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On Acoustic Jumping Conditions for a Duct with an Area Expansion and a Non-zero Mean Flow

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***Abstract*—In this paper we study the jump conditions for a one-dimensional acoustic model of a duct which has an abrupt area expansion and a non-zero mean flow. We investigate on the compatibility of the three (mass, energy and momentum) jump conditions across an area expansion and then propose to use either the mass-momentum or the energy-momentum jump conditions for a proper one-dimensional acoustic modeling. The combination of mass and energy jump condition is shown to be inappropriate.**

***Keywords*—Acoustic Model, Jump Condition, Resonance Frequency**

I. Introduction

One-dimensional modeling of a simple duct seems to be the most elementary problem in an acoustics course. Basically for an acoustic model one simply uses a standard wave solution of the Hermholtz equation and then applies two boundary conditions at the inlet and the outlet of the duct, e.g., see [1], [2].

This simple one-dimensional modeling problem however becomes a bit confusing when there is a sudden area expansion in the duct and the duct is subject to non-zero mean flow. The confusion comes from the fact that there are several options in choosing jump conditions across the area expansion.

More specifically, a standard one-dimensional solution of the Hermholtz equation leaves us only two undetermined wave functions but there are many jump conditions including the next three conditions;

- mass conservation
- energy conservation
- momentum relation

Authors of [3], [4] proposed to model an area expansion as a loss of stagnation pressure instead of the energy conservation.

As will be shown in this paper, we have to choose only two jump conditions among many jump conditions in the course of one-dimensional modeling. This redundancy of jump condition is neither physically appealing nor easily justifiable, considering that those jump conditions should be somehow connected.

A complete resolution comes if some of the jump conditions become dependent and only two independent conditions are left. In fact, this is the case of the zero mean flow and the three (mass, energy and momentum) conditions, as will be shown.

Another way for a resolution is to justify a certain choice of jump conditions based on physical grounds. One may claim that energy conservation should be discarded as a sudden area expansion does work and thus propose to choose the mass and momentum conditions instead.

One criticism for this selection is that there is still controversy on a proper momentum relation across an area expansion. For an example, authors of [5] and [6] (p. 396) proposed different momentum relations. Furthermore there are important engineering applications in which the energy relation should appear. For an example, in a thermo-acoustic modeling of gas turbines, a flame located at the point of area expansion serves as an energy source and therefore a thermo-acoustic model needs the energy jump condition.

It seems that the problem of how to choose jump conditions has no definite answers, which motivates our investigation on the relations of three (mass, energy, momentum) jump conditions in this paper.

II. Acoustic Model

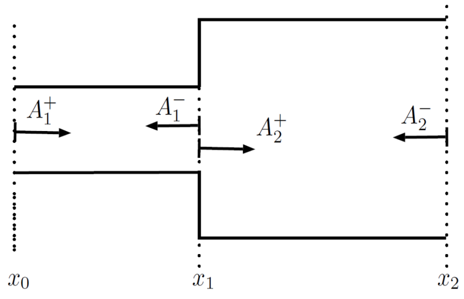

Fig. 1: Schematic of a Combustor

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A. Wave functions

We consider a duct composed of two chambers as shown in Fig. 1. Each chamber $[x_{k-1}, x_k]$, $k \in [1,2]$ has downstream and upstream waves A_k^+ and A_k^- waves and there exist a sudden area expansion between two chambers at $x = x_1$.

