On bĝ – Continuous Maps and bĝ – Open Maps in Topological Spaces

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Abstract

Recently the author[19] defined bĝ–Closed sets and studied many basic properties. In this paper a new class of maps namely bĝ– Continuous map and bĝ– Open map were introduced in Topological Spaces and we find some of its basic properties. Further a new class of bĝ– homeomorphisms are also introduced and studied some of their relationship among other homeomorphisms.

1. Introduction

In 1996, Andrijevic[14] introduced one such new version called b-open sets. Levine[5] introduced the concept of generalized closed sets and studied their properties. By considering the concept of g–closed sets many concepts of topology have been generalized and interesting results have been obtained by several mathematician. Veerakumar[12] introduced ĝ–closed sets. Recently R.Subasree and M.MariaSingam[19] introduced bĝ–closed sets.

Balachandran et al[17] introduced the concept of generalized continuous maps in topological spaces. The purpose of this paper is to introduce a new version of maps called bĝ–continuous map and bĝ–open map. Moreover we introduce the concept of bĝ–homeomorphism and we investigated the properties of all such transformations.

2. Preliminaries

Throughout this paper (X, τ) (or simply X) and (Y, σ) (or simply Y) represents topological spaces on which no separation axioms are assumed unless otherwise mentioned. Let us recall the following definitions.

Definition 2.1 : A subset A of a space (X,τ) is called a

i) Semi–open set if A ⊑ cl[Int (A)]

ii) α–open set if A ⊑ Int[cl(Int (A))]

ii) b–open set if A ⊑ cl[Int (A)] ⋃ Int [cl(A)]

The complement of a semi–open (resp. α–open, b–open) set is called semi-closed (resp. α–closed, b–closed) set.

The intersection of all semi-closed (resp. α-closed, b-closed) sets of X containing A is called the semi-closure (resp. α-closure, b-closure) and is denoted by scl(A) (resp. αcl(A), bcl(A)). The family of all semi-open (resp. α-open, b-open) subsets of a space X is denoted by SO(X), (resp. αO(X), bO(X)).

Definition 2.2: A subset A of a space (X,τ) is called a

i) **generalized closed** (briefly g-closed) set[5] if cl(A)⊑U whenever A⊑U and U is open set in (X,τ).

ii) **semi-generalized closed** (briefly sg-closed) set[2] if scl(A)⊑U whenever A⊑U and U is a semi-open set in (X,τ).

iii) **generalized semi-closed** (briefly gs-closed) set[1] if scl(A) ⊑U whenever A⊑U and U is open set in (X,τ).

iv) **α-generalized closed** (briefly αg-closed) set[7] if αcl(A) ⊑U whenever A⊑U and U is open set in (X,τ).

v) **generalized α-closed** (briefly gα-closed) set[6] if αcl(A) ⊑U whenever A⊑U and U is α-open set in (X,τ).

vi) **δ-generalized closed** (briefly δg-closed) set[3] if clδ(A)⊑U whenever A⊑U and U is open set in (X,τ).

vii) **ĝ -closed** set[12] if cl(A) ⊑U whenever A⊑U and U is a semi-open set in (X,τ).

viii) **αĝ -closed** set[9] if αcl(A) ⊑U whenever A⊑U and U is ĝ-open set in (X,τ).

ix) **gb-closed** set[15] if bcl(A) ⊑U whenever A⊑U and U is open set in (X,τ).

The complement of a g-closed(resp. sg-closed, gs-closed, αg-closed, gα-closed, δg-closed, ĝ –closed and αĝ–closed) set is called g-open (resp. sg-open, gs-open, αg-open, gα-open, δg-open, ĝ –open and αĝ –open) set .

Definition 2.3: A function f: (X,τ) $\longrightarrow$ (Y, σ) is called

i) **Continuous**[12] if $f^{-1}(V)$ is closed in (X,τ) for every closed set V in (Y, σ)

ii) **g–continuous** [17] if $f^{-1}(V)$ is g –closed in (X,τ) for every closed set V in (Y, σ)

iii) **ĝ –continuous**[20] if $f^{-1}(V)$ is ĝ –closed in (X,τ) for every closed set V in (Y, σ)

iv) **αg –continuous**[7] if $f^{-1}(V)$ is αg–closed in (X,τ) for every closed set V in (Y, σ)

v) **αĝ –continuous** if $f^{-1}(V)$ is αĝ –closed in (X,τ) for every closed set V in (Y, σ)

vi) **b –continuous** [25] if $f^{-1}(V)$ is b –closed in (X,τ) for every closed set V in (Y, σ)

vii) **gb–continuous**[25] if $f^{-1}(V)$ is gb–closed in (X,τ) for every closed set V in (Y, σ)

viii) **gs–continuous**[22] if $f^{-1}(V)$ is gs –closed in (X,τ) for every closed set V in (Y, σ)

ix) **sg–continuous**[24] if $f^{-1}(V)$ is sg–closed in (X,τ) for every closed set V in (Y, σ)

