Published by :
https://www.ijert.org/
An International Peer-Reviewed Journal

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 15 Issue 07 , July - 2026

On Chromatic Degree Partition Number of a Graph

N. Malathi[1], M. Bhuvaneshwari[2], Selvam Avadayappan[3]

[1]Department of Mathematics, V.V.Vanniaperumal College for Women, Virudhunagar, India

[2,3]Research Department of Mathematics, VHN Senthikumara Nadar College,

(Affiliated to Madurai Kamaraj University)

Virudhunagar, India

Abstrat - Major Corporations rely on partitioning certain group of individuals or components to ensure the systematic functioning of the projects in some situations. The Graph Theoretical approach makes our work easier. We introduce the concept chromatic similar degree partitioning. A partition of the vertex set $V(G)$ is defined to be a chromatic similar degree partition of G if each partition class is independent and the absolute difference of the sum of the degrees of all vertices between any two partition classes is at most 1. The maximum cardinality k and the minimum cardinality k' taken over all the chromatic similar degree partition number of the graph G are called the Max – Chromatic Degree Partition Number and Min – Chromatic Degree Partition of G and are denoted by $\chi_{\vee}\psi_D(G)$ and $\chi_{\wedge}\psi_D(G)$ respectively. In this paper we attempt to prove some theorems on these parameters.

AMS Subject Classification Code (2010): 05C (Primary)

Keywords: Degree Partition Number, Partitions in Graphs, Colour Partitioning, Chromatic Partitioning

1 INTRODUCTION

In this we paper, we consider only undirected simple connected graphs. For the basic definitions and notations of graph theory, we refer the text book by Harary [2]. A finite non-empty set of objects called vertices together with a set of unordered pairs of distinct vertices of G called edges is called a graph G. Throughout this paper, the vertex set and edge set of G are denoted by $V(G)$ and $E(G)$ respectively. The number of vertices n and the number of edges m in a graph G are called its order and its size respectively.

The number of edges incident with a vertex v is called the degree of the vertex v and it is denoted by $\deg v$. The vertex of degree $n-1$ is called *full vertex*. The sum of the degree of all the vertices in a set $W \in V(G)$ is denoted by $D(W)$.

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 15 Issue 07 , July - 2026

If all the vertices are of same degree say r, then the graph is said to be an $r-regular$ graph. Suppose that the degree of every vertex of a graph G is either s or t, then G is called an $(s,t)-biregular$ graph. The $(n-1)-regular$ graph is called the Complete Graph denoted by K_n and $0-regular$ graph is called totally disconnected which is denoted by $\overline{K_n}$. A $2-regular$ graph of order n is called cycle C_n and the graph obtained by removing one of the edges of C_n is called the path P_n. The minimum and maximum degree among the vertices of G are denoted by $\delta(G)$ and $\Delta(G)$ respectively.

The given two graphs G and H with same order and size are said to be isomorphic if there exists a bijection between $V(G)$ and $V(H)$ preserving the adjacency. In this case we write $G \cong H$.

A vertex partition of a graph G is a collection of disjoint subsets of $V(G)$ whose union is $V(G)$. Also, each smaller sets are called the partition classes of $V(G)$. Each subset in the partition is called a partition class of $V(G)$. A vertex partition is said to be an *isolated partition* if each partition class contains exactly one vertex and the class which contains exactly one vertex is called an *isolated partition class*.

A subset S of $V(G)$ is called an *independent set* of G if no two vertices in S are adjacent. A graph is known to be a *bipartite graph* if the vertex set of G can be partitioned into two independent sets say V_1 and V_2. The complete bipartite graph $K_{m,n}$ is nothing but the bipartite graph with $|V_1| = m$ and $|V_2| = n$ in which every vertex in V_1 is adjacent to every vertex in V_2. The graph $K_{1,n}$ is called the *star graph*.

Whenever the vertex of a graph G can be partitioned into k independent sets, G is called a *k-partite* graph. The *Turan graph* $T(n,k)$ is the complete $k-partite$ graph on n vertices whose partite sets are as nearly equal in cardinality as possible. It is noticeable that $T(n,1) \cong \overline{K_n}$. Always, the number of vertices in each partite set in $T(n,k)$ is either $\left\lfloor\frac{n}{k}\right\rfloor$ or $\left\lceil\frac{n}{k}\right\rceil$ and the number of vertices in each partite set in $T(n,k)$ is same if and only if $n \equiv 0(mod\ k)$.