A solution of the Hermholt equation gives that the perturbations of pressure, velocity and density at each chamber can be represented as combinations of wave functions in (1) below. Note that, for each $k \in [1,2]$, the *three* functions of pressure, velocity and density waves are given as combinations of *two* independent function $A_k^\pm$ and therefore we need *four unknown* functions to describe pressure, velocity and density waves over the full range $[x_0, x_2]$.

$$\begin{aligned} p_k(x,t) &:= \bar{p}_k + A_k^+\left(t - \frac{x - x_{k-1}}{c + \bar{u}_k}\right) + A_k^-\left(t - \frac{x_k - x}{c - \bar{u}_1}\right) \\ u_k(x,t) &:= \bar{u}_k + \frac{1}{\bar{\rho} c}\left[A_k^+\left(t - \frac{x - x_{k-1}}{c + \bar{u}_k}\right) - A_k^-\left(t - \frac{x_k - x}{c - \bar{u}_k}\right)\right], \quad (k = 1,2) \\ \rho_k(x,t) &:= \bar{\rho} + \frac{1}{c^2}\left[A_k^+\left(t - \frac{x - x_{k-1}}{c + \bar{u}_k}\right) + A_k^-\left(t - \frac{x_k - x}{c - \bar{u}_k}\right)\right] \end{aligned} \tag{1}$$

Define

$$\begin{aligned} I_1(t) &:= A_1^+\left(t - \tau_1^+\right) \\ J_1(t) &:= A_1^-\left(t\right) \\ I_2(t) &:= A_2^+\left(t\right) \\ J_2(t) &:= A_2^-\left(t - \tau_2\right) \\ \tau_i^\pm &:= \frac{x_i - x_{i-1}}{c_i \pm \bar{u}_i} \quad (i = 1,2) \end{aligned} \tag{2}$$

Each wave function $A_k^\pm(x,t)$ has two independent variables : x (space) and t (time). However, as we already know how those two variables $\{x, t\}$ are combined to appear as a single variable in the wave function $A_k^\pm$ in (1), we have only to know $\{I_k(t), J_k(t)\}$ in (2) which have one time variable only, for a complete construction of $A_k^\pm(x,t)$. In other words, by fixing one variable $x = x_1$, we lose nothing. The same is true with the pressure, velocity and density functions.

Note that, with $x = x_1$ in (1), we have

$$\begin{aligned} p_k(x_1,t) - \bar{p}_k &= p_k'(x_1,t) = I_k + J_k \\ \bar{\rho} c[u_k(x_1,t) - \bar{u}_k] &= \bar{\rho} c u_k'(x_1,t) = I_k - J_k \\ c^2[\rho_k(x_1,t) - \bar{\rho}] &= c^2 \rho_k'(x_1,t) = I_k + J_k \end{aligned} \tag{3}$$

B. Boundary Conditions

Acoustic boundary conditions at $x \in \{x_0, x_2\}$ are characterized by *reflection coefficients* which are transfer functions between incident and reflected waves at both boundaries.

Two reflection coefficients are defined as

$$\begin{aligned} R_1 &:= \frac{\widetilde{A_1^+}}{\widetilde{A_1^-} e^{-\tau_1^- s}} = \frac{\widetilde{A_1^+} e^{-\tau_1^+ s}}{\widetilde{A_1^-} e^{-(\tau_1^+ + \tau_1^-)s}} = \frac{\widetilde{I_1}}{\widetilde{J_1} e^{-\delta_1 s}} \\ R_2 &:= \frac{\widetilde{A_2^-}}{\widetilde{A_2^+} e^{-\tau_2^+ s}} = \frac{\widetilde{A_2^-} e^{-\tau_2^- s}}{\widetilde{A_2^+} e^{-(\tau_2^+ + \tau_2^-)s}} = \frac{\widetilde{J_2}}{\widetilde{I_2} e^{-\delta_2 s}} \end{aligned} \tag{4}$$

where $\{\widetilde{I_k}(s), \widetilde{J_k}(s)\}$ and $\widetilde{A_k^\pm}(s)$ denotes the Laplace transform of $\{I_k, J_k\}$ and $A_k^\pm(t)$, respectively, and the quantities δ_1, δ_2 are defined

$$\begin{aligned} \delta_1 &:= \tau_1^+ + \tau_1^- = (x_1 - x_0)\left[\frac{1}{c + \bar{u}_1} + \frac{1}{c - \bar{u}_1}\right], \\ \delta_2 &:= \tau_2^+ + \tau_2^- = (x_2 - x_1)\left[\frac{1}{c + \bar{u}_2} + \frac{1}{c - \bar{u}_2}\right]. \end{aligned} \tag{5}$$