Definition 2.4: A function f: (X,τ) $\longrightarrow$ (Y, σ) is called a

i) **open map**[12] if $f(V)$ is open in (Y, σ) for every open set V in (X,τ)

ii) **g–open map**[23] if $f(V)$ is g–open in (Y, σ) for every open set V in (X,τ)

iii) **ĝ –open map**[12] if $f(V)$ is ĝ –open in (Y, σ) for every open set V in (X,τ)

iv) **gs–open map**[21] if $f(V)$ is gs–open in (Y,σ) for every open set V in (X,τ)

v) **sg– open map**[21] if $f(V)$ is sg–open in (Y,σ) for every open set V in (X,τ)

Definition 2.5: A function f: (X,τ) $\longrightarrow$ (Y, σ) is called a

i) **Homeomorphism**[12] if f is both continuous map and open map.

ii) **g–homeomorphism**[23] if f is both g–continuous map and g–open map.

iii) **ĝ–homeomorphism**[12] if f is both ĝ–continuous map and ĝ–open map.

iv) **sg–homeomorphism**[22] if f is both sg–continuous map and sg–open map.

v) gs–homeomorphism[22] if f is both gs–continuous map and gs–open map

3. bĝ – Continuous functions

We introduce the following definitions:

Definition 3.1: A function f: (X,τ) $\longrightarrow$ (Y, σ) is said to be bĝ – continuous map if f^{-1}(V) is bĝ-closed in (X,τ) for every closed set V of (Y, σ).

Example 3.2: Let X=Y={a,b,c}
τ = {X, Φ, {a}} and
σ = {Y, Φ, {a}, {b}, {a,b},{a,c}}
Define a function f: (X,τ) $\longrightarrow$ (Y, σ) by f(a) = a, f(b) = b, f(c) = c. Then f is bĝ – continuous, since the inverse images of a closed sets {b}, {b,c}, {a,c}, {c} in (Y, σ) are {b},{b,c}, {a,c}, {c} respectively which are bĝ – Closed in (X,τ).

Theorem 3.3: Every continuous map is bĝ – continuous.

Proof: Let V be a closed set in (Y, σ). Since f is continuous, then f^{-1}(V) is closed in (X,τ). Since from[19] Remark 3.23 "Every closed set is bĝ – Closed". Then f^{-1}(V) is bĝ – Closed in (X,τ). Hence f is bĝ – continuous.

Remark 3.4: The converse of the above theorem need not be true.
(i.e) Every bĝ – continuous need not be a continuous map as shown in the following example.

Example 3.5: Let X=Y={a,b,c}
τ = {X, Φ, {a}} and σ = {Y, Φ, {b}}
Define a function f: (X,τ) $\longrightarrow$ (Y, σ) by f(a) = a, f(b) = b, f(c) = c.
Then f is bĝ – continuous, but not continuous, since the inverse image of a closed set {a,c}in (Y, σ) is {a,c} which is bĝ – closed but not closed in (X,τ).

Theorem 3.6: Every g-continuous map is bĝ – continuous.

Proof: Let V be a closed set in (Y, σ). Since f is g-continuous, then f^{-1}(V) is g-closed in (X,τ). Since from[19] Proposition 3.6, "Every g-closed set is bĝ – Closed". Then f^{-1}(V) is bĝ –Closed in (X,τ). Hence f is bĝ-continuous.

Remark 3.7: The converse of the above theorem need not be true.
(i.e) Every bĝ – continuous need not be a g-continous map as shown in the following example.

Example 3.8: Let X=Y={a,b,c}
τ = {X, Φ,{a},{b},{a,b}}and σ = {Y, Φ,{a,c}}
Define a function f: (X,τ) $\longrightarrow$ (Y, σ) by f(a) = a, f(b) = b, f(c) = c.
Then f is bĝ – continuous, but not g-continuous, since the inverse image of a closed set {b} in (Y, σ) is {b} which is bĝ – closed but not g-closed in (X,τ).

