Published by :
http://www.ijert.org

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 8 Issue 07, July-2019

On $\bar{g}$-Closed Mappings in Topological Spaces

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***Abstract*:- In this paper we introduce a new class of closed maps namely $\bar{g}$-closed maps which settled in between the class of closed maps and the class of g-closed maps and then we study many basic properties of $\bar{g}$-closed maps together with the relationships of some other maps.**

2000 Mathematics Subject Classification: 54c10, 54c20.
Key words and phrases: $\bar{g}$-closed maps, $\bar{g}$*-closed maps

1. INTRODUCTION

Malghan[22] and Devi et al[8] introduced the concept of generalized closed maps and semi generalized closed maps respectively in topological spaces. Manoj et al[23] introduced the concept of $\bar{g}$-closed sets in topological spaces. In this paper we introduce a new class of closed maps namely $\bar{g}$-closed maps and $\bar{g}$*-closed maps.

2. PRELIMINARIES

Throughout this paper (X, τ), (Y, σ) and (Z, η) represent topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of space (X, τ) the cl(A), int(A) and A^c denote the closure of A, the interior of A and the complement of A in X respectively.

We recall the following definitions:

Definition 2.01: A subset A of a topological space (X, τ) is called g-closed[2] (resp. *g-closed[13], g*-closed[13], **g-closed[17], $\hat{\hat{g}}$-closed[12]) set if cl(A) ⊆ U (resp. cl(A) ⊆ U, cl(A) ⊆ U, scl(A) ⊆ U, cl(A) ⊆ U) whenever A ⊆ U and U is open (resp. $\hat{g}$-open, g-open, $\hat{g}$-open, sg-open) set in (X, τ).

Definition 2.02 : A map f : (X, τ) → (Y, σ) is called g-closed[20] (resp. *g-closed[13], g*-closed[24], **gs-closed[8], $\hat{\hat{g}}$-closed[12]) map if the image of each closed set in (X, τ) is g-closed (resp. *g-closed, g*-closed, **gs-closed, $\hat{\hat{g}}$-closed) in (Y, σ).

3. $\bar{g}$-CLOSED MAPS

In this section we introduce the following definitions.

Definition 3.01: A map f : (X, τ) → (Y, σ) is called $\bar{g}$-closed (resp. $\bar{g}$-open) map if f(A) is $\bar{g}$-closed (resp. $\bar{g}$-open) set in (Y, σ) for every closed (open) set A of (X, τ).

Definition 3.02: Let (X, τ) be a topological space and A ⊆ X. We define the $\bar{g}$-interior of A (briefly $\bar{g}$-int(A)) to be the union of all $\bar{g}$-open sets contained in A.

Theorem 3.03: Every closed map, *g-closed map, g*-closed map and $\hat{\hat{g}}$-closed map is $\bar{g}$-closed map.

Next examples show that the converse of the above theorem is not true in general.

Example 3.04: Let X = Y = {a, b, c}, τ = {ϕ, {c}, {a, c}, {b, c}, X} and σ = {ϕ, {a}, {b, c}, Y}. Define f : (X, τ) → (Y, σ) by f(a) = b, f(b) = a and f(c) = c, then f is not closed map, *g-closed map, g*-closed map and $\hat{\hat{g}}$-closed map however f is $\bar{g}$-closed map.

Theorem 3.05: Every $\bar{g}$-closed map is g-closed map and **gs-closed map.

Example 3.06: Let X = Y = {a, b, c}, τ = {ϕ, {a}, {a, b}, {a, c}, X} and σ = {ϕ, {a}, {a, b}, Y}. Define f : (X, τ) → (Y, σ) by identity mapping, then f is not g-closed map and **gs-closed map however f is $\bar{g}$-closed map.

Therefore the class of $\bar{g}$-closed maps properly contains the class of closed maps, the class of *g-closed maps, the class of $\hat{\hat{g}}$-closed maps and the class of g*-closed maps and properly contained in class of g-closed maps and the class of **gs-closed maps.

Theorem 3.07: If $f : (X, \tau) \to (Y, \sigma)$ be a closed map and $g : (Y, \sigma) \to (Z, \eta)$ be a $\overline{g}$-closed map then their composition $gof : (X, \tau) \to (Z, \eta)$ is $\overline{g}$-closed map.

Remark 3.08: The following example shows that the composition of two $\overline{g}$-closed maps need not be $\overline{g}$-closed map.

Example 3.09: Let $X = \{a, b, c\}$, $\tau = \{\phi, \{a\}, \{b, c\}, X\}$, $\sigma = \{\phi, \{b\}, X\}$ and $\eta = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$. Define $f : (X, \tau) \to (X, \sigma)$ by $f(a) = b$, $f(b) = c$ and $f(c) = a$. Define $g : (X, \sigma) \to (X, \eta)$ by identity mapping then f and g both are $\overline{g}$-closed maps but their composition $gof : (X, \tau) \to (X, \eta)$ is not a $\overline{g}$-closed map.

Theorem 3.10: If $f : (X, \tau) \to (Y, \sigma)$ and $g : (Y, \sigma) \to (Z, \eta)$ be two mappings such that their composition $gof : (X, \tau) \to (Z, \eta)$ be a $\overline{g}$-closed map then the following are true

(i) If f is continuous and surjective, then g is $\overline{g}$-closed map.

(ii) If g is $\overline{g}$-irresolute and injective, then f is $\overline{g}$-closed map.

Theorem 3.11: For any bijective $f : (X, \tau) \to (Y, \sigma)$ the following statements are equivalent.

(i) $f^{-1} : (Y, \sigma) \to (X, \tau)$ is $\overline{g}$-continuous.

(ii) f is $\overline{g}$-open map and

(iii) f is $\overline{g}$-closed map.

Definition 3.12: A map $f : (X, \tau) \to (Y, \sigma)$ is said to be a $\overline{g}$*-closed (resp. $\overline{g}$*-open) if the image f(A) is $\overline{g}$-closed (resp. $\overline{g}$-open) set in (Y, σ) for every $\overline{g}$-closed (resp. $\overline{g}$-open) set A in (X, τ).

Theorem 3.13: Every $\overline{g}$*-closed map is $\overline{g}$-closed map.

The converse is not true in general as it can be seen from the following example.

Example 3.14: Let $X = Y = \{a, b, c\}$, $\tau = \{\phi, \{a\}, \{b, c\}, X\}$ and $\sigma = \{\phi, \{b\}, Y\}$. Define $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = b$, $f(b) = c$ and $f(c) = a$ then f is $\overline{g}$-closed map but not $\overline{g}$*-closed map.

Theorem 3.15: For any bijection $f : (X, \tau) \to (Y, \sigma)$ the following are equivalent

(i) $f^{-1} : (Y, \sigma) \to (X, \tau)$ is $\overline{g}$-irresolute,

(ii) f is a $\overline{g}$*-open map and

(iii) f is a $\overline{g}$*-closed map.

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