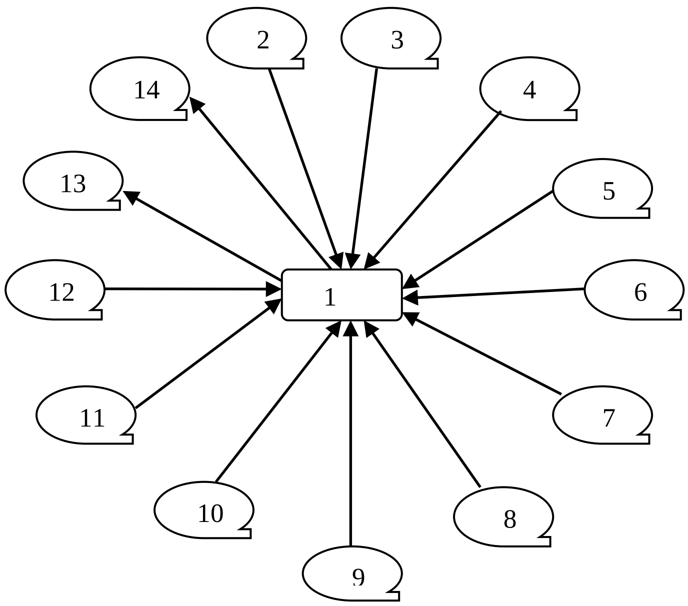
Example 3.27: Let X = {a,b,c,d} and τ = {X,ϕ,{a,c},{a,b,c},{a,c,d}}. gsp-C(X) = {X,ϕ,{a},{b},{c},{d},{a,b},{a,d},{b,c},{b,d},{c,d},{b,c,d},{a,b,d}} and ψg*-C(X) = {X,ϕ,{b},{d},{b,d},{b,c,d},{a,b,d}}. Here, {a},{c},{a,b},{a,d},{b,c},{c,d} are gsp-closed sets but not ψg*-closed sets.

Proposition 3.28: Every ψg*-closed set is ψg-closed set.
Proof: Let A be any ψg*-closed set in X. Let U be any open set such that A ⊆ U. Since, "Every open set is g*-open set", ψCl(A) ⊆ U where U is open. Hence, A is ψg-closed set.

The reverse implication does not hold as shown in the following example.

Example 3.29: Let X = {a,b,c,d} and τ = {X,ϕ,{d},{a,b},{a,b,d}}. ψg-C(X) = {X,ϕ,{a},{b},{c},{d},{a,b},{a,c},{b,c},{c,d},{a,b,c},{b,c,d},{a,c,d}} and ψg*-C(X) = {X,ϕ,{c},{d},{a,b},{a,c},{b,c},{c,d},{a,b,c},{b,c,d},{a,c,d}}. Here, {a},{b} are ψg-closed sets but not ψg*-closed sets.

Remark 3.30: The following diagram shows the relationship of ψg*-closed sets with other known existing sets. A → B represents A implies B but not conversely.



1. ψg*-closed	2. Closed	3. semi-closed
4. α-closed	5. regular-closed	6. g-closed
7. αg-closed	8. gr*-closed	9. g*-closed
10.g**-closed	11.g*s-closed	12.(gs)*-closed
13.gsp-closed	14.ψg-closed.	

4. CHARACTERIZATION

Lemma 4.1: The finite union of ψg*-closed sets need not be ψg*-closed set.

Example 4.2: Let X = {a,b,c} and τ = {X,ϕ,{a},{b},{a,b}}. ψg*-C(X) = {X,ϕ,{a},{b},{c},{b,c},{a,c}}. Here, {a}   {b} = {a,b} is not ψg*-closed set.

Lemma 4.3: The finite intersection of ψg*-closed sets need not be ψg*-closed set.

Example 4.4: Let X = {a,b,c} and τ = {X,ϕ,{a}}. ψg*-C(X) = {X,ϕ,{b},{c},{a,b},{b,c},{a,c}}. Here,   {a,b}   {a,c} = {a} is not ψg*-closed set.

Proposition 4.5: Let A be a ψg*-closed set of X. Then ψCl(A)-A does not contain a non-empty g*-closed set.
Proof: Suppose A is a ψg*-closed set. Let F be a g*-closed set contained in ψCl(A)-A. Now F^c is a g*-open set of X such that A ⊆ F^c . Since A is ψg*-closed, we have ψCl(A) ⊆ F^c. Hence, F ⊆ $(\psi Cl(A))^c$. Also, F ⊆ ψCl(A)-A. Therefore, F ⊆ (ψCl(A)-A) ∩ $(\psi Cl(A))^c$ ⊆ ψCl(A) ∩ $(\psi Cl(A))^c$ = ϕ. Hence, F must be ϕ.

