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On Contra sbĝ – Continuous functions in Topological Spaces

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Abstract **- In this paper a new class of functions called contra sbĝ-continuous function is introduced and its properties are studied. Some characterization and several properties concerning Contra sbĝ-continuity are obtained. Also, Contra sbĝ-irresolute function and Perfectly Contra sbĝ-irresolute function are introduced.**

Keywords: sbĝ-closed sets, sbĝ-continuous, Contra sbĝ-continuous, Contra sbĝ-irresolute.

AMS Mathematics Subject Classification : 54C08, 54C10.

1. INTRODUCTION

In 1996, Dontchev[7] introduced and investigated a new notion of continuity called contra – continuity. Follwing this, many authors introduced various types of new generalizations of contra continuity called as contra α-continuity, contra semi-continuity[3], contra b-continuity[12], contra sg-continuity[5], contra gs-continuity[5], contra gb-continuity[16], contra g*b-continuity[16], contra bĝ-continuity[15] and so on and they investigated their properties. In 2015, we introduced sbĝ-closed sets[3] in Topological spaces.

In this paper, we introduce and investigate some of the properties of contra sbĝ-continuous, contra sbĝ-irresolute functions and we obtain some of its characterization.

2. PRELIMINARIES

Throughout this paper (X, τ) (or simply X) represents topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of (X,τ), Cl(A), Int(A) and A^c denote the closure of A, interior of A and the complement of A respectively. We are giving some definitions.

Definition 2.1: A subset A of a topological space (X,τ) is called

1. a semi-open set[5] if A ⊆ Cl(Int(A)).
2. an α-open set[8] if A ⊆ Int(Cl(Int(A))).
3. a b-open set[1] if A ⊆ Cl(Int(A)) ∪ Int(Cl(A)).
4. a regular open[14] set if A = Int(Cl(A)).

The complement of a semi–open (resp.α–open, b–open, regular-open) set is called semi-closed (resp.α–closed, b–closed, regular-closed) set.

The intersection of all semi-closed (resp.α-closed, b-closed, regular-closed) sets of X containing A is called the semi-closure (resp.α-closure, b-closure, regular closure) of A and is denoted by sCl(A) (resp.αCl(A), bCl(A), rCl(A)). The family of all semi-open (resp. α-open, b-open, regular-open) subsets of a space X is denoted by SO(X) (resp. αO(X), bO(X), rO(X)).

Definition 2.2: A subset A of a topological space (X,τ) is called a

1) a sg-closed set[5] if sCl(A) ⊆ U whenever A ⊆ U and U is semi-open in X.
2) a gs-closed set[5] if sCl(A) ⊆ U whenever A ⊆ U and U is open in X.
3) a gb-closed set[16] if bCl(A) ⊆ U whenever A ⊆ U and U is open in X.
4) a g*b-closed set[16] if bCl(A) ⊆ U whenever A ⊆ U and U is g-open in X.
5) a bĝ-closed set[15] if bCl(A) ⊆ U whenever A ⊆ U and U is ĝ-open in X.
6) a sbĝ-closed set[3] if sCl(A) ⊆ U whenever A ⊆ U and U is bĝ-open in X.

The complement of a sg-closed (resp. gs-closed, gb-closed, g*b–closed and bĝ-closed) set is called sg-open (resp. gs-open, gb-open, g*b–open and bĝ-open) set.

Definition 2.3: A space (X,τ) is called a

i. $T_{sb\hat{g}}$ – space[3] if every sbĝ-closed set in X is closed.
ii. $T^{\alpha}_{sb\hat{g}}$ – space[3] if every sbĝ-closed set in X is α-closed.

Definition 2.4: A function f: (X,τ) → (Y,σ) is called a

i. sbĝ – continuous map[4] if f^{-1}(V) is sbĝ-closed in (X,τ) for every closed set V in (Y,σ).
ii. sbĝ – irresolute map[4] if f^{-1}(V) is sbĝ-closed in (X,τ) for every closed set V in (Y,σ).

Definition 2.5: A function f: (X,τ) → (Y,σ) is called a

i. Contra continuous map[7] if f^{-1}(V) is closed in (X,τ) for every open set V in (Y,σ).

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Vol. 5 Issue 02, February-2016

ii. Contra semi-continuous map[6] if f^{-1}(V) is semi-closed in (X,τ) for every open st V in (Y,σ).
iii. Contra α-continuous map[8] if f^{-1}(V) is α-closed in (X,τ) for every open set V in (Y,σ).
iv. Contra b-continuous map[15] if f^{-1}(V) is b-closed in (X,τ) for every open set V in (Y,σ).
v. Contra sg-continuous map[5] if f^{-1}(V) is sg-closed in (X,τ) for every open set V in (Y,σ).
vi. Contra gs-continuous map[5] if f^{-1}(V) is gs-closed in (X,τ) for every open set V in (Y,σ).
vii. Contra gb-continuous map[16] if f^{-1}(V) is gb-closed in (X,τ) for every open set V in (Y,σ).
viii. Contra g*b-continuous map[16] if f^{-1}(V) is g*b-closed in (X,τ) for every open set V in (Y,σ).
ix. Contra bĝ-continuous map[15] if f^{-1}(V) is bĝ-closed in (X,τ) for every open set V in (Y,σ).

Definition 2.6:[15] A space (X, τ) is said to be locally indiscrete if every open subset of X is closed in X.

Definition 2.7:[5] A topological space (X,τ) is said to be Urysohn space if for each pair of distinct points x and y in X, there exists two open sets U and V in X such that x ∈ U, y ∈ V and Cl(U) ∩ Cl(V) = ϕ.