The relations $\widetilde{I_1} = R_1 e^{-\delta_1 s} \widetilde{J_1}$ and $\widetilde{J_2} = R_2 e^{-\delta_2 s} \widetilde{I_2}$ in (4) can be written in a matrix form as

$$\begin{bmatrix} 1 & -R_1 e^{-\delta_1 s} & 0 & 0 \\ 0 & 0 & -R_2 e^{-\delta_2 s} & 1 \end{bmatrix} \widetilde{X} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{6}$$

where

$$X := \begin{bmatrix} I_1 & J_1 & I_2 & J_2 \end{bmatrix}. \tag{7}$$

C. Jump Conditions

As we have four unknown functions $\{I_k, J_k\}$ $(k = 1,2)$ for a description of acoustic model, from two boundary conditions (6), we need two additional relations between functions. Therefore we need to have two jump conditions across the area expansion at $x = x_1$ in Fig. 1.

We assume incompressive flows and thus the sound speed c and mean density $\bar{\rho}$ are constants over the full range $[x_0, x_2]$.

The mass, momentum, energy rate (denoted m, f, e, respectively) are given

$$\begin{aligned} m &= \mathcal{A} \rho u \\ f &= \mathcal{A}(p + \rho u^2) \\ e &= \mathcal{A}(\eta p u + \rho u^3/2), \quad \eta := \frac{\gamma}{\gamma - 1}. \end{aligned} \tag{8}$$

where $\mathcal{A}$ denotes cross sectional area.

Remark 1: Throughout this paper, by saying *mass, momentum and energy* without the term *rate* explicitly, we mean the rates of those quantities across $x = x_1$.

The perturbations of quantities in (8), denoted by m', f', e', become

$$\begin{aligned} m'/\mathcal{A} &= \overline{\rho} u' + \overline{u}\rho' \\ f'/\mathcal{A} &= p'_k + 2\overline{\rho}\,\overline{u}u' + \overline{u}^2\rho' \\ e'/\mathcal{A} &= (\eta\overline{p} + 3\overline{u}^2\overline{\rho}/2)u' + \eta\overline{u}p' + \overline{u}^3\rho'/2 \end{aligned} \tag{9}$$

where we used the facts $\gamma\overline{p} = c^2\overline{\rho}$ and $c^2\rho' = p'$.

Let us introduce a new notation. From now on two subscripts $\{1,2\}$ of functions $\{m, f, e, \mathcal{A}\}$ of x represent their magnitude at $x \to x_1 \uparrow$ (left limit) and $x \to x_1 \downarrow$ (right limit), respectively.

We assume that the mean flow is much slower than the sound speed. That is, $M_k := \overline{u}_k/c \ll 1$.

Making use of (3), the perturbation form (9) can be rewritten as, for $k = 1, 2$,

$$\begin{aligned} \frac{cm'_k}{\mathcal{A}_k} &= (I_k - J_k) + M_k(I_k + J_k) \\ \frac{f'_k}{\mathcal{A}_k} &= (I_k + J_k) + 2M_k(I_k - J_k) \\ \frac{e'_k}{c\mathcal{A}_k} &= \frac{1}{\gamma - 1}(I_k - J_k) + \eta M_k(I_k + J_k) \end{aligned} \tag{10}$$

where for simplicity we assumed $M_k^2 = 0$. This expression can be written in a matrix form as

$$\frac{1}{\mathcal{A}_k}\begin{bmatrix} c\,m'_k \\ f'_k \\ e'_k/c \end{bmatrix} = \begin{bmatrix} 1 + M_k & -1 + M_k \\ 1 + 2M_k & 1 - 2M_k \\ \frac{\gamma M_k + 1}{\gamma - 1} & \frac{\gamma M_k - 1}{\gamma - 1} \end{bmatrix} \begin{bmatrix} I_k \\ J_k \end{bmatrix} \tag{11}$$