Theorem 3.9: Every b-continuous map is bĝ – continuous.

Proof: Let V be a closed set in (Y, σ). Since f is b-continuous, then f^{-1}(V) is b-closed in (X,τ). Since from[19] Proposition 3.3, "Every b-closed set is bĝ – Closed". Then f^{-1}(V) is bĝ –Closed in (X,τ).Hence f is bĝ - continuous.

Remark 3.10: The converse of the above theorem need not be true.
(i.e) Every bĝ – continuous need not be a b-continous map as shown in the following example.

Example 3.11: Let X=Y={a,b,c}
τ = {X, Φ, {a}} and
σ = {Y, Φ, {a}, {b},{a,b},{a,c}}
Define a function f: (X,τ) ⟶ (Y, σ) by f(a) = a, f(b) = b, f(c) = c.

Then f is bĝ-continuous, but not b-continuous, since the inverse image of a closed set {a,c} in (Y, σ) is {a,c} which is bĝ – Closed but not b-closed in (X,τ).

Theorem 3.12: Every gb-continuous map is bĝ – continuous.

Proof: Let V be a closed set in (Y, σ). Since f is gb-continuous, then f^{-1}(V) is gb-closed in (X,τ). Since from[19] Proposition 3.18, "Every gb-closed set is bĝ – Closed". Then f^{-1}(V) is bĝ – Closed in (X,τ). Hence f is bĝ – continuous.

Corollary 3.13: The converse of the above theorem is also true.
(i.e) Every bĝ – continuous is gb-continous.

Proof: Let V be a closed set in (Y, σ). Since f is bĝ-continuous, then f^{-1}(V) is bĝ -closed in (X,τ). Since from[19] Corollary 3.19, "Every bĝ -closed set is gb– Closed". Then f^{-1}(V) is gb– Closed in (X,τ). Hence f is gb-continuous.

Remark 3.14: The following example shows the relationship between bĝ – continuous map and gb-continous map.

bĝ – continuous ⇄ gb-continous

Example 3.15: Let X=Y={a,b,c}
τ = {X, Φ, {a,c}}and
σ = {Y, Φ, {a}, {b}, {a,b}, {a,c}}
Define a function f: (X,τ) ⟶ (Y, σ) by f(a) = a, f(b) = b, f(c) = c.
bĝ-closed set in X = {X, Φ, {a}, {b}, {c}, {a,b}, {b,c}}
gb-closed set in X = {X, Φ,{a},{b},{c},{a,b}, {b,c}}
closed sets in Y = {Y, Φ,{b},{c},{a,c},{b,c}}
Clearly f is both bĝ – continuous and gb-continuous.

Theorem 3.16: Every ĝ-continuous map is bĝ – Continuous.

Proof: Let V be a closed set in (Y, σ). Since f is ĝ-continuous, then f^{-1}(V) is ĝ-closed in (X,τ). Since from[19] Proposition 3.9, "Every ĝ-closed set is bĝ – Closed". Then f^{-1}(V) is bĝ – Closed in (X,τ). Hence f is bĝ – Continuous.

Remark 3.17: The converse of the above theorem need not be true.
(i.e) Every bĝ – continuous need not be ĝ-continous map as shown in the following example.

Example 3.18: Let X=Y={a,b,c}
τ = {X, Φ, {a,c}}and
σ = {Y, Φ, {a}, {b}, {a,b}, {a,c}}
Define a function f: (X,τ) ⟶ (Y, σ) by
f(a) = b, f(b) = a, f(c) = a.
bĝ-closed set in X = {X, Φ,{a},{b},{c},{a,b}, {b,c}}
ĝ-closed set in X = {X, Φ, {b}, {a,b}, {b,c}}

Then f is bĝ – continuous, but not ĝ-continuous, since the inverse image of a closed set {b,c} in (Y, σ) is {a} which is bĝ – closed but not ĝ-closed in (X,τ).

Theorem 3.19: Every gs-continuous map is bĝ – continuous.

Proof: Let V be a closed set in (Y, σ). Since f is gs-continuous, then f^{-1}(V) is gs-closed in (X,τ). Since from[19] Proposition 3.12 "Every gs-closed set is bĝ – Closed". Then f^{-1}(V) is bĝ – closed in (X,τ). Hence f is bĝ – Continuous.

Remark 3.20: The converse of the above theorem need not be true.
(i.e) Every bĝ – continuous need not be gs-continous map as shown in the following example.