Definition 2.8:[5] For a map f : X → Y, the subset {(x,f(x)) : x ∈ X} ⊂ X × Y is called the graph of f and is denoted by G(f).

3. CONTRA sbĝ-CONTINUOUS FUNCTIONS

We introduce the following definition.
Definition 3.1: A function f: (X,τ) → (Y,σ) is called Contra sbĝ-continuous if f^{-1}(V) is sbĝ-closed in (X,τ) for every open set V in (Y,σ).

Example 3.2: Let X = Y = {a,b,c} with topologies
τ = {X,ϕ,{a},{b},{a,b} and σ = {Y,ϕ,{a},{a,b}}
Define a function f: (X,τ) → (Y,σ) by f(a) = a, f(b) = c, f(c) = b
sbĝ – C(X) = {X,ϕ,{a},{b},{c},{a,c},{b,c}}
Here, the inverse image of open sets {a} and {a,b} in Y are {a} and {a,c} respectively which are sbĝ-closed sets in X. Hence, f is contra sbĝ-continuous.

Theorem 3.3:
a) Every contra continuous function is contra sbĝ-continuous function.
b) Every contra α-continuous function is contra sbĝ-continuous function.

Proof:
a) Let V be any open set in (Y,σ). Since f is contra continuous, f^{-1}(V) is closed in (X,τ). By Proposition 3.4 in [3], f^{-1}(V) is sbĝ-closed in (X,τ). Hence, f is contra sbĝ-continuous function.
b) Let V be any open set in (Y,σ). Since f is contra α-continuous, f^{-1}(V) is α-closed in (X,τ). By Proposition 3.7 in [3], f^{-1}(V) is sbĝ-closed in (X,τ). Hence, f is contra sbĝ-continuous function.

The following examples show that the converse of the above proposition need not be true.

Example 3.4:
a) Let X = Y = {a,b,c} with topologies
τ = {X, ϕ, {a},{b},{a,b}} and σ = {Y, ϕ, {a},{b,c}}
Define a function f: (X,τ) → (Y,σ) by f(a) = a, f(b) = b, f(c) = c.
sbĝ – C(X) = {X,ϕ,{a},{b},{c},{a,c},{b,c}}
C(X) = {X,ϕ,{c},{a,c},{b,c}}
Here the inverse image of an open set {a} in (Y,σ) is {a} which is sbĝ-closed but not closed in (X,τ). Hence, f is contra sbĝ-continuous but not contra continuous.
b) Let X = Y = {a,b,c} with topologies
τ = {X, ϕ, {a},{c},{a,c},{b,c}} and σ = {Y, ϕ, {b}}
Define a function f: (X,τ) → (Y,σ) by f(a) = a, f(b) = b, f(c) = c.
sbĝ – C(X) = {X,ϕ,{a},{b},{a,b},{b,c}}
α - C(X) = {X,ϕ,{a},{a,b},{b,c}}
Here the inverse image of an open set {b} in (Y,σ) is {b} which is sbĝ-closed but not α-closed in (X,τ). Hence, f is contra sbĝ-continuous but not contra α-continuous.

Theorem 3.5:
a) Every contra sbĝ-continuous function is contra b-continuous function
b) Every contra sbĝ-continuous function is contra sg-continuous function
c) Every contra sbĝ-continuous function is contra gs-continuous function
d) Every contra sbĝ-continuous function is contra gb-continuous function
e) Every contra sbĝ-continuous function is contra g*b-continuous function
f) Every contra sbĝ-continuous function is contra bĝ-continuous function.

Proof:
a) Let V be any open set in (Y,σ). Since f is contra sbĝ-continuous, f^{-1}(V) is sbĝ-closed in (X,τ). By Proposition 3.11 in [3], f^{-1}(V) is b-closed in (X,τ). Hence, f is contra b-continuous function.
b) Let V be any open set in (Y,σ). Since f is contra sbĝ-continuous, f^{-1}(V) is sbĝ-closed in (X,τ). By Proposition 3.13 in [3], f^{-1}(V) is sg-closed in (X,τ). Hence, f is contra sg-continuous function.
c) Let V be any open set in (Y,σ). Since f is contra sbĝ-continuous, f^{-1}(V) is sbĝ-closed in (X,τ). By Proposition 3.15 in [3], f^{-1}(V) is gs-closed in (X,τ). Hence, f is contra gs-continuous function.
d) Let V be any open set in (Y,σ). Since f is contra sbĝ-continuous, f^{-1}(V) is sbĝ-closed in (X,τ). By

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Vol. 5 Issue 02, February-2016

Proposition 3.17 in [3], f^{-1}(V) is gb-closed in (X,τ). Hence, f is contra gb-continuous function.

e) Let V be any open set in (Y,σ). Since f is contra sbĝ-continuous, f^{-1}(V) is sbĝ-closed in (X,τ). By Proposition 3.21in [3], f^{-1}(V) is g*b-closed in (X,τ). Hence, f is contra g*b-continuous function.

f) Let V be any open set in (Y,σ). Since f is contra sbĝ-continuous, f^{-1}(V) is sbĝ-closed in (X,τ). By Proposition 3.23 in [3], f^{-1}(V) is bĝ-closed in (X,τ). Hence, f is contra bĝ-continuous function.

The converse of the above theorem need not be true as shown in the following example.