From this representation, the mass conservation condition $m'_1 - m'_2 = 0$ can be written as

$$\begin{bmatrix} 1 + M_1 & -1 + M_1 & -\beta - M_1 & \beta - M_1 \end{bmatrix} X = 0 \tag{12}$$

where $\beta := \mathcal{A}_2/\mathcal{A}_1$ denotes the area ratio and we used $M_1 = \beta M_2$ from the mass continuity $\overline{u}_1\mathcal{A}_1 = \overline{u}_2\mathcal{A}_2$.

In a similar way, the energy conservation condition $e'_1 - e'_2 = 0$ can be written as

$$\begin{bmatrix} 1 + \gamma M_1 & -1 + \gamma M_1 & -\beta - \gamma M_1 & \beta - \gamma M_1 \end{bmatrix} X = 0. \tag{13}$$

Momentum balance across an area expansion can be modeled as

$$f_2 - f_1 = (\mathcal{A}_2 - \mathcal{A}_1)p_1 = \mathcal{A}_1(\beta - 1)p_1, \tag{14}$$

e.g., see [6](p. 396 Eq. (13.67c)).

By substituting the representation of a momentum perturbation in (10) to a perturbation of the equality (14), it follows that

$$\begin{aligned} &\mathcal{A}_1(\beta - 1)p_1 + f_1 - f_2 \\ &= \left[\ \beta + 2M_1 \ \vdots\ \beta - 2M_1 \ \vdots\ -\beta(1 + 2M_2) \right. \\ &\qquad \left. \vdots\ -\beta(1 - 2M_2)\ \right] X = 0. \end{aligned} \tag{15}$$

By dividing both sides with β and using $M_2 = M_1/\beta$, the momentum relation can be written as

$$\begin{aligned} &\left[\ \beta + 2M_1 \ \vdots\ \beta - 2M_1 \ \vdots \right. \\ &\qquad \left. \vdots\ -\beta - 2M_1 \ \vdots\ -\beta + 2M_1\ \right] X = 0. \end{aligned} \tag{16}$$

Up to now we have found four jump conditions corresponding to the mass, energy, and momentum relations across a sudden area expansion, which can be summarized in a 3×4 matrix form as

$$\begin{array}{l} \text{(mass)} \\ \text{(energy)} \\ \text{(momentum)} \end{array} \left[\begin{array}{c:c:c:c} 1 + M_1 & -1 + M_1 & -\beta - M_1 & \beta - M_1 \\ 1 + \gamma M_1 & -1 + \gamma M_1 & -\beta - \gamma M_1 & \beta - \gamma M_1 \\ \beta + 2M_1 & \beta - 2M_1 & -\beta - 2M_1 & -\beta + 2M_1 \end{array}\right] X = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{17}$$

Note that in the case of zero mean flow, i.e., $M_1 = 0$, this matrix has a form

$$\begin{bmatrix} 1 & -1 & -\beta & \beta \\ 1 & -1 & -\beta & \beta \\ \beta & \beta & -\beta & -\beta \end{bmatrix} X = \begin{bmatrix} 0 \\ 0 \\ 01 \end{bmatrix}. \tag{18}$$

Thus the first (mass) and second (energy) rows become identical, leaving us only two independent relations. An implication of this result is that, when there is no mean flow, we have only two independent jump conditions and there is no ambiguity in choosing jump conditions.