Example 3.21: Let X=Y={a,b,c}
τ = {X, Φ, {a,c}}and σ = {Y, Φ, {a},{a,b}}
Define a function f: (X,τ) ⟶ (Y, σ) by

f(a) = c, f(b) = a, f(c) = a.
bĝ-closed set in X = {X, Φ, {a}, {b}, {c}, {a,b}, {b,c}}
gs-closed set in X = {X, Φ, {b}, {a,b}, {b,c}}

Then f is bĝ – continuous, but not gs-continuous, since the inverse image of a closed set {c} in (Y, σ) is {a} which is bĝ – closed but not gs-closed in (X,τ).

Theorem 3.22: Every sg-continuous map is bĝ – continuous.
Proof: Let V be a closed set in (Y, σ). Since f is sg-continuous, then f^{-1}(V) is sg-closed in (X,τ). Since from [19] Remark 3.23 "Every sg-closed is gs-closed" and from [19] proposition (3.12) "Every gs-closed set is bĝ – closed", we have "Every sg-closed set is bĝ – closed" . Hence f^{-1}(V) is bĝ – closed in (X,τ). Thus f is bĝ – Continuous.

Remark 3.23: The converse of the above theorem need not be true.
(i.e) Every bĝ – continuous need not be sg-continous map as shown in the following example.

Example 3.24: Let X=Y={a,b,c}
τ = {X, Φ, {a}}and σ = {Y, Φ, {a,b},{c}}
Define a function f: (X,τ) ⟶ (Y, σ) by
f(a) =a, f(b) = b, f(c) = c.
bĝ-closed set in X = {X, Φ, {b}, {c}, {a,b}, {a,c}, {b,c}}
sg-closed set in X = {X, Φ, {b}, {c}, {b,c}}

Then f is bĝ – continuous, but not sg-continuous, since the inverse image of a closed set {a,b} in (Y, σ) is {a,b} which is bĝ – closed but not sg-closed in (X,τ).

Theorem 3.25: Every αg-continuous map is bĝ – continuous.
Proof: Let V be a closed set in (Y, σ). Since f is αg-continuous, then f^{-1}(V) is αg-closed in (X,τ). Since from[19] Proposition [3.15] "Every αg-closed is bĝ – closed", we have f^{-1}(V) is bĝ – closed in (X,τ).Thus f is bĝ – Continuous.

Remark 3.26: The converse of the above theorem need not be true.
(i.e) Every bĝ – continuous need not be αg-continous map as shown in the following example.

Example 3.27: Let X=Y={a,b,c}
τ = {X, Φ, {a}, {b}, {a,b}} and
σ = {Y, Φ, {b}}
Define a function f: (X,τ) ⟶ (Y, σ) by
f(a) =a, f(b) = b, f(c) = b.
bĝ-closed set in X = {X, Φ, {a}, {b}, {c}, {a,c}, {b,c}}
αg-closed set in X = {X, Φ, {c}, {a,c}, {b,c}}

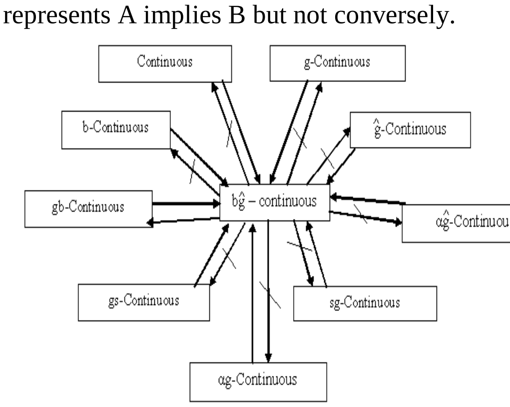
Then f is bĝ – continuous, but not αg-continuous, since the inverse image of a closed set {a,c} in (Y, σ) is {a} which is bĝ – closed but not αg-closed in (X,τ).

Theorem 3.28: Every αĝ-continuous map is bĝ – continuous.
Proof: Let V be a closed set in (Y, σ). Since f is αĝ-continuous, then f^{-1}(V) is αĝ-closed in (X,τ). Since from[19] Proposition [3.20] "Every αĝ-closed is bĝ – closed", we have f^{-1}(V) is bĝ – closed in (X,τ). Thus f is bĝ – Continuous.

Remark 3.29: The converse of the above theorem need not be true.
(i.e) Every bĝ – continuous need not be αĝ-continous map as shown in the following example.