Example 3.6:

a) Let X = Y = {a,b,c} with topologies
τ = {X, ϕ, {a,b},{c}} and σ = {Y, ϕ, {a},{a,b}}
Define a function f: (X,τ) → (Y,σ) by f(a) = a, f(b) = b, f(c) = c.
sbĝ – C(X) = {X,ϕ,{c},{a,b}}
b - C(X) = {X,ϕ,{a},{b},{c},{a,b},{a,c},{b,c}}

Here the inverse image of an open set {a} in (Y,σ) is {a} which is b-closed but not sbĝ -closed in (X,τ). Hence, f is contra b-continuous but not contra sbĝ-continuous.

b) Let X = Y = {a,b,c} with topologies
τ = {X, ϕ, {a},{b,c}} and σ = {Y, ϕ, {a},{a,b},{a,c}}
Define a function f: (X,τ) → (Y,σ) by f(a) = a, f(b) = b, f(c) = c.
sbĝ – C(X) = {X,ϕ,{a},{b,c}}
sg - C(X) = {X,ϕ,{a},{b},{c},{a,b},{a,c},{b,c}}

Here the inverse image of the open sets {a,b} and {a,c} in (Y,σ) are {a,b} and {a,c} which are sg-closed but not sbĝ -closed in (X,τ). Hence, f is contra sg-continuous but not contra sbĝ-continuous function.

c) Let X = Y = {a,b,c,d} with topologies
τ = {X, ϕ, {a},{a,c},{a,b,d}} and σ = {Y, ϕ, {b},{a,b},{b,c,d}}
Define a function f: (X,τ) → (Y,σ) by f(a) = d, f(b) = b, f(c) = c, f(d) = a.
sbĝ-C(X)={X,ϕ,{b},{c},{d},{b,c},{c,d},{b,d}, {b,c,d}}
gs-C(X)={X,ϕ,{b},{c},{d},{b,c},{c,d},{b,d}, {a,b,c},{a,c,d},{b,c,d}}

Here the inverse image of an open set {b,c,d} in (Y,σ) is {a,b,c} which is gs-closed set but not sbĝ -closed set in (X,τ). Hence, f is contra gs-continuous but not contra sbĝ-continuous function.

d) Let X = Y = {a,b,c,d} with topologies
τ = {X, ϕ, {a},{a,c},{a,b,d}} and σ = {Y, ϕ, {b},{a,b},{b,c,d}}
Define a function f: (X,τ) → (Y,σ) by f(a) = c, f(b) = b, f(c) = a, f(d) = d.
sbĝ–C(X)={X,ϕ,{b},{c},{d},{b,c},{c,d},{b,d}, {b,c,d}}
gb-C(X)={X,ϕ,{b},{c},{d},{b,c},{c,d},{b,d}, {a,b,c},{a,c,d},{b,c,d},{a,b,d}}

Here the inverse image of an open set {b,c,d} in (Y,σ) is {a,b,d} which is gb-closed but not sbĝ -closed in (X,τ). Hence, f is contra gb-continuous but not contra sbĝ-continuous.

e) Let X = Y = {a,b,c} with topologies
τ = {X, ϕ,{a,c}} and σ = {Y, ϕ, {a},{a,b},{a,c}}
Define a function f: (X,τ) → (Y,σ) by f(a) = b, f(b) = a, f(c) = c.
sbĝ – C(X) = {X,ϕ,{b}}
gb - C(X) = {X,ϕ,{a},{b},{c},{a,b},{b,c}}

Here the inverse image of the open sets {a,b} and {a,c} in (Y,σ) are {a,b} and {b,c} which are g*b-closed but not sbĝ -closed in (X,τ). Hence, f is contra g*b-continuous but not contra sbĝ-continuous.

f) Let X = Y = {a,b,c,d} with topologies
τ = {X, ϕ, {b},{a,b},{b,c,d}} and σ = {Y, ϕ, {a},{a,c},{a,b,d}}
Define a function f: (X,τ) → (Y,σ) by f(a) = a, f(b) = b, f(c) = c, f(d) = d.
sbĝ–C(X)={X,ϕ,{a},{c},{d},{a,c},{a,d},{c,d}, {a,c,d}}
bĝ-C(X)={X,ϕ,{a},{c},{d},{c,d},{a,d},{a,c}, {a,b,c},{a,c,d},{a,b,d}}

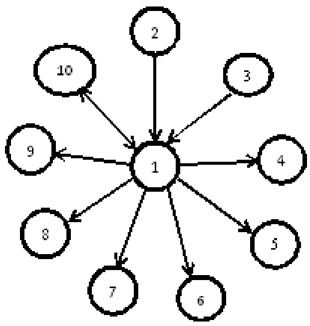
Here the inverse image of an open set {a,b,,d} in (Y,σ) is {a,b,d} which is bĝ-closed but not sbĝ -closed in (X,τ). Hence, f is contra bĝ-continuous but not contra sbĝ-continuous.

Theorem 3.7: If f: (X,τ) → (Y,σ) is contra semi-continuous function if and only if f is contra sbĝ-continuous function.

Proof: Let V be any open set in (Y,σ). Since f is contra semi-continuous, f^{-1}(V) is semi-closed in (X,τ). Since from Proposition 3.6 in [3], f^{-1}(V) is sbĝ-closed in (X,τ). Hence, f is contra sbĝ-continuous.

Conversely, Let V be any open set in (Y,σ). Since f is contra sbĝ-continuous, f^{-1}(V) is sbĝ-closed in (X,τ). Since from Proposition3.6 in [3], f^{-1}(V) is semi-closed in (X,τ). Hence, f is contra semi-continuous.

Remark 3.8: The following diagram shows the relationships of contra sbĝ-continuous function with other known existing functions.