Proposition 4.6: If A is both g*-open and ψg*-closed set of X, then A is ψ-closed.
Proof: Since A is both g*-open and ψg*-closed, we have ψCl(A) ⊆ A. Therefore, A = ψCl(A). Hence, A is ψ-closed.

Proposition 4.7: The intersection of a ψg*-closed set and a ψ-closed set of X is always ψg*-closed set.
Proof: Let A be a ψg*-closed set and B be a ψ-closed set. Since A is ψg*-closed, ψCl(A) ⊆ U whenever A ⊆ U and U is g*-open. Let B be such that A   B ⊆ U where U is g*-open. Now, ψCl(A   B) ⊆ ψCl(A)   ψCl(B) ⊆ U   B ⊆ U. Hence, A   B is ψg*-closed set. Therefore, intersection of any ψg*-closed set and a ψ-closed set of X is always ψg*-closed set.

Proposition 4.8: For x   X, the set X-{x} is ψg*-closed or g*-open.
Proof: Suppose X-{x} is not g*-open then X is the only g*-open set containing X-{x} and ψCl{X-{x}}⊆ X. Hence X-{x} is ψg*-closed in X.

Proposition 4.9: If A is ψg*-closed and A ⊆ B ⊆ ψCl(A), then B is ψg*-closed.
Proof: Let U be a g*-open set of X such that B ⊆ U then A ⊆ U. Since A is ψg*-closed, then ψCl(A) ⊆ U. Now ψCl(B) ⊆ ψCl(A) ⊆ U. Therefore B is ψg*-closed in X.

Proposition 4.10: Let A ⊆ Y ⊆ X and suppose that A is ψg*-closed in X, then A is ψg*-closed relative to Y.
Proof: Given that A ⊆ Y ⊆ X and A is ψg*-closed in X. To show that A is ψg*-closed relative to Y. Let A ⊆ Y   U, where U is g*-open in X. Since A is ψg*-closed, A ⊆ U, implies ψCl(A) ⊆ U. Therefore, Y   ψCl(A) ⊆ Y   U. Thus A is ψg*-closed relative to Y.

Proposition 4.11: Suppose that B ⊆ A ⊆ X, B is ψg*-closed relative to A and that A is both g*-open and ψg*-closed set of X, then B is ψg*-closed relative to X.
Proof: Let B ⊆ G and G be an g*-open set in X. But given that B ⊆ A ⊆ X, therefore B ⊆ A   G. since B is ψg*-closed relative to A, A   ψCl(B)   A   G. Hence A   ψCl(B) ⊆ G. Thus A   (ψCl(B)) $(\psi Cl(B))^c$ ⊆ G ∪ $\psi Cl(B)^c$. Since A is both g*-open and ψg*-closed set in X, by proposition 4.6, A is ψ-closed, we have ψCl(A) = A   G $\psi Cl(B)^c$. Also B ⊆ A implies ψCl(B)   ψCl(A). Thus ψCl(B)   ψCl(A)   G $\psi Cl(B)^c$. Therefore, ψCl(B)   G. Since ψCl(A) is not contained in $(\psi Cl(B))^c$. Thus B is ψg*-closed relative to X.

5. APPLICATIONS

As an applications of ψg*-closed sets, we introduce four new spaces namely, $T_{\psi g^*}$-space and $_gT_{\psi g^*}$-space.

Definition 5.1: A Space (X,τ) is called a $T_{\psi g^*}$-space if every ψg*-closed set in it is closed.

Definition 5.2: A Space (X,τ) is called a $_gT_{\psi g^*}$-space if every ψg*-closed set in it is g-closed.

Definition 5.3: A Space (X,τ) is called a $_{g^*}T_{\psi g^*}$-space if every ψg*-closed set in it is g*-closed.

Definition 5.4: A Space (X,τ) is called a $_{\alpha g}T_{\psi g^*}$-space if every ψg*-closed set in it is αg-closed.

Proposition 5.5: Every $T_{\psi g^*}$-space is $_gT_{\psi g^*}$-space.
Proof: Let (X,τ) be $T_{\psi g^*}$-space. Let A be ψg*-closed set in (X,τ). Since (X, τ) is $T_{\psi g^*}$-space, A is closed. But every closed set is g-closed set. Therefore, A is g-closed. Hence, (X,τ) is $_gT_{\psi g^*}$-space.