Example 3.30: Let X=Y={a,b,c}
τ = {X, Φ, {a},{b,c}}and
σ = {Y, Φ, {a},{b},{a,b}}
Define a function f: (X,τ) ⟶ (Y, σ) by
f(a) =a, f(b) = b, f(c) = c.
bĝ-closed set in X = {X, Φ, {a}, {b}, {c}, {a,b},{a,c}, {b,c}}
αĝ-closed set in X = {X, Φ, {a}, {b,c}}
Then f is bĝ – continuous, but not αĝ-continuous, since the inverse image of a closed set {c} in (Y, σ) is {c} which is bĝ – closed but not αĝ-closed in (X,τ).

Remark 3.31: The following diagram shows the relationships of bĝ – continuous map with other known existing maps. A ⟶ B represents A implies B but not conversely.



4 Applications

Remark 4.1: The composition of two bĝ – continuous functions need not be bĝ – continuous. For we consider the following example.

Example 4.2: Let X={a,b,c}
τ = {X, Φ, {a},{b},{a,b}}, σ = {X, Φ, {b}} and
η = {X, Φ, {a}}
Define a function f: (X,τ)⟶ (X, σ) by
f(a) =a, f(b) = b, f(c) = c and
Define a function g: (X,σ)⟶ (X,η) by
g(a) =b, g(b) = c, g(c) = a.
Clearly f and g are bĝ – continuous.
But for a closed set {b,c} in (X,η)
$(f\ o\ g)^{-1}$\{b,c\} = $g^{-1}[f^{-1}\{b,c\}] = g^{-1}\{b,c\}$
= {a,b} which is not bĝ – closed in (X,τ). Hence f o g is not bĝ-continuous.

Definition 4.3: A function f: (X,τ) ⟶(Y, σ) is said to be bĝ – irresolute if f^{-1}(V) is bĝ-closed in (X,τ) for every bĝ-closed set V of (Y, σ).

Remark 4.4: The composition of two bĝ –irresolute functions is again bĝ – irresolute.

5 bĝ – open maps and bĝ – closed maps

We introduce the following definitions:

Definition 5.1: Let X and Y be two topological spaces. A map f: (X,τ) ⟶(Y, σ) is called bĝ –open map if the image of every open set in X is bĝ-open in (Y, σ).

Definition 5.2: Let X and Y be two topological spaces. A map f: (X,τ) ⟶(Y, σ) is called bĝ – closed map if the image of every closed set in X is bĝ-closed in (Y, σ).

Theorem 5.3: Every open map is bĝ-open map.
Proof: Let f: (X,τ)⟶(Y, σ) is a open map and V be a open set in X, then f(V) is a open set in Y. Since[19] Proposition(3.3), "Every open set is bĝ-open set", we have f(V) is a bĝ-open set in Y. Thus f is bĝ-open map.

Remark 5.4: The converse of the above theorem need not be true.
(i.e) Every bĝ-open map need not be a open map as shown in the following example.

Example 5.5: Let X=Y={a,b,c}
τ = {X, Φ, {a,c}}and σ = {Y, Φ,{b}}
Define a function f: (X,τ) ⟶(Y, σ) by
f(a) =b, f(b) = b, f(c) = c.
Open sets in X = {X, Φ, {a,c}}
bĝ– open set in Y = {Y, Φ, {a}, {b}, {c}, {a,b},{b,c}}
Here f is bĝ – open map, but not a open map, since the image of a open set {a,c} in (X, τ) is {b,c} which is bĝ – open but not open in (Y, σ) .

Theorem 5.6: Every g–open map is bĝ–open map.
Proof: Let f: (X,τ) ⟶ (Y, σ) is a g–open map and V be a open set in X, then f(V) is a g–open set in Y. Since from[19] Proposition(3.6), "Every g–open set is bĝ–open set", we have f(V) is a bĝ–open set in Y. Thus f is a bĝ–open map.

Remark 5.7: The converse of the above theorem need not be true.
(i.e) Every bĝ–open map need not be a g–open map as shown in the following example.

Example 5.8: Let X=Y={a,b,c}
τ = {X, Φ, {a}, {b},{a,b},{a,c}}and
σ = {Y, Φ,{a,c}}
Define a function f: (X,τ)⟶ (Y, σ) by
f(a) =b, f(b) = c, f(c) = b.
Open sets in X = {X, Φ, {a}, {b},{a,b},{a,c}}
bĝ– open set in Y = {Y, Φ, {a}, {c}, {a,b},{a,c},{b,c}}
g–open set in Y = {Y, Φ, {a}, {c}, {a,c}}
Here f is bĝ – open map, but not a g–open map, since the image of a open set {a,c} in (X,τ) is {a,b} which is bĝ – open but not g–open in (Y, σ) .