1. Contra sbĝ-continuous 2. Contra continuous
3. Contra α-continuous 4. Contra b-continuous
5. Contra sg-continuous 6. Contra gs-continuous
7. Contra gb-continuous 8. Contra g*bcontinuous
9.Contra bĝ-continuous

Theorem 3.9: The following are equivalent for a function f: (X,τ) → (Y,σ),

a. f is contra sbĝ-continuous function
b. For every closed subset F of Y, f^{-1}(F) is sbĝ-open in X
c. For each x ∈ X and each closed subset F of Y with f(x) ∈ F there exists a sbĝ-open set U of X with x ∈ U, f(U) ⊆ F.

Proof:

(a) $\Rightarrow$ (b):

Let F be any closed set in Y. Then F^c is an open set in Y. Since f is contra sbĝ-continuous, $f^{-1}(F^c)$ is sbĝ-closed set in X. Then $[f^{-1}(F)]^c$ is sbĝ-closed set in X. Therefore $f^{-1}(F)$ is sbĝ-open in X.

(b) $\Rightarrow$ (a):

Let F be an open set in Y. Then F^c is closed set in Y. By (b), $f^{-1}(F^c)$ is sbĝ-open set in X. Then $[f^{-1}(F)]^c$ is sbĝ-open set in X. So $f^{-1}(F)$ is sbĝ-closed set in X. Therefore, f is contra sbĝ-continuous function

(b) $\Rightarrow$ (c):

Let F be any closed subset of Y and let f(x) ∈ F where x ∈ X. Then by (b), f^{-1}(F) is sbĝ-open in X. Also, x ∈ f^{-1}(F). Take U = f^{-1}(F). Then U is a sbĝ-open set containing x and f(U) ⊆ F.

(c) $\Rightarrow$ (b):

Let F be any closed subset of Y. If x ∈ f^{-1}(F) then f(x) ∈ F. By (c), there exists a sbĝ-open set U_x of X with x ∈ U_x such that f(U_x) ⊆ F. Then f^{-1}(F) = ∪ { U_x : x ∈ f^{-1}(F)}. Hence, f^{-1}(F) is sbĝ – open in X.

Theorem 3.10: If X is $T_{sbĝ}$ – space, then for the function f: (X,τ) → (Y,σ), the following statements are equivalent.

i. f is contra continuous function
ii. f is contra sbĝ-continuous function.

Proof:

(i) $\Rightarrow$ (ii):

Let V be any open set in Y. Since f is contra continuous, f^{-1}(V) is closed in X. From [3] proposition 3.4, f^{-1}(V) is sbĝ-closed in X. Therefore, f is contra sbĝ-continuous.

(ii) $\Rightarrow$ (i):

Let V be any open set in Y. Since f is contra sbĝ-continuous, f^{-1}(V) is sbĝ-closed in X. Also, Since X is $T_{sbĝ}$ – space, f^{-1}(V) is closed in X. Therefore, f is contra continuous.

Theorem 3.11: If a function f: (X,τ) → (Y,σ) is contra sbĝ-continuous and X is $T^{\alpha}_{sbĝ}$ – space, then f is contra α-continuous.

Proof: Let V be any open set in Y. Since f is contra sbĝ-continuous, f^{-1}(V) is sbĝ-closed in X. Also, Since X is $T^{\alpha}_{sbĝ}$ – space, f^{-1}(V) is α-closed in X. Therefore, f is contra α-continuous.

Theorem 3.12: Let f: (X,τ) → (Y,σ) be a function then the following statements are equivalent,

i. f is sbĝ-continuous function
ii. For each point x ∈ X and each open set of Y with f(x) ∈ V, there exist a sbĝ-open set U of X such that x ∈ U, f(U) ⊆ V.

Proof:

(i) $\Rightarrow$ (ii):

Let f(x) ∈ V, then x ∈ f^{-1}(V). Since f is sbĝ-continuous, f^{-1}(V) is sbĝ-open in X. Let U = f^{-1}(V), then x ∈ U and f(U) ⊆ V.

(ii) $\Rightarrow$ (i):

Let V be any open set in Y and x ∈ f^{-1}(V). Then f(x) ∈ V. From (ii), there exists a sbĝ-open set U_x of X such that x ∈ U_x ⊆ f^{-1}(V) and f^{-1}(V) = ∪{ U_x}. Then f^{-1}(V) is sbĝ-open in X. Hence, f is sbĝ-continuous.

Theorem 3.13: If a function f: (X,τ) → (Y,σ) is contra sbĝ-continuous and Y is regular, then f is sbĝ-continuous.

Proof: Let x ∈ X and V be an open set in Y with f(x) ∈ V. Since Y is regular, there exists an open set W in Y such that f(x) ∈ W and Cl(W) ⊆ V. Since f is contra sbĝ-continuous and Cl(W) is a closed subset of Y with f(x) ∈ Cl(W). By theorem 3.9, there exist a sbĝ-open set U of X with x ∈ U such that f(U) ⊆ Cl(W). That is, f(U) ⊆ V. By theorem 3.12, f is sbĝ-continuous.

Definintion 3.14: A function f: (X,τ) → (Y,σ) is said to be strongly sbĝ-continuous if f^{-1}(V) is closed in (X,τ) for every sbĝ-closed set V in (Y,σ).

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Example 3.15: Let X = Y = {a,b,c} with topologies
τ = {X, ϕ, {a},{b},{a,b},{a,c}} and σ = {Y, ϕ, {a},{a,b},{a,c}}
Define a function f: (X,τ) → (Y,σ) by f(a) = a, f(b) = b, f(c) = c.
sbĝ – C(Y) = {Y,ϕ,{b},{c},{b,c}}

Here the inverse image of sbĝ-closed sets {b},{c} and {b,c} in (Y,σ) are {b},{c} and {b,c} respectively which are closed in (X,τ). Hence, f is strongly sbĝ-continuous.