The *join* of two graphs G and H, denoted by $G \vee H$ is obtained from $G \cup H$, by joining each vertex of G to every vertex of H by means of an edge. The *wheel graph* $W_n (n \geq 4)$ is nothing but $K_1 \vee C_{n-1}$.

The graph $I_n\ (n \geq 4)$ on n vertices with degree sequence $n-1, n-2, \dots, \left\lfloor\frac{n}{2}\right\rfloor, \left\lfloor\frac{n}{2}\right\rfloor, \dots, 2, 1$ is called the *irregular most graph* [3].

The partitioning of the vertex set by means of distinct independent sets is called proper colour partition of G. The proper colour partition with minimum possible number of

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 15 Issue 07 , July - 2026

independent sets is called the minimum proper colour partition and such number is called the chromatic number of G, $\chi(G)$. Many researchers studied on this concept which can be referred from [1], [6], [7] and [8].

The similar degree partition of a graph G is defined as the partition $\{V_1, V_2, \dots, V_k\}$ of $V(G)$ such that $|D(V_i) - D(V_j)| \leq 1$ for every $1 \leq i, j \leq k$. The similar degree partition of G with the maximum possible number of partition classes is called the max-similar degree partition of G. The cardinality of the max-similar degree partition is called the degree partition number of G denoted by $\psi_D(G)$. More results on this parameter can be found in [3], [4] and [5].

A proper colour partition of G is said to be chromatic similar degree partition if the absolute difference between the degree sum of any two colour classes is at most 1. The maximum value k and the minimum value k' among all the chromatic similar degree partition of the graph G are called the max-chromatic degree partition number $\chi_\vee\psi_D(G)$ and the min-chromatic degree partition number $\chi_\wedge\psi_D(G)$ respectively, if such partition exists. Otherwise $\chi_\vee\psi_D(G)$ and $\chi_\wedge\psi_D(G)$ are defined to be infinity. The chromatic similar degree partitions with $\chi_\vee\psi_D(G)$ and $\chi_\wedge\psi_D(G)$classes are called max-chromatic similar degree partition and min-chromatic similar degree partition respectively. A graph is said to have unique chromatic similar degree partition if $V(G)$ is uniquely partitioned into k chromatic similar degree partition classes.

2 Main Results

In this section, we establish some interesting facts and results on our parameters.

Observation 1 For any graph G, $\chi_\wedge\psi_D(G) \leq \chi_\vee\psi_D(G)$, if such partition exists. □

Observation 2 For any graph G, $\chi(G) \leq \chi_\wedge\psi_D(G)$, if such partition exists. □

Observation 3 For any graph G, $\chi_\vee\psi_D(G) \leq \psi_D(G)$, if such partition exists.

Proof $\psi_D(G)$ gives the maximum possible number of similar degree partition classes. So, $\chi_\vee\psi_D(G)$ can be at most $\psi_D(G)$ if such number exists. □

Observation 4 $\chi_\wedge\psi_D(G) = 1$ *if and only if* $G \cong \overline{K_n}$.

Proof The result follows as $\chi(G) = 1$ *if and only if* $G \cong \overline{K_n}$. □

Observation 5 For any connected non-trivial graph G, $\chi_\vee\psi_D(G) > 1$ and $\chi_\wedge\psi_D(G) > 1$

Proof The result is obvious as $\chi(G) > 1$ if and only if $G \ncong \overline{K_n}$. □

Observation 6 The full vertex of any graph of order n forms an isolated partition in any chromatic similar degree partition if such partition exists.

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 15 Issue 07 , July - 2026

Proof Any subset of $V(G)$ with at least two vertices containing the full vertex cannot be independent. So, the full vertex must form an isolated partition class in any chromatic similar degree partition if such partition exists. □

Observation 7 If G is a non-trivial graph such that $\psi_D(G) = 1$, then both $\chi_\vee\psi_D(G)$ and $\chi_\wedge\psi_D(G)$ cannot be finite.