Theorem 5.9: Every ĝ–open map is bĝ–open map.
Proof: Let f: (X,τ) ⟶ (Y, σ) is a ĝ–open map and V be a open set in X, then f(V) is a ĝ– open set in Y. Since from[19] Proposition(3.9), "Every ĝ–open set is bĝ–open set", we have f(V) is a bĝ–open set in Y. Thus f is a bĝ–open map.
Remark 5.10: The converse of the above theorem need not be true.
(i.e) Every bĝ–open map need not be a ĝ–open map as shown in the following example.

Example 5.11: Let X=Y={a,b,c}
τ = {X, Φ, {a}, {b},{a,b}}and σ = {Y, Φ,{a}}
Define a function f: (X,τ)⟶ (Y, σ) by
f(a) =a, f(b) = b, f(c) = c.
Open sets in X = {X, Φ, {a}, {b},{a,b}}
bĝ– open sets in Y = {Y, Φ, {a},{b}, {c}, {a,b},{a,c}}
ĝ–open sets in Y = {Y, Φ, {a}}

Here f is bĝ – open map, but not a ĝ–open map, since the image of a open set {a,b} in (X,τ) is {a,b} which is bĝ – open set but not ĝ–open set in (Y, σ) .

Theorem 5.12: Every sg–open map is a bĝ–open map.
Proof: Let f: (X,τ) ⟶ (Y, σ) is a sg–open map and V be a open set in X, then f(V) is a sg– open set in Y. Since from[19] Remark(3.23), "Every sg–open set is bĝ–open set", we have f(V) is a bĝ–open set in Y. Thus f is a bĝ–open map.

Remark 5.13: The converse of the above theorem need not be true.
(i.e) Every bĝ–open map need not be sg–open map as shown in the following example.

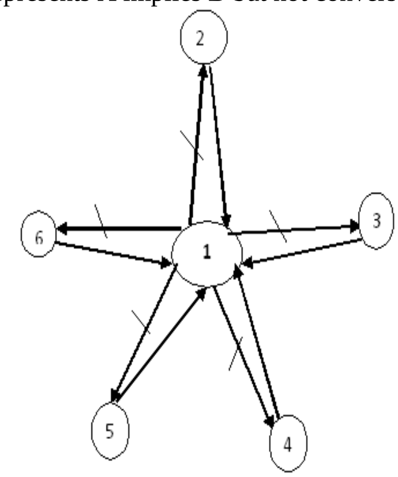
Example 5.14: Let X=Y={a,b,c}
τ = {X, Φ,{a,c}}and σ = {Y, Φ,{a}}
Define a function f: (X,τ)⟶ (Y, σ) by
f(a) =b, f(b) = a, f(c) = b.
Open sets in X = {X, Φ,{a,c}}
bĝ– open sets in Y = {Y, Φ, {a},{b}, {c}, {a,b},{a,c}}
sg–open sets in Y = {Y, Φ, {a},{a,b},{a,c}}
Here f is bĝ – open map, but not a sg–open map, since the image of a open set {a,c} in (X,τ) is {b} which is bĝ – open set but not sg–open set in (Y, σ) .

Theorem 5.15: Every gs–open map is a bĝ–open map.
Proof: Let f: (X,τ) ⟶ (Y, σ) is a gs–open map and V be a open set in X, then f(V) is a gs– open set in Y. Since from[19] Proposition(3.12),"Every gs–open set is bĝ–open set", we have f(V) is a bĝ–open set in Y. Thus f is a bĝ–open map.

Remark 5.16: The converse of the above theorem need not be true.
(i.e) Every bĝ–open map need not be a gs–open map as shown in the following example.

Example 5.17: Let X=Y={a,b,c}
τ = {X, Φ,{a},{b,c}}and σ = {Y, Φ,{a,c}}
Define a function f: (X,τ)⟶ (Y, σ) by
f(a) =a, f(b) =b, f(c) = c.
Open sets in X = {X, Φ,{a},{b,c}}
bĝ– open sets in Y = {Y, Φ, {a},{c}, {a,b},{a,c},{b,c}}
gs–open sets in Y = {Y, Φ, {a},{c},{a,c}}
Here f is bĝ – open map, but not a gs–open map, since the image of a open set {b,c} in (X,τ) is {b,c} which is bĝ – open set but not gs–open set in (Y, σ) .