The converse of the above proposition need not be true as shown in the following example.

Example 5.6: Let X = {a,b,c} and τ = {X,ϕ,{a,c},{a,b,c},{a,c, d}}
ψg*-C(X) = {X,ϕ,{b},{d},{b,d},{b,c,d},{a,b,d}}
g-C(X) = {X,ϕ,{b},{d},{b,d},{b,c,d},{a,b,d}}
C(X) = {X,ϕ,{b},{d},{b,d}}
Here, (X,τ) is $_gT_{\psi g^*}$-space but not $_gT_{\psi g^*}$-space.

Proposition 5.7: Every $T_{\psi g^*}$-space is $T_{1/2}$-space.
Proof: Let (X,τ) be $T_{\psi g^*}$-space. Let A be g-closed set in (X,τ). By proposition 3.12, A is ψg*-closed set. Since (X,τ) is $T_{\psi g^*}$-space, A is closed. Hence, (X,τ) is $T_{1/2}$-space.

The converse of the above proposition need not be true as shown in the following example.

Example 5.8: Let X = {a,b,c} and τ = {X,ϕ,{a},{b},{a,b}}
ψg*-C(X) = {X,ϕ,{a},{b},{c},{b,c},{a,c}}
g-C(X) = {X,ϕ,{c},{b,c},{a,c}}
C(X) = {X,ϕ,{c},{b,c},{,a,c}}
Here, (X,τ) is $T_{1/2}$-space but not $T_{\psi g^*}$-space.

Proposition 5.9: Every $T_{\psi g^*}$-space is $T^*_{1/2}$-space.
Proof: Let (X,τ) be $T_{\psi g^*}$-space. Let A be g*-closed set in (X,τ). By proposition 3.18, A is ψg*-closed set. Since (X,τ) is $T_{\psi g^*}$-space, A is closed. Hence, (X,τ) is $T^*_{1/2}$-space.

The converse of the above proposition need not be true as shown in the following example.

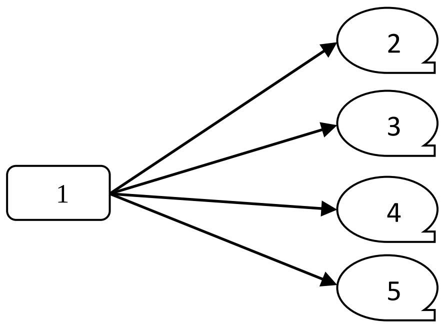
Example 5.10: Let X = {a,b,c} and τ = {X,ϕ,{a},{b},{a,b}}
ψg*-C(X) = {X,ϕ,{a},{b},{c},{b,c},{a,c}}
g*-C(X) = {X,ϕ,{c},{b,c},{a,c}}
C(X) = {X,ϕ,{c},{b,c},{,a,c}}
Here, (X,τ) is $T^*_{1/2}$-space but not $T_{\psi g^*}$-space.

Proposition 5.11: Every $T_{\psi g^*}$-space is ${}_{\alpha}T_b$-space.
Proof: Let (X,τ) be $T_{\psi g^*}$-space. Let A be αg-closed set in (X,τ). By proposition 3.14, A is ψg*-closed set. Since (X,τ) is $T_{\psi g^*}$-space, A is closed. Hence, (X,τ) is ${}_{\alpha}T_b$-space.

The converse of the above proposition need not be true as shown in the following example.

Example 5.12: Let X = {a,b,c} and τ = {X,ϕ,{a},{b},{a,b}}
ψg*-C(X) = {X,ϕ,{a},{b},{c},{b,c},{a,c}}
αg-C(X) = {X,ϕ,{c},{b,c},{a,c}}
C(X) = {X,ϕ,{c},{b,c},{,a,c}}
Here, (X,τ) is ${}_{\alpha}T_b$-space but not $T_{\psi g^*}$-space.

Remark 5.13: The following diagram shows the relationship about $T_{\psi g^*}$-space, ${}_{g}T_{\psi g^*}$-space, ${}_{g^*}T_{\psi g^*}$ and ${}_{\alpha g}T_{\psi g^*}$-space with other known existing spaces. $A \rightarrow B$ represents A implies B but not conversely.



1. $T_{\psi g^*}$-space 2. ${}_{g}T_{\psi g^*}$-space 3. $T_{1/2}$-space
4. $T^*_{1/2}$-space 5. ${}_{\alpha}T_b$-space.

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