Proof In viewing observations 1, 3 and 5, the result is quite possible. □

Observation 8 If G is either K_n or an $(n-1, n-2)-$ biregular graph with n vertices where $n \geq 4$, $\chi_\wedge\psi_D(G) = n$.

Proof As G possess at least one full vertex, the degree sum of any partition class must range from $n-2$ to n. But there cannot be any partition class with degree sum n. So, the only possible chromatic similar degree partition is the isolated partition. □

Theorem 9 $\chi_\wedge\psi_D(G) = 2$ if and only if G is bipartite.

Proof As $\chi(G) = 2$ if and only if G is bipartite, the theorem holds. □

Theorem 10 $\chi_\vee\psi_D(G) = n$ if and only if G is either $r-regular$ or $(r, r+1)-biregular$.

Proof As $\psi_D(G) = n$ if and only if G is either $r-regular$ or $(r, r+1)-biregular$[3], the result follows easily. □

Observation 11 $\chi_\vee\psi_D(T(n,k)) = n$.

Proof As $T(n,k)$ is a $\left(\left\lfloor\frac{n}{k}\right\rfloor, \left\lfloor\frac{n}{k}\right\rfloor + 1\right) - biregular$ graph, the result follows easily. □

Theorem 12 Let k denote the smallest prime number such that $k|n$. Then $\chi_\wedge\psi_D(C_n) = k$.

Proof As C_n is $2-regular$, $\chi_\wedge\psi_D(C_n)$ must be the smallest factor of n greater than 1.
So, $\chi_\wedge\psi_D(C_n) \geq k$.
Also, since $n \equiv 0(mod\ k)$, we can partition the vertex set of C_n into k independent sets, each consisting of $\frac{n}{k}$ vertices as follows.

$$V_1 = \{v_1, v_{k+1}, \dots, v_{n-k+1}\},$$
$$V_2 = \{v_2, v_{k+2}, \dots, v_{n-k+2}\}, \dots,$$
$$V_k = \{v_k, v_{2k}, \dots, v_n\}$$

Then $\pi_k = \{V_1, V_2, \dots, V_k\}$ constitute our required min-chromatic degree partition of C_n. □

Theorem 13 Let G contain $t(\leq n)$ full vertices. Then $\left\lceil\frac{D(V)+t}{n}\right\rceil \leq \chi_\wedge\psi_D(G) \leq \left\lfloor\frac{D(V)-t}{n-2}\right\rfloor$ if such partition exists.

Proof Let $v_1, v_2, \dots, v_t$ be the full vertices of G. Then in any chromatic similar degree partition (if exists), these vertices should form t isolated partition classes.

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 15 Issue 07 , July - 2026

Suppose $\chi_{\wedge}\psi_D(G) = k$. Then t classes namely $V_1, V_2, \dots, V_t$ must be isolated. Also, $D(V_i) = n-1$ for $1 \le i \le t$. Therefore, for the remaining $k-t$ classes $V_{t+1}, V_{t+2}, \dots, V_k$, $D(V_j)$ is either n or $n-2$ for $t+1 \le j \le k$.

As the degree sum of these $k-t$ classes is at least $n-2$,

Thus, $D(V) = \sum_{i=1}^{k} D(V_i) \ge t(n-1) + (k-t)(n-2)$

$$= (n-2)k + t$$

This forces that $k \le \frac{D(V)-t}{n-2} \Rightarrow k \le \left\lfloor \frac{D(V)-t}{n-2} \right\rfloor$

The other side of the inequality follows in a similar way. □

The analogous of this result for max-chromatic similar partition can be stated as follows:

Theorem 14 Let G contain $t(\le n)$ full vertices. Then $\left\lceil \frac{D(V)+t}{n} \right\rceil \le \chi_{\vee}\psi_D(G) \le \left\lfloor \frac{D(V)-t}{n-2} \right\rfloor$ if such partition exists.