Remark 5.18: The following diagram shows the relationships of bĝ – open map with other known existing open maps. A ⟶ B represents A implies B but not conversely.



1. bĝ – open map 2. Open map
3. g–open map 4. ĝ – open map
5. sg–open map 6. gs – open map

6 bĝ – Homeomorphisms

Definition 6.1:A bijection f: (X,τ)⟶(Y,σ) is called a bĝ – homeomorphism if f is both bĝ – continuous map and bĝ – open map.

Example 6.2: Let X=Y={a,b,c}
τ = {X, Φ,{a}}and σ = {Y, Φ,{b}}
Define a function f: (X,τ)⟶ (Y, σ) by
f(a) =c, f(b) = a, f(c) = b.
bĝ –closed sets in X = {X,Φ,{b},{c},{a,b}, {a,c},{b,c}}
bĝ –open sets in Y = {Y,Φ,{a},{b},{c},{a,b},{b,c}}
sg –closed sets in X = {X,Φ,{b},{c},{b,c}}
sg –open sets in Y = {Y,Φ,{b},{a,b},{b,c}}
Here the inverse image of a closed set {a,c} in Y is {a,b} which is bĝ –closed in X and the image of a open set {a} in X is {c} which is bĝ –open in Y . Hence f is bĝ – homeomorphism.

Theorem 6.3: Every homeomorphism is a bĝ – homeomorphism
Proof: Follows from theorem 3.3 "Every Continuous map is bĝ – continuous" and from theorem 5.3 "Every open map is bĝ – open map.

Remark 6.4: The converse of the above theorem need not be true.
(i.e) Every bĝ–homeomorphism need not be a homeomorphism as shown in the following example.

Example 6.5: Let X=Y={a,b,c}
τ = {X, Φ,{a},{b,c}}and σ = {Y, Φ,{a,c}}
Define a function f: (X,τ)⟶ (Y, σ) by
f(a) =a, f(b) = b, f(c) = c.
bĝ –closed sets in X
={X,Φ,{a},{b},{c},{a,b}, {a,c},{b,c}}
open sets in Y = {Y,Φ,{a,c}}
closed sets in X = {X,Φ,{a},{b,c}}
bĝ –open sets in Y =
{Y,Φ,{a},{c},{a,b},{a,c},{b,c}}

Here the inverse image of a closed set {b} in Y is {b} which is bĝ –closed in X but not closed in X and the image of a open set {a} in X is {a} which is bĝ –open in Y but not open in Y.

Hence f is bĝ – homeomorphism, but not a homeomorphism, since f is not a open-map and not a continuous map.

Theorem 6.6: Every sg–homeomorphism is a bĝ – homeomorphism
Proof: Follows from theorem 3.22 "Every sg–continuous map is bĝ – continuous" and by theorem 5.12 "Every sg–open map is bĝ – open map.

Remark 6.7: The converse of the above theorem need not be true.
(i.e) Every bĝ–homeomorphism need not be a sg–homeomorphism as shown in the following example.

Example 6.8: Let X=Y={a,b,c}
τ = {X, Φ,{a}}and σ = {Y, Φ,{b}}
Define a function f: (X,τ)⟶ (Y, σ) by
f(a) =c, f(b) = a, f(c) = b.
bĝ –closed sets in X = {X,Φ,{b},{c},{a,b}, {a,c},{b,c}}
bĝ –open sets in Y =
{Y,Φ,{a},{b},{c},{a,b},{b,c}}
sg –closed sets in X = {X,Φ,{b},{c},{b,c}}
sg –open sets in Y = {Y,Φ,{b},{a,b},{b,c}}

Here the inverse image of a closed set {a,c} in Y is {a,b} which is bĝ –closed in X but not sg–closed in X and the image of a open set {a} in X is {c} which is bĝ –open in Y but not sg–open in Y.

Hence f is bĝ – homeomorphism, but not sg–homeomorphism, since f is not sg–continuous and sg–open map.

Theorem 6.9: Every gs–homeomorphism is a bĝ – homeomorphism
Proof: Follows from theorem 3.19 "Every gs–continuous map is bĝ – continuous" and by theorem 5.15 "Every gs–open map is bĝ – open map.

Remark 6.10: The converse of the above theorem need not be true.
(i.e) Every bĝ–homeomorphism need not be a gs–homeomorphism as shown in the following example.

