Odd-graceful labeling of corona graph $C_{2n}^*K_1$.

* Veena Shinde-Deore.
  Research Scholar, JJT University.
  Head, Department of Mathematics
  And Statistics,
  Bhavan’s H.S. College, Mumbai.

* Dr (Mrs.) Manisha M. Acharya
  Research Guide, JJT University.
  Associate Professor and Head, Department of Mathematics, Maharshi Dayanand College,
  Parel, Mumbai.

Abstract
The research in Graph theory had lead to one of the important area which involves
labeling of graphs. There are different types of labelings such as graceful
labeling, magic labeling, prime labeling etc applied to various classes of graphs.
In this paper, odd-gracefulness of the corona graph $C_{2n}^*K_1$ for $n \geq 2$ is
obtained.

Keywords:
Graph theory, Graceful graph, labeling of graphs, corona graph, odd-graceful
labeling.

Introduction

Definition 1: Graph: A graph $G$ is a pair $(V(G), E(G))$ where $V(G)$ is a
nonempty finite set of elements known as vertices and $E(G)$ is family of
unordered pairs of elements of $V(G)$ known as edges.

Definition 2: Corona graph: The corona $G_1^*G_2$ of two graphs $G_1$ and $G_2$ is a graph $G$
obtained by taking one copy of $G_1$ which has $p_1$-vertices and $p_1$-copies of $G_2$ and then joining
$i^{th}$ vertex of $G_1$ to every vertex in the $i^{th}$ copy of $G_2$.

Figure 1: Different graphs.

Figure 2: Corona $C_3^*K_1$

Definition 3: Difference vertex labeling: A difference vertex labeling of
graph $G$ is an assignment $f$ of labels to
the vertices of G that induces for each edge uv the weight \(| f(u) - f(v) | \).

**Figure 3:** Difference vertex labeling.

**Definition 4: Labeled graph:** When the dots or lines in a graph are labeled with numbers we call it a ‘labeled graph’. Labeling of the vertices of a graph G is assignment of distinct natural numbers to vertices of G. This labeling induces a natural labeling of the edges called edge labels or edge weights.

**Figure 4:** Labeling of graphs.

**Definition 5: Graceful graph:** Let G be a graph with q edges. Let f be labeling of G such that the set of labels of vertices is a subset of \{ 0,1,2,3,……..,q \} and the set of the edge labels is from set \{ 1,3,…….,2n-1 \}. Then the labeling f is said to be graceful and graph G is called graceful graph.

**Figure 5:** Graceful graph.

**Definition 6: Odd-graceful graph:** A difference vertex labeling of graph G of size n is odd-graceful if f is an injection from V(G) to \{0,1,…….,2n-1\} such that the induced weights are \{1,3,…….,2n-1\}. The graph with odd-graceful labeling is called odd-graceful graph.

**Figure 6:** Odd-graceful graph.

**Odd-gracefulness of \(C_{2n}\ast K_1\).**

**Theorem:** The graphs \(C_{2n}\ast K_1\) are odd graceful, for \(n \geq 2\).

**Proof :-** Number of vertices of \(C_{2n}\ast K_1\) = \(p(C_{2n}\ast K_1) = 4n\)

Number of edges of \(C_{2n}\ast K_1\) = \(q(C_{2n}\ast K_1) = 4n\)

Let vertex set of \(C_{2n}\ast K_1\) be \(V(C_{2n}\ast k_1) = \{u_1,u_2,...........,u_{2n}; v_1,v_2,...........,v_{2n}\}\) where vertices \(u_1,u_2,...........,u_{2n}\) are vertices of cycle \(C_{2n}\), ‘\(v_i\)’ is the pendant vertex adjacent to \(u_i\); \(1 \leq i \leq 2n\).
There are two cases: (i) $n \equiv 0 \pmod{2}$  
(ii) $n \equiv 1 \pmod{2}$

In both the cases we show that edge weight set is $W = \{1,3,5,\ldots,8n-1\}$ with vertex set labeling $\{0,1,2,3,4,5,\ldots,8n-1\}$.

In both cases, the labeling function $f$ is given in two parts viz Part-I and Part-II.

Part-I describes the labeling function for the vertices $(u_i,v_i)$ where $1 \leq i \leq n$ and Part-II describes labeling function for the vertices $(u_i,v_i)$ where $n + 1 \leq i \leq 2n$. Further in each case, Part-II is divided into three subparts namely S-1, S-2 and S-3.

**Part-I**: The labeling function $f$ for vertices $u_i$ and $v_i$ where $1 \leq i \leq n$ is given as follows:

$f(v_{2i-1}) = 4i - 4$ and $f(u_{2i-1}) = 8n - (4i - 3)$  
for $1 \leq i \leq \frac{n}{2}$ when $n \equiv 0 \pmod{2}$  
and  
for $1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor + 1$ when $n \equiv 1 \pmod{2}$

$f(v_{2i}) = 8n - (4i - 1)$ and $f(u_{2i}) = 4i - 2$  
for $1 \leq i \leq \frac{n}{2}$ when $n \equiv 0 \pmod{2}$  
and  
for $1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor$ when $n \equiv 1 \pmod{2}$

The edge weights covered in Part-I are all odd numbers in descending order from $8n - 1$ to $4n + 3$ i.e. from $8n - 1$ to $8n - (4n - 3)$.

**Part-II**: As mentioned earlier, Part-II is divided into three subparts S-1, S-2 and S-3.

**Subpart S-1**: In this subpart we have to consider only two vertices $u_{n+1}$ and $v_{n+1}$. The labeling function for this subpart is as follows:

$f(u_{n+1}) = 2n - 1$ and $f(v_{n+1}) = 6n - 2$  
when $n \equiv 0 \pmod{2}$

$f(u_{n+1}) = 6n - 2$ and $f(v_{n+1}) = 2n - 