III. Compatibility of Jump Conditions

A. Mass and Energy Relations

Suppose we take the following mass and momentum relations in (17) for granted. Then a rewriting of (17) gives the next equality

$$\begin{aligned} &\left[\begin{array}{c:c} 1 + M_1 & -1 + M_1 \\ 1 + \gamma M_1 & -1 + \gamma M_1 \end{array}\right] \begin{bmatrix} I_1 \\ J_1 \end{bmatrix} = \\ &\qquad \left[\begin{array}{c:c} \beta + M_1 & -\beta + M_1 \\ \beta + \gamma M_1 & -\beta + \gamma M_1 \end{array}\right] \begin{bmatrix} I_2 \\ J_2 \end{bmatrix}. \end{aligned} \tag{19}$$

Our aim is to compare the momentum relation in (17) with a new momentum relation which comes from the mass and energy conditions.

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Firstly we express $\{I_2, J_2\}$ in terms of $\{I_1, J_1\}$ from the matrix relation (19) assuming $M_k^2 = 0$ and obtain

$$\begin{aligned}\frac{f_2' - f_1'}{\mathcal{A}_1} &= (\beta-1)(I_1+J_1) - 2(M_1 - M_2)(I_1 - J_1)\\ &= (\beta-1)p_1' - 2\frac{(\beta-1)}{\beta}M_1\overline{\rho}cu_1'\\ &= (\beta-1)\left(p_1' - \frac{2}{\beta}\overline{\rho}\,\overline{u_1}u_1'\right).\end{aligned} \tag{20}$$

Then, by recovering an unperturbed form from (20) to have

$$f_2 - f_1 = \mathcal{A}_1(\beta-1)\left(p_1 - \frac{1}{\beta}\overline{\rho}u_1^2\right). \tag{21}$$

A comparison of this result with the momentum relation (14) reveals that our new momentum relation which is a pure consequence of the mass and energy conservation in (19), has an additional term.

In order words, the mass conservation condition being taken for granted, the energy conservation condition is equivalent to having a smaller momentum increase than expected from the momentum relation (14).

This difference however is not significant. To see this, note firstly that the additional term in (20) can be written

$$\frac{2}{\beta}\overline{\rho}\,\overline{u_1}u_1' = \frac{2M_1}{\beta}\,\overline{\rho}cu_1'. \tag{22}$$

Secondly, we know from (3) that $\overline{\rho}c\,u_1'$ has the same order of p_1'. Hence if $M_1 \ll \beta$ then the momentum difference $f_2' - f_1'$ in (21) is dominated by the term p_1', recovering the momentum relation (14).

B. *Mass and Momentum Relations*

In this section we assume that the mass and momentum relations hold and, under that assumption, develop a new energy relation and then compare it with the energy conservation relation in (17).

The mass and momentum conditions (the first and third rows of the matrix (17) can be written as

$$\begin{aligned}&\begin{bmatrix}1+M_1 & -1+M_1\\ \beta+2M_1 & \beta-2M_1\end{bmatrix}\begin{bmatrix}I_1\\ J_1\end{bmatrix}\\ &\quad= \begin{bmatrix}\beta+M_1 & -\beta+M_1\\ \beta+2M_1 & \beta-2M_1\end{bmatrix}\begin{bmatrix}I_2\\ J_2\end{bmatrix}.\end{aligned} \tag{23}$$

By expressing $\{I_2, J_2\}$ in terms of $\{I_1, J_1\}$ from this equality, the energy difference can be written as a linear combination of $\{I_2, J_2\}$ as

$$\begin{aligned}e_2' - e_1' &= -c\mathcal{A}_1\frac{\beta(\beta-1)(3\beta-1)}{2}M_2^2(I_2 - J_2)\\ &= -\mathcal{A}_1\frac{\beta(\beta-1)(3\beta-1)}{2}\overline{\rho}\,\overline{u_2}^2u_2'\end{aligned} \tag{24}$$

and the corresponding unperturbed form

$$e_2 - e_1 = -\mathcal{A}_1\frac{\beta(\beta-1)(3\beta-1)}{6}\overline{\rho}u_2^3 \tag{25}$$

can be obtained.

As the term in the right hand side of (25) is always negative for all $\beta \geq 1$, it follows that a sudden area expansion causes an energy loss.