Example 6.11: Let X=Y={a,b,c}
τ = {X, Φ,{a,c}}and σ = {Y, Φ,{a},{b,c}}
Define a function f: (X,τ)⟶ (Y, σ) by
f(a) =a, f(b) = b, f(c) = c.
bĝ –closed sets in X =
{X,Φ,{a},{b},{c},{a,b},{b,c}}
bĝ –open sets in Y =
{Y,Φ,{a},{b},{c},{a,b},{a,c},{b,c}}
gs –closed sets in X = {X,Φ,{b},{a,b},{b,c}}
gs –open sets in Y =
{Y,Φ,{a},{b},{c},{a,b},{a,c},{b,c}}

Here the inverse image of a closed set {a} in Y is {a} which is bĝ –closed in X but not gs–closed in X hence f is not gs–continuous, however f is a gs–open map. Hence f is a bĝ –homeomorphism but not gs–homeomorphism.

Theorem 6.12: Every g–homeomorphism is a bĝ – homeomorphism
Proof: Follows from theorem 3.6 "Every g–continuous map is bĝ – continuous" and by theorem 5.6 "Every g–open map is bĝ – open map.

Remark 6.13: The converse of the above theorem need not be true.
(i.e) Every bĝ–homeomorphism need not be a g–homeomorphism as shown in the following example.

Example 6.14: Let X=Y={a,b,c}
τ = {X, Φ,{a},{a,b}}and σ = {Y, Φ,{a,c}}
Define a function f: (X,τ)⟶ (Y, σ) by
f(a) =a, f(b) = b, f(c) = c.
bĝ –closed sets in X=
{X,Φ,{b},{c},{a,c},{b,c}}
bĝ –open sets in Y =
{Y,Φ,{a},{c},{a,b},{a,c},{b,c}}
g –closed sets in X = {X,Φ,{c},{a,c},{b,c}}
g –open sets in Y = {Y,Φ,{a},{c},{a,c}}

Here the inverse image of a closed set {b} in Y is {b} which is bĝ –closed in X but not g–closed in X and for the image of a open set {a,b} in X is {a,b} which is bĝ –open in Y but not g–open in Y hence f is not g–continuous and g–open map. Thus f is a bĝ – homeomorphism but not g–homeomorphism .

Theorem 6.15: Every ĝ–homeomorphism is a bĝ – homeomorphism

Proof: Follows from theorem 3.16 "Every ĝ–continuous map is bĝ – continuous" and by theorem 5.9 "Every ĝ–open map is bĝ – open map".

Remark 6.16: The converse of the above theorem need not be true.

(i.e) Every bĝ–homeomorphism need not be a ĝ–homeomorphism as shown in the following example.

Example 6.17: Let X=Y={a,b,c}

τ = {X, Φ,{a},{a,b}}and

σ = {Y, Φ,{a},{b},{a,b}}

Define a function f: (X,τ) ⟶ (Y, σ) by

f(a) =b, f(b) = a, f(c) = c.

bĝ –closed sets in X = {X,Φ,{b},{c},{a,c},{b,c}}

bĝ –open sets in Y = {Y,Φ,{a},{b},{a,b},{a,c},{b,c}}

ĝ –closed sets in X = {X,Φ,{c},{b,c}}

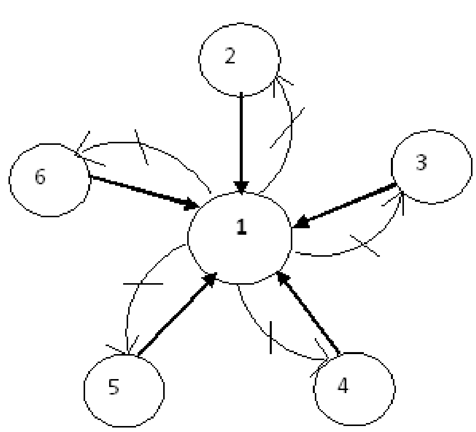
ĝ –open sets in Y = {Y,Φ,{a},{b},{a,b}}

Here the inverse image of a closed set {b,c} in Y is {a,c} which is bĝ –closed in X but not ĝ–closed in X, hence f is not ĝ–continuous, however f is a ĝ–open in Y.

Thus f is a bĝ –homeomorphism but not ĝ–homeomorphism.

Remark 6.18: The following diagram shows the relationships of bĝ – homeomorphism with other known existing homeomorphisms.

A ⟶ B represents A implies B but not conversely.



1. bĝ –homeomorphism
2. homeomorphism
3. g – homeomorphism
4. ĝ – homeomorphism
5. sg – homeomorphism
6. gs – homeomorphism

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