The proof is very similar to Theorem 13. □

Theorem 15 $\chi_{\wedge}\psi_D(W_n) = \chi_{\vee}\psi_D(W_n) = \begin{Bmatrix} 5 & if\ n = 5 \\ 4 & if\ n \equiv 1(mod\ 3) \\ \infty & otherwise \end{Bmatrix}$

Proof Clearly $W_4 \cong K_4$ and W_5 is $(4,3) - regular$.

So, $\chi_{\wedge}\psi_D(W_4) = \chi_{\vee}\psi_D(W_4) = 4$ and $\chi_{\wedge}\psi_D(W_5) = \chi_{\vee}\psi_D(W_5) = 5$

Now, we consider $n \ge 6$. Let $V(W_n) = \{v_1, v_2, \dots, v_n\}$ where v_n be the full vertex.

Since $D(V) = 4n - 4$, $\left\lceil \frac{D(V)+1}{n} \right\rceil = \left\lfloor \frac{D(V)-1}{n-2} \right\rfloor = 4$ for $n \ge 6$.

Thus by Theorems 13 and 14, $\chi_{\wedge}\psi_D(W_n) = \chi_{\vee}\psi_D(W_n) = 4$ if such partition exists.

We already see that, in a similar degree partition of W_n, the full vertex can form an isolated partition only when $n \equiv 1(mod\ 3)$ for $n \ge 6$. [5]

Thus, the chromatic similar degree partition exists only when $n \equiv 1(mod\ 3)$.

Hence, $\chi_{\wedge}\psi_D(W_n) = \chi_{\vee}\psi_D(W_n) = \infty$ whenever $n \not\equiv 1(mod\ 3)$ and $n \ge 6$.

When $n \equiv 1(mod\ 3)$, $V_1 = \{v_1, v_4, \dots, v_{n-3}\}$, $V_2 = \{v_2, v_5, \dots, v_{n-2}\}$, $V_3 = \{v_3, v_6, \dots, v_{n-1}\}$ and $V_4 = \{v_n\}$ is the required chromatic similar degree partition.

So, $\chi_{\wedge}\psi_D(W_n) = \chi_{\vee}\psi_D(W_n) = 4$ whenever $n \equiv 1(mod\ 3)$. □

Theorem 16 For $n \ge 3$, $\chi_{\wedge}\psi_D(I_n) = \chi_{\vee}\psi_D(I_n) = \left\lfloor \frac{n}{2} \right\rfloor + 1$.

Proof The degree sequence of I_n is $\left(n-1, n-2, \dots, \left\lfloor \frac{n}{2} \right\rfloor, \left\lfloor \frac{n}{2} \right\rfloor, \dots, 2, 1\right)$ and it has one full vertex.

Now, $D(V) = \frac{(n-1)n}{2} + \left\lfloor \frac{n}{2} \right\rfloor = \frac{n^2-n}{2} + \left\lfloor \frac{n}{2} \right\rfloor$

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 15 Issue 07 , July - 2026

So, $D(V) = \begin{cases} \frac{n^2}{2} & if\ n\ is\ even \\ \frac{n^2-1}{2} & if\ n\ is\ odd \end{cases}$

Now $\left\lceil\frac{D(V)+1}{n}\right\rceil = \begin{cases} \left\lceil\frac{n^2+2}{2n}\right\rceil & if\ n\ is\ even \\ \left\lceil\frac{n^2+1}{2n}\right\rceil & if\ n\ is\ odd \end{cases}$

$$= \begin{cases} \left\lceil\frac{n}{2}+\frac{1}{n}\right\rceil & if\ n\ is\ even \\ \left\lceil\frac{n}{2}+\frac{1}{2n}\right\rceil & if\ n\ is\ odd \end{cases}$$

$$= \begin{cases} \frac{n}{2}+1 & if\ n\ is\ even \\ \left\lceil\frac{n}{2}\right\rceil & if\ n\ is\ odd \end{cases} \text{ as } n \geq 3$$