Definition 3.16: A function f: (X,τ) → (Y,σ) is said to be perfectly sbĝ-continuous if f^{-1}(V) is clopen in (X,τ) for every sbĝ-closed set V in (Y,σ).

Example 3.17 : Let X = Y = {a,b,c} with topologies
τ = {X, ϕ, {a},{b},{a,b},{a,c}} and σ = {Y, ϕ,{a,c}}
Define a function f: (X,τ) → (Y,σ) by f(a) = a, f(b) = b, f(c) = c.
sbĝ – C(Y) = {Y,ϕ,{b}}

Here the inverse image of sbĝ-closed set {b} in (Y,σ) is {b} which is both open and closed in (X,τ). Hence, f is perfectly sbĝ-continuous.

Definition 3.18: A topological space (X,τ) is said to be sbĝ-Hausdorff (or sbĝ-T_2 space) if for each pair of distinct points x and y in X, there exists sbĝ-open subsets U and V of X containing x and y respectively such that U ∩ V = ϕ.

Example 3.19: Let X = {a,b,c} with a topology τ = {X, ϕ, {a},{b},{a,b}}
sbĝ – O(X) = {X,ϕ,{a},{b},{a,b},{a,c},{b,c}}
Clearly (X,τ) is sbĝ-Hausdorff space.

Theorem 3.20: Let f: (X,τ) → (Y,σ) be surjective, closed and contra sbĝ-continuous. If X is $T_{sbĝ}$ – space, then Y is locally indiscrete.
Proof: Let V be any open set in (Y,σ). Since f is contra sbĝ-continuous, f^{-1}(V) is sbĝ-closed in (X,τ). Also, Since X is $T_{sbĝ}$ – space, f^{-1}(V) is closed in (X,τ). By hypothesis, f is closed and surjective, $f(f^{-1}$(V)) = V is closed in (Y,σ). Hence, Y is locally indiscrete.

Theorem 3.21: If a function f: (X,τ) → (Y,σ) is continuous and (X,τ) is locally indiscrete space, then f is contra sbĝ-continuous.
Proof: Let V be any open set in (Y,σ). Since f is continuous, f^{-1}(V) is open in (X,τ). Since X is locally indiscrete, f^{-1}(V) is closed in (X,τ). By Proposition 3.4 in [3], f^{-1}(V) is sbĝ-closed in (X,τ). Hence, f is contra sbĝ-continuous.

Theorem 3.22: If a function f: (X,τ) → (Y,σ) is contra sbĝ-continuous, injective and Y is Urysohn space, then the topological space X is sbĝ-Hausdorff.
Proof: Let x_1 and x_2 be two distinct points of X. Suppose y_1= f(x_1) and y_2= f(x_2). Since f is injective, $x_1 \neq x_2$ then $y_1 \neq y_2$. Since Y is Urysohn, there exist open sets V_1 and V_2 containing y_1 and y_2 respectively in Y such that Cl(V_1) ∩ Cl(V_2) = ϕ. Since f is contra sbĝ-continuous. By theorem 3.12, there exists sbĝ-open sets U_1 and U_2 containing x_1 and x_2 respectively in X such that f(U_1) ⊆ Cl(V_1) and f(U_2) ⊆ Cl(V_2). Since Cl(V_1) ∩ Cl(V_2) = ϕ, $U_1 \cap U_2$ = ϕ. Hence, X is sbĝ-Hausdorff space.

Theorem 3.23: Let f: (X,τ) → (Y,σ) be a function and g: X → X× Y be a graph function of f defined by g(x) = (x,f(x)) for every x ∈ X. If g is contra sbĝ-continuous, then f is contra sbĝ-continuous.
Proof: Let V be closed subset of Y. Then X× V is a closed subset of X× Y. Since g is contra sbĝ-continuous, g^{-1} (X × V)is sbĝ-open subset of X. Also, g^{-1} (X × V) = f^{-1}(V) which is sbĝ-open subset of X. Hence, f is contra sbĝ-continuous.

Definition 3.24: A space X is said to be locally sbĝ-indiscrete if every sbĝ-open set of X is closed in X.

Example 3.25: Let X = {a,b,c} with topology
τ = {X, ϕ,{c},{a,b}}
sbĝ – O(X) = {X,ϕ,{c},{a,b}}
Here every sbĝ-open set in X is closed in X. Hence, X is locally sbĝ-indiscrete space.

Theorem 3.26: If f: (X,τ) → (Y,σ) is contra sbĝ-continuous with X as locally sbĝ-indiscrete, then f is continuous.
Proof: Let V be any open set in (Y,σ). Then f^{-1}(V) is sbĝ-closed in X. Since X is locally sbĝ-indiscrete space, f^{-1}(V) is open in X. Thus, f is continuous.

4. CONTRA sbĝ-IRRESOLUTE FUNCTIONS

Definition 4.1: A function f: (X,τ) → (Y,σ) is called contra sbĝ-irresolute, if f^{-1}(V) is sbĝ-closed in (X,τ) for every sbĝ-open set V in (Y,σ).