In addition, we represent the energy terms $\{e_1, e_2\}$ of (24) explicitly to have

$$\begin{aligned}\overline{\rho}\mathcal{A}_2u_2\left[\frac{\eta p_2}{\overline{\rho}} + \frac{1}{2}u_2^2\right] - \overline{\rho}\mathcal{A}_1u_1\left[\frac{\eta p_1}{\overline{\rho}} + \frac{1}{2}u_1^2\right]\\ = -\overline{\rho}\mathcal{A}_2u_2\frac{(\beta-1)(3\beta-1)}{6}u_2^2,\end{aligned} \tag{26}$$

which is equal to

$$\frac{\eta p_2}{\overline{\rho}} + \frac{1}{2}u_2^2 + \frac{1}{2}K_eu_2^2 = \frac{\eta p_1}{\overline{\rho}} + \frac{1}{2}u_1^2 \tag{27}$$

after a cancellation of the mass rate where

$$K_e := \frac{(\beta-1)(3\beta-1)}{3} > 0. \tag{28}$$

The stagnation enthalpy is defined

$$H_k := \eta\frac{p_k}{\overline{\rho}} + \frac{1}{2}u_k^2 \tag{29}$$

and thus the energy loss in (27) across a sudden area expansion can be seen as a loss of stagnation enthalpy as follows;

$$H_2 + \frac{1}{2}K_eu_2^2 = H_1. \tag{30}$$

The quantity K_e can be called as a *stagnation enthalpy loss* coefficient associated with an area expansion.

In conclusion the combination of mass and momentum relations automatically results in an energy loss which contradicts with the energy conservation in (17).

IV. Resonant Frequency

By substituting the boundary conditions (6) to the mass, energy and momentum relations across an area expansion, we obtain a complex valued 4×2 matrix

$$\begin{array}{l} \text{(mass)} \\ \text{(energy)} \\ \text{(momentum)} \end{array}\left[\begin{array}{cc}(1+M_1)R_1e^{-\delta_1 s} - 1 + M_1 & -\beta - M_1 + (\beta - M_1)R_2e^{-\delta_2 s}\\ (1+\gamma M_1)R_1e^{-\delta_1 s} - 1 + \gamma M_1 & -\beta - \gamma M_1 + (\beta - \gamma M_1)R_2e^{-\delta_2 s}\\ (\beta+2M_1)R_1e^{-\delta_1 s} + \beta - 2M_1 & -\beta - 2M_1 - (\beta - 2M_1)R_2e^{-\delta_2 s}\end{array}\right]\begin{bmatrix}\widetilde{J}_1\\ \widetilde{I}_2\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}. \tag{31}$$

Depending on which jump conditions in (31) we choose, a homogeneous 2×2 square matrix, call it $\Delta(s)$,

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is determined and the corresponding matrix equation allows non-zero solutions $\widetilde{J}_1, \widetilde{I}_2$ only when the determinant $|\Delta(s)|$ is zero. As the determinant $|\Delta(s)|$ is nothing but the characteristic equation of a differential equation associated with the matrix equation (31), let us call the roots of $|\Delta(s)| = 0$ as *characteristic roots*.

Characteristic roots are complex numbers in general but typically they are close to the imaginary axis. Hence the image of a real-valued function $|\Delta(jw)|$ defined on the real axis $w \in \mathbb{R}$, has a local minimum at $w = w^*$ such that $s = \sigma + jw^*$ is a characteristic root for some real number σ. The frequency w^* or $f^* := w^*/2\pi$ is commonly called as a resonance frequency.

There are three possible ways in choosing two jump conditions among the three rows of the matrix (31). However, it turns out that the mass-energy combination (first and second rows) should be avoided. An obvious reason is that, as we have already observed, the first and second row becomes identical when $M_1 = 0$ and hence a singularity occurs. A hidden but more fundamental drawback of the mass-energy combination is that characteristic roots are independent of mean flow. In fact, elementary calculations with (31) can reveal that the equation $|\Delta(s)| = 0$ of this case is given

$$R_1 R_2 e^{-(\delta_1+\delta_2)s} - \frac{\beta - 1}{\beta + 1}\left(R_1 e^{-\delta_1 s} - R_2 e^{-\delta_2 s}\right) - 1 = 0 \tag{32}$$

which is independent of the mean flow M_1.