Also, $\left\lfloor\frac{D(V)-1}{n-2}\right\rfloor = \begin{cases} \left\lfloor\frac{n^2-2}{2(n-2)}\right\rfloor & if\ n\ is\ even \\ \left\lfloor\frac{n^2-3}{2(n-2)}\right\rfloor & if\ n\ is\ odd \end{cases}$

$$= \begin{cases} \left\lfloor\frac{n^2-2n+2n-4+2}{2(n-2)}\right\rfloor & if\ n\ is\ even \\ \left\lfloor\frac{n^2-2n+2n-4+1}{2(n-2)}\right\rfloor & if\ n\ is\ odd \end{cases}$$

$$= \begin{cases} \left\lfloor\frac{n}{2}+1+\frac{1}{n-2}\right\rfloor & if\ n\ is\ even \\ \left\lfloor\frac{n}{2}+1+\frac{1}{2(n-2)}\right\rfloor & if\ n\ is\ odd \end{cases}$$

$$= \begin{cases} \frac{n}{2}+1 & if\ n\ is\ even \\ \left\lfloor\frac{n}{2}+1\right\rfloor & if\ n\ is\ odd \end{cases} \text{ as } n \geq 3$$

$$= \begin{cases} \frac{n}{2}+1 & if\ n\ is\ even \\ \left\lfloor\frac{n+2}{2}\right\rfloor & if\ n\ is\ odd \end{cases}$$

$$= \begin{cases} \frac{n}{2}+1 & if\ n\ is\ even \\ \frac{n+1}{2} & if\ n\ is\ odd \end{cases}$$

$$= \begin{cases} \frac{n}{2}+1 & if\ n\ is\ even \\ \left\lceil\frac{n}{2}\right\rceil & if\ n\ is\ odd \end{cases}$$

Hence, $\chi_\wedge\psi_D(I_n) = \chi_\vee\psi_D(I_n) = \left\lfloor\frac{n}{2}\right\rfloor + 1$ if such partition exists.

We label the vertices of I_n as $v_1, v_2, \dots, v_n$ such that $\deg v_i = \begin{cases} i & if\ 1 \leq i \leq n-1 \\ \left\lfloor\frac{n}{2}\right\rfloor & if\ i = n \end{cases}$

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 15 Issue 07 , July - 2026

It may be noted that for $1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor$, v_i is adjacent to $v_{n-1}, v_{n-2}, \dots, v_{n-i}$, v_{n-i} is adjacent to $v_{n-1}, v_{n-2}, \dots, v_{n-i+1}, v_{n-i-1}, \dots, v_{i-1}, v_i$ and v_n is adjacent to $v_{n-1}, v_{n-2}, \dots, v_{\left\lfloor \frac{n}{2} \right\rfloor+1}$

Case i Let n be even.

Consider $\pi_{\frac{n}{2}+1} = \left\{V_1, V_2, \dots, V_{\frac{n}{2}+1}\right\}$ where $V_1 = \{v_1, v_{n-3}\}$,

$V_2 = \{v_2, v_{n-4}\}, \dots, V_{\frac{n}{2}-2} = \left\{v_{\frac{n}{2}-2}, v_{\frac{n}{2}}\right\}, V_{\frac{n}{2}-1} = \left\{v_{\frac{n}{2}-1}, v_n\right\}, V_{\frac{n}{2}} = \{v_{n-1}\}$ and $V_{\frac{n}{2}+1} = \{v_{n-2}\}$

which is the required chromatic similar degree partition.

Case ii Let n be odd.

In this case we take, $\pi_{\frac{n+1}{2}} = \left\{V_1, V_2, \dots, V_{\frac{n+1}{2}}\right\}$ where $V_1 = \{v_1, v_{n-2}\}$,

$V_2 = \{v_2, v_{n-3}\}, \dots, V_{\frac{n-3}{2}} = \left\{v_{\frac{n-3}{2}}, v_{\frac{n+1}{2}}\right\}, V_{\frac{n-1}{2}} = \left\{v_{\frac{n-1}{2}}, v_n\right\}$ *and* $V_{\frac{n+1}{2}} = \{v_{n-1}\}$ which is the required chromatic similar degree partition.