Example 4.2: Let X = Y = {a,b,c} with topologies
τ = {X, ϕ, {a},{b},{a,b}} and σ = {Y, ϕ, {b}}
Define a function f: (X,τ) → (Y,σ) by f(a) = a, f(b) = c, f(c) = b.
sbĝ – C(X) = {X,ϕ,{a},{b},{c},{a,c},{b,c}}
sbĝ – O(Y) = {Y,ϕ,{b},{b,c},{a,b}}
Here the inverse image of sbĝ-open set {b},{b,c} and {a,b} in (Y,σ) are {c},{b,c} and {a,c} respectively which are sbĝ-closed set in (X,τ). Hence, f is contra sbĝ-irresolute function.

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Remark 4.3: The following example shows that the concepts of sbĝ-irresolute function and contra sbĝ-irresolute are independent of each other.

Example 4.4:

1. Let X = Y = {a,b,c,d} with topologies

τ = {X, ϕ, {a},{a,c},{a,b,d}} and σ = {Y, ϕ, {b},{a,b},{b,c,d}}

Define a function f: (X,τ) → (Y,σ) by f(a) = b, f(b) = a, f(c) = c, f(d) =d.

sbĝ–C(X)={X,ϕ,{b},{c},{d},{b,c},{c,d},{b,d}, {b,c,d}}

sbĝ–O(Y)={Y,ϕ,{a},{c},{d},{a,c},{a,d},{c,d}, {a,c,d}}

sbĝ–C(Y)={Y,ϕ,{b},{a,b},{b,c},{b,d},{a,b,c}, {a,b,d},{b,c,d}}

Clearly f is contra sbĝ-irresolute but not sbĝ-closed set {b}, {a,b}, {b,c}, {b,d}, {a,b,c}, {a,b,d} and {b,c,d} in Y are {a}, {a,b}, {a,c}, {a,d}, {a,b,c}, {a,b,d} and {a,c,d} respectively which are not sbĝ-closed in (X,τ).

2. Let X = Y = {a,b,c} with topologies

τ = {X, ϕ, {a},{b},{a,b},{a,c}} and σ = {Y, ϕ, {a},{a,b}}

Define a function f: (X,τ) → (Y,σ) by f(a) = b, f(b) = a, f(c) = c.

sbĝ – C(X) = {X,ϕ,{b},{c},{a,c},{b,c}}

sbĝ – O(Y) = {Y,ϕ,{a},{a,b},{a,c}}

sbĝ – C(Y) = { Y,ϕ,{b},{c},{b,c}}

Here, the inverse image of sbĝ-open sets {a},{a,b} in (Y,σ) are {a},{a,b} which are not sbĝ-closed set in (X,τ). Hence, the f is sbĝ-irresolute but not contra sbĝ-irresolute.

Theorem 4.5: Every contra sbĝ-irresolute function is contra sbĝ-continuous.

Proof: Let V be any open set in (Y,σ). By proposition 3.4 in [3], V is sbĝ-open in Y, Since f is contra sbĝ-irresolute, V is sbĝ-closed in (X,τ). Hence, f is contra sbĝ-continuous.

The converse of the above theorem need not be true as shown in the following example.

Example 4.6: Let X = Y = {a,b,c} with topologies

τ = {X, ϕ, {a},{c},{a,c},{b,c}} and σ = {Y, ϕ, {a}}

Define a function f: (X,τ) → (Y,σ) by f(a) = a, f(b) = b, f(c) = c.

sbĝ – C(X) = {X,ϕ,{a},{b},{a,b},{b,c}}

sbĝ – O(Y) = {Y,ϕ,{a},{a,b},{a,c}}

Here, the inverse image of sbĝ-open set {a,c} in (Y,σ) are {a,c} which is not sbĝ-closed set in (X,τ). Hence, the f is contra sbĝ-continuous but not contra sbĝ-irresolute.

Remark 4.7: The following example shows that the concepts of sbĝ-continuous and contra sbĝ-continuous are independent of each other.

Example 4.8:

1. Let X = Y = {a,b,c,d} with topologies

τ = {X, ϕ, {a},{a,c},{a,b,d}} and σ = {Y, ϕ, {b},{a,b},{b,c,d}}

Define a function f: (X,τ) → (Y,σ) by f(a) = b, f(b) = a, f(c) = d, f(d) = c.

sbĝ–C(X)={X,ϕ,{b},{c},{d},{b,c},{c,d},{b,d}, {b,c,d}}

Since the inverse image of open sets {b},{a,b} and {b,c,d} in (Y,σ) are {a},{a,b} and {a,c,d} respectively which are not sbĝ-closed in (X,τ), f is not contra sbĝ-continuous. Since the inverse image of closed sets {a},{c,d} and {a,c,d} in (Y,σ) are {b},{c,d} and {b,c,d} respectively which are sbĝ-closed in (X,τ), f is sbĝ-continuous. Hence, f is sbĝ-continuous but not contra sbĝ-continuous.

2. Let X = Y = {a,b,c} with topologies

τ = {X, ϕ,{b}} and σ = {Y, ϕ,{a,c}}

Define a function f: (X,τ) → (Y,σ) by f(a) = a, f(b) = b, f(c) = c.

sbĝ – C(X) = {X,ϕ,{a},{c},{a,c}}

Since the inverse image of an open set {a,c} in Y is {a,c} which is sbĝ-closed in X, f is contra sbĝ-continuous. Also, since the inverse image of a closed set {b} in Y is {b} which is not sbĝ-closed in X, f is not sbĝ-continuous. Hence, f is contra sbĝ-continuous but not sbĝ-continuous.

5. PERFECTLY CONTRA sbĝ-IRRESOLUTE FUNCTION

Definition 5.1: A function f: (X,τ) → (Y,σ) is called perfectly contra sbĝ-irresolute function if f^{-1}(V) is sbĝ-clopen in (X,τ) for every sbĝ-open set V in (Y,σ).