With the mass-energy combination excluded, there are only two cases ; mass-momentum and energy-momentum combinations.

Note that the only difference between the mass (first) and energy (second) row of the matrix (31) is that M_1 of the first row is replaced with γM_1 in the second. As a result, the two choices do not make much difference as long as the mean flow is small. Roughly speaking, with a fixed momentum relation, the energy relation emphasizes the effect of mean flow γ times than the mass relation.

Finally, let us consider s simple numerical example with parameters

$$L_1 = 1.0 \text{ (m)}, \quad L_2 = 1.5 \text{ (m)}, \quad \beta = 5,$$
$$c = 443 \text{ (m/s)}, \quad \gamma = 1.4, \quad R_1 = R_2 = 1. \tag{33}$$

For a set of mean flow $0 \le M_1 \le 0.5$, the characteristic roots of both the mass-momentum and energy-momentum combinations are numerically found and shown in Fig. 2. In overall the two combinations have similar characteristic roots but show some differences as mean flow increases.

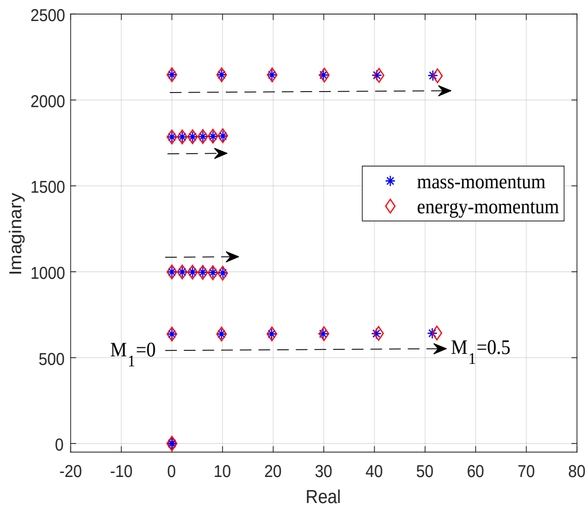

Fig. 2: Characteristic Roots ($0 \le M_1 \le 0.5$)

V. Conclusion

We considered a one-dimensional acoustic model of a duct which has an abrupt area expansion and subject to a non-zero mean flow. We investigated on the relations between three (mass, energy and momentum) jump conditions across an area expansion and found that the combination of mass-energy jump condition is not a proper choice for an acoustic modeling. It was also shown that the combination of either the mass-momentum or the energy-momentum jump condition result in a similar model for reasonably small mean flow.

References

[1] L. E. Kinsler, A. R. Frey, A. B. Coppens, and J. V. Sanders, *Fundamentals of Acoustics*, 4th ed. John Wiley & Sons, 2000.

[2] S. W. Rienstra and A. Hirschberg, *An Introduction to Acoustics*. Eindhoven University of Technology, 2017.

[3] V. B. Panicker and M. L. Munjal, "Aero-acoustic analysis of straight-through mufflers with simple and extended tube expansion chambers," *Journal of the Indian Institute of Science*, vol. 63, no. 1, pp. 1–19, 1981.

[4] M. L. Munjal, *Acoustics of Ducts and Mufflers*, 2nd ed. Wiley, 2014.

[5] P. Davies, "Practical flow duct acoustics," *Journal of Sound and Vibration*, vol. 124, no. 1, pp. 91 – 115, 1988.

[6] T. Lieuwen and V. Yang, *Combustion instabilities in gas turbine engines: operational experience, fundamental mechanisms and modeling*, ser. Progress in astronautics and aeronautics. American Institute of Aeronautics and Astronautics, 2005.