Hence, for any $n \geq 3$, $\chi_{\wedge}\psi_D(I_n) = \chi_{\vee}\psi_D(I_n) = \left\lfloor \frac{n}{2} \right\rfloor + 1$ □

Theorem 17 Let $G \cong K_1 \vee P_{n-1}$ where $n \geq 4$. Then

$$\chi_{\wedge}\psi_D(G) = \begin{Bmatrix} 3 & if\ n = 4,5 \\ 4 & if\ n > 5\ and\ n \equiv 1,2(mod\ 3) \\ \infty & otherwise \end{Bmatrix}$$

and $\chi_{\vee}\psi_D(G) = \begin{Bmatrix} 4 & n \equiv 1,2(mod\ 3) \\ \infty & otherwise \end{Bmatrix}$.

Proof The degree sequence of G is $(n-1, 3^{n-3}, 2^2)$ and G has one full vertex.

Clearly, $D(V) = n - 1 + 3(n-3) + 2(2) = 4n - 6$ and so

$$\left\lceil \frac{D(V)+1}{n} \right\rceil = \left\lceil \frac{4n-5}{n} \right\rceil = \left\lceil 4 - \frac{5}{n} \right\rceil = \begin{Bmatrix} 3 & if\ n = 4,5 \\ 4 & if\ n > 5 \end{Bmatrix}$$

and $\left\lfloor \frac{D(V)-1}{n-2} \right\rfloor = \left\lfloor \frac{4n-7}{n-2} \right\rfloor = \left\lfloor 4 + \frac{1}{n-2} \right\rfloor = 4$ as $n \geq 4$.

So, if chromatic similar degree partition exists, $\chi_{\wedge}\psi_D(G) \geq \begin{Bmatrix} 3 & if\ n = 4,5 \\ 4 & if\ n > 5 \end{Bmatrix}$ and

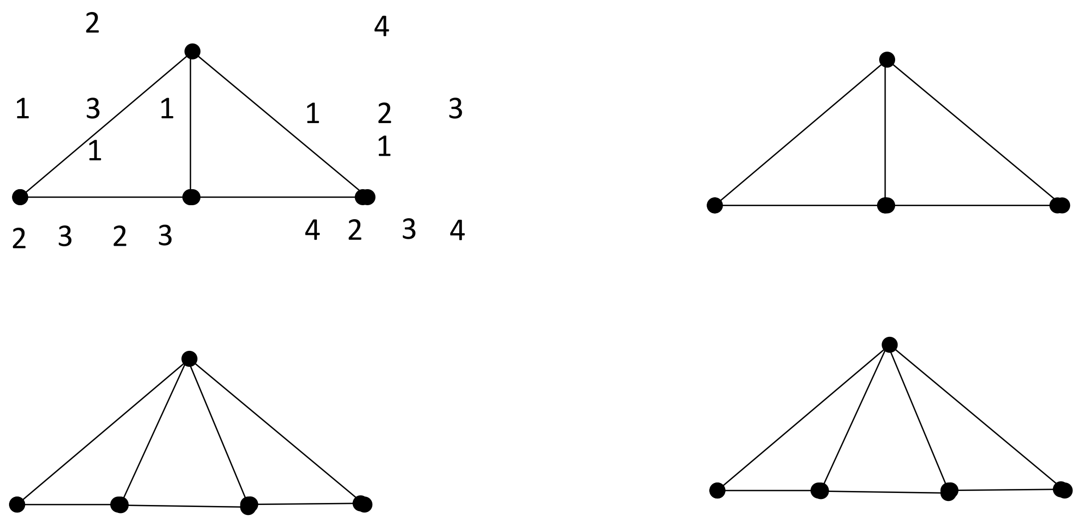
$\chi_{\vee}\psi_D(G) \leq 4$ for any $n \geq 4$.

The chromatic similar degree partition, for the case 4 and 5 are illustrated in Figure 1.

Now, for $n \geq 6$, $\chi_{\wedge}\psi_D(G) = \chi_{\vee}\psi_D(G) = 4$ if any chromatic similar degree partition exists.

Let $V(G) = \{v_1, v_2, \dots, v_n\}$ such that $\deg v_i = \begin{Bmatrix} 3 & if\ 2 \leq i \leq n-2 \\ 2 & if\ i = 1, n-1 \\ n-1 & if\ i = n \end{Bmatrix}$.