Example 5.2: Let X = Y = {a,b,c} with topologies

τ = {X, ϕ, {a},{b,c}} and σ = {Y, ϕ, {a,b},{c}}

Define a function f: (X,τ) → (Y,σ) by f(a) = c, f(b) = a, f(c) = b.

sbĝ – O(X) = {X,ϕ,{a},{b,c}}

sbĝ – O(Y) = {Y,ϕ,{c},{a,b}}

Since the inverse images of all sbĝ-open sets in (Y,σ) are sbĝ-clopen set in (X,τ), f is perfectly contra sbĝ-irresolute function.

Theroem 5.3:

1) Every perfectly contra sbĝ-irresolute map is contra sbĝ-irresolute map.
2) Every perfectly contra sbĝ-irresolute map is sbĝ-irresolute map.

Proof:

(1) and (2) directly follows from the definitions 2.4, 4.1 and 5.1.

The converse of the above theorem need not be true as shown in the following example.

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Example 5.4:

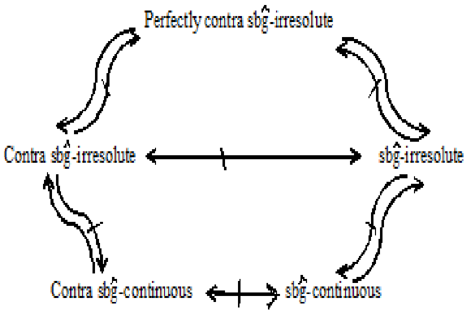
1) Let X = Y = {a,b,c} with topologies
τ = {X, ϕ, {a},{b},{a,b}} and σ = {Y, ϕ, {b}}
Define a function f: (X,τ) → (Y,σ) by f(a) = a, f(b) = c, f(c) = b.
sbĝ – O(X) = {X,ϕ,{a},{b},{a,b},{a,c},{b,c}}
sbĝ – O(Y) = {Y,ϕ,{b},{a,b},{b,c}}

Since the inverse image of sbĝ-open set {b} in (Y,σ) is {c} which is sbĝ-closed set in (X,τ) but not sbĝ-open set in X, f is contra sbĝ –irresoulute but not perfectly contra sbĝ-irresolute function.

2) Let X = Y = {a,b,c} with topologies
τ = {X, ϕ, {a},{b},{a,b},{a,c}} and σ = {Y, ϕ,{a},{a,b},{a,c}}
Define a function f: (X,τ) → (Y,σ) by f(a) = a, f(b) = b, f(c) = c.
sbĝ – O(X) = {X,ϕ,{a},{b},{a,b},{a,c}}
sbĝ – O(Y) = {Y,ϕ,{a},{a,b},{a,c}}

Since the inverse image of sbĝ-open sets {a} and {a,b} in (Y,σ) are {a} and {a,b} respectively which are sbĝ-open set in (X,τ) but not sbĝ-closed set in (X,τ), f is sbĝ–irresolute but not perfectly contra sbĝ-irresolute function.

Remark 5.5: From the above discussions and known results, we have the following diagram.



In this diagram, A → B means A implies B but not conversely. A ↮ B means A and B are independent of each other.

6. COMPOSITION OF TWO MAPS

The following example shows that the composition of two contra sbĝ-continuous function need not be contra sbĝ-continuous.

Example 6.1: Let X = Y = Z = {a,b,c} with topologies
τ = {X, ϕ, {a},{c},{a,c},{b,c}}, σ = {Y, ϕ, {b}} and η = {Z, ϕ,{a,c}}
Define a function f: (X,τ) → (Y,σ) by f(a) = a, f(b) = b, f(c) = c and g: (Y,σ) → (Z,η) by g(a) = a, g(b) = b, g(c) = c.
sbĝ – C(X) = {X,ϕ,{a},{b},{a,b},{b,c}}
sbĝ – C(Y) = {Y,ϕ,{a},{c},{a,c}}

Clearly f and g are contra sbĝ-continuous. But their composition is not contra sbĝ-continuous, since $(g∘f)^{-1}$ of an open set {a,c} in (Z,η) is {a,c} which is not sbĝ-closed in (X,τ). Hence, g∘f is not contra sbĝ-continuous.

Theorem 6.2: The composition of two strongly sbĝ-continuous function is strongly sbĝ-continuous function.

Proof: Let f: (X,τ) → (Y,σ) and g: (Y,σ) → (Z,η) be strongly sbĝ-continuous functions. Let V be sbĝ-closed set in (Z,η). Since g is strongly sbĝ-continuous, $g^{-1}(V)$ is closed in (Y,σ). By Propositon 3.4 in [3], $g^{-1}(V)$ is sbĝ-closed in (Y,σ). Since f is strongly sbĝ-continuous, $f^{-1}(g^{-1}(V)) = (g ∘ f)^{-1}(V)$ is closed in (X,τ). Therefore, g∘f is strongly sbĝ-continuous.

Theorem 6.3: The composition of two perfectly sbĝ-continuous function is perfectly sbĝ-continuous function.

Proof: Let f: (X,τ) → (Y,σ) and g: (Y,σ) → (Z,η) be perfectly sbĝ-continuous functions. Let V be sbĝ-closed set in (Z,η). Since g is perfectly sbĝ-continuous, $g^{-1}(V)$ is clopen in (Y,σ). By Propositon 3.4 in [3], $g^{-1}(V)$ is sbĝ-clopen in (Y,σ). Since f is perfectly sbĝ-continuous, $f^{-1}(g^{-1}(V)) = (g ∘ f)^{-1}(V)$ is clopen in (X,τ). Therefore, g∘f is perfectly sbĝ-continuous.

The following example shows that the composition of two contra sbĝ-irresolute function need not be contra sbĝ-irresolute.

Example 6.4: Let X = Y = {a,b,c} with topologies
τ = {X, ϕ, {a},{b},{a,b}}, σ = {Y, ϕ, {b}} and η = {Z, ϕ,{a,c}}
Define a function f: (X,τ) → (Y,σ) by f(a) = a, f(b) = c, f(c) = b and g: (Y,σ) → (Z,η) by g(a) = a, g(b) = b, g(c) = c.
sbĝ – C(X) = {X,ϕ,{a},{b},{c},{a,c},{b,c}}
sbĝ – O(Y) = {Y,ϕ,{b},{b,c},{a,b}}
sbĝ- O(Z) = {Z, ϕ, {a,c}.

Clearly f and g are contra sbĝ-irresolute function. But their composition is not contra sbĝ-irresolute, since $(g∘f)^{-1}$ of an open set {a,c} in (Z,η) is {a,c} which is not sbĝ-closed in (X,τ). Hence, g∘f is not contra sbĝ-continuous.

Theorem 6.5: The composition of two perfectly contra sbĝ-irresolute function is perfectly contra sbĝ-irresolute function.

Proof: Let f: (X,τ) → (Y,σ) and g: (Y,σ) → (Z,η) be perfectly contra sbĝ-irresolute functions. Let V be any sbĝ-open set in (Z,η). Since g is perfectly contra sbĝ-irresolute, $g^{-1}(V)$ is sbĝ-clopen in (Y,σ). Since f is perfectly contra sbĝ-irresolute, $f^{-1}(g^{-1}(V)) = (g ∘ f)^{-1}(V)$ is sbĝ-clopen in (X,τ). Therefore, g∘f is perfectly contra sbĝ-irresolute.

Theorem 6.6: If a function f: (X,τ) → (Y,σ) is strongly sbĝ-continuous function and g: (Y,σ) → (Z,ɳ) is contra sbĝ-continuous function then g ∘ f: (X,τ) → (Z,ɳ) is contra continuous.

Proof: Let V be any open set in (Z,ɳ). Since g is contra sbĝ-continuous, g^{-1}(V) is sbĝ-closed in (Y,σ). Since f is strongly sbĝ-continuous, $f^{-1}\left(g^{-1}(V)\right) = (g \circ f)^{-1}(V)$ is closed in (X,τ). Hence, g ∘ f is contra continuous.

Theorem 6.7: If a function f: (X,τ) → (Y,σ) is contra sbĝ-irresolute function and g: (Y,σ) → (Z,ɳ) is sbĝ-irresolute function then g ∘ f: (X,τ) → (Z,ɳ) is contra sbĝ-irresolute.

Proof: Let V be any sbĝ-open set in (Z,ɳ). Since g is sbĝ-irresolute, g^{-1}(V) is sbĝ-open in (Y,σ). Since f is contra sbĝ-irresolute, $f^{-1}\left(g^{-1}(V)\right) = (g \circ f)^{-1}(V)$ is sbĝ-closed in (X,τ). Hence, g ∘ f is contra sbĝ-irresolute.

Theorem 6.8: If a function f: (X,τ) → (Y,σ) is contra sbĝ-irresolute function and g: (Y,σ) → (Z,ɳ) is sbĝ-continuous function then g ∘ f: (X,τ) → (Z,ɳ) is contra sbĝ-continuous.

Proof: Let V be any open set in (Z,ɳ). Since g is sbĝ-continuous, g^{-1}(V) is sbĝ-open in (Y,σ). Since f is contra sbĝ-irresolute, $f^{-1}\left(g^{-1}(V)\right) = (g \circ f)^{-1}(V)$ is sbĝ-closed in (X,τ). Hence, g ∘ f is contra sbĝ-continuous.

Theorem 6.9: If a function f: (X,τ) → (Y,σ) is sbĝ-irresolute function with Y as locally indiscrete space and g: (Y,σ) → (Z,ɳ) is contra sbĝ-continuous function then g ∘ f: (X,τ) → (Z,ɳ) is sbĝ-continuous.

Proof: Let V be any open set in (Z,ɳ). Since g is contra sbĝ-continuous, g^{-1}(V) is sbĝ-open in (Y,σ). But Y is locally sbĝ-indiscrete, g^{-1}(V) is closed in (Y,σ). By Proposition 3.4 in [3], g^{-1}(V) is sbĝ-closed in (Y,σ). Since f is sbĝ-irresolute, $f^{-1}\left(g^{-1}(V)\right) = (g \circ f)^{-1}(V)$ is sbĝ-closed in (X,τ). Hence, g ∘ f is contra sbĝ-continuous.

Theorem 6.10: If a function f: (X,τ) → (Y,σ) is sbĝ-irresolute function and g: (Y,σ) → (Z,ɳ) is contra sbĝ-continuous function then g ∘ f: (X,τ) → (Z,ɳ) is contra sbĝ-continuous.

Proof: Let V be any open set in (Z,ɳ). Since g is contra sbĝ-continuous, g^{-1}(V) is sbĝ-closed in (Y,σ). Since f is sbĝ-irresolute, $f^{-1}\left(g^{-1}(V)\right) = (g \circ f)^{-1}(V)$ is sbĝ-closed in (X,τ). Hence, g ∘ f is contra sbĝ-continuous.

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