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 15 Issue 07 , July - 2026



Min-chromatic Similar Degree Partition   Max-chromatic Similar Degree Partition

Figure 1

Clearly, $\{v_n\}$ forms an isolated partition in chromatic similar degree partition. So, we need there more independent classes from $\{v_1, v_2, .., v_{n-1}\}$ which are $V_1 = \{v_1, v_4, ...\}, V_2 = \{v_2, v_5, ...\}$ and $V_3 = \{v_3, v_6, ...\}$.

If $n \equiv 0(mod\ 3)$, then $v_{n-1} \in V_2$ and $D(V_1) = n-1, D(V_2) = n-1$ and $D(V_3) = n-3$, which is not a similar degree partition.

So, $\chi_\wedge \psi_D(G) = \chi_\vee \psi_D(G) = \infty$ if $n \equiv 0(mod\ 3)$

Next, if $n \equiv 1(mod\ 3)$, then $v_{n-1} \in V_3$ and $D(V_1) = n-2, D(V_2) = n-1$ and $D(V_3) = n-2$.

Also, if $n \equiv 2(mod\ 3)$, then $v_{n-1} \in V_1$ and $D(V_1) = n-1, D(V_2) = n-2$ and $D(V_3) = n-2$.

Hence, $\chi_\wedge \psi_D(G) = \chi_\vee \psi_D(G) = 4$ if $n \equiv 1,2(mod\ 3)$. □

Theorem 18 $\chi_\wedge \psi_D\big(T(n,k)\big) = \begin{Bmatrix} k & if\ n \equiv 0(mod\ k)\ or\ k = 1,2\ or\ n = 3,4 \\ n & otherwise \end{Bmatrix}$ for $n \geq 3$ and $k \geq 1$.

Proof By Observation 2, $\chi_\wedge \psi_D(G) \geq k$.

Also, the result is obvious when $k = 1\ or\ 2$ or $n = 3\ or\ 4\ or\ 5$.

Now, the colour classes in the k-colouring partition $n \geq 6$ consists of $\left\lfloor \frac{n}{k} \right\rfloor$ vertices or $\left\lceil \frac{n}{k} \right\rceil$ vertices.

Without loss of generality, we assume that the colour classes $V_1, V_2, \dots, V_r$ $(r \leq k)$ contains $\left\lfloor \frac{n}{k} \right\rfloor$ vertices and $V_{r+1}, V_{r+2}, \dots, V_k$ contains $\left\lceil \frac{n}{k} \right\rceil$ vertices.

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 15 Issue 07 , July - 2026

$$\deg v = \begin{cases} n - \left\lfloor \frac{n}{k} \right\rfloor & if\ v \in V_i, 1 \le i \le r \\ n - \left\lceil \frac{n}{k} \right\rceil & if\ v \in V_j, r+1 \le j \le k \end{cases}$$

So, $D(V_i) = \begin{cases} \left\lfloor \frac{n}{k} \right\rfloor \left(n - \left\lfloor \frac{n}{k} \right\rfloor\right) & if\ 1 \le i \le r \\ \left\lceil \frac{n}{k} \right\rceil \left(n - \left\lceil \frac{n}{k} \right\rceil\right) & if\ r+1 \le i \le k \end{cases}$

By taking $\left\lfloor \frac{n}{k} \right\rfloor = t$, for any $1 \le i \le r$ and $r+1 \le j \le k$,

$$D(V_j) - D(V_i) = \begin{cases} t(n-t) - t(n-t) & if\ n \equiv 0 (mod\ k) \\ (t+1)(n-(t+1)) - t(n-t) & otherwise \end{cases}$$

$$= \begin{cases} 0 & if\ n \equiv 0 (mod\ k) \\ n - 2t - 1 & otherwise \end{cases}$$

But the case $n - 2t - 1 = 1$ is impossible when $k \ge 3$ and $n \ge 6$.

Thus $\chi_{\wedge}\psi_D(T(n,k)) = k$ whenever $n \equiv 0 (mod\ k)$.

In addition, for any $n \ge 6, k \ge 3$ and $n \not\equiv 0(mod\ k)$, the only possible chromatic similar degree partition is isolated partition and hence $\chi_{\wedge}\psi_D(T(n,k)) = n$. □

Corollary 19 Let G be a $(n-2) - regular$ graph, $n \ge 3$. Then $\chi_{\wedge}\psi_D(G) = \frac{n}{2}$.

Proof As G is $(n-2) - regular$, n is even.

Each vertex in G is not adjacent to exactly one vertex. So, G is $T\left(n, \frac{n}{2}\right)$ graph.

Hence by the above theorem, $\chi_{\wedge}\psi_D(G) = \frac{n}{2}$. □

Theorem 20 Let G be a $(n-3) - regular$ graph where $n \ge 4$. Then

$$\chi_{\wedge}\psi_D(G) = \begin{cases} \frac{n}{3} & if\ n \equiv 0(mod\ 3)\ and\ G \cong T\left(n, \frac{n}{3}\right) \\ \frac{n}{2} & else\ if\ n \equiv 0(mod\ 2) \\ n & otherwise \end{cases}$$

Proof G can be obtained by removing a spanning cycle or a vertex disjoint union of cycles which covers all the vertices of G, from K_n.

Case i Let G be obtained by removing vertex disjoint union of C_3's.

In this case, $n \equiv 0\ (mod\ 3)$ and $G \cong T\left(n, \frac{n}{3}\right)$.

So, $\chi_{\wedge}\psi_D(G) = \frac{n}{3}$

Case ii Let G be obtained by removing at least one cycle of length at least 4.

Without loss of generality, let $v_0, v_1, v_2, \dots, v_r, v_0$ $(r \ge 3)$ be a cycle removed to obtain G.

Clearly v_i is non-adjacent to v_{i-1} and v_{i+1} where $0 \le i \le r$ and the addition and subtraction are taken over modulo r.

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 15 Issue 07 , July - 2026

Also, v_{i-1} and v_{i+1} are adjacent in G.

So, from these r vertices $v_1, v_2, \dots, v_{i-1}, v_{i+1}, \dots, v_r$, a maximum of two vertices can be in a chromatic similar degree partition whose degree sum will be $2(n-3)$.

In order to attain the required degree sum, all the vertices in G should be partitioned into $\frac{n}{2}$ classes of non-adjacent vertices.

So, whenever $n \equiv 0 \ (mod\ 2)$, this partition is possible and so $\chi_\wedge \psi_D(G) = \frac{n}{2}$.

And suppose $n \not\equiv 0 (mod\ 2)$ or G is obtained from K_n by removal of a cycle of length at least 4, $\chi_\wedge \psi_D(G) = n$ as the only possible chromatic similar degree partition is isolated partition.

3 REFERENCES

[1] Alexey Pokrovskiy, (2012). *Partitioning edge-coloured complete graphs into monochromatic cycles and paths*, Journal of Combinatorial Theory, Series B.

[2] Harary. F, Graph Theory, (1972). Addison-Wesly, Reading Mass.

[3] Malathi. N, M. Bhuvaneshwari and Selvam Avadayappan, (2021). *A Note on Degree Partition Number of a Graph,* Journal of Emerging Technologies and Innovative Research, Volume 8, Issue 7, Pages a717-a722.

[4] Malathi. N, M. Bhuvaneshwari and Selvam Avadayappan, (2023). *More Results on Degree Partition Number*, Advances and Applications in Mathematical Sciences, Volume 22, Issue 8, Pages 1905 – 1914.

[5] Malathi. N, M. Bhuvaneshwari and Selvam Avadayappan, (2025). *Degree Partition Number of Some Derived Graphs*, IAENG International Journal of Applied Mathematics, Volume 55, Issue 6, Pages1865-1872.

[6] Sampathkumar. E, (1976). *Partition graphs and coloring numbers of a graph*. Discrete Mathematics - DM. 16. 57-60. 10.1016/0012-365X (76) 90093-5.

[7] Shoham Letzter, (2019). *Monochromatic cycle partitions of 2-coloured graphs with minimum degree 3n/4*, the electronic journal of combinatorics, Volume 26, Issue 1.

[8] Yegnanarayanan. V, (2001). *Graph colourings and partitions*, Theoretical Computer Science, Volume 263, Issues 1–2, Pages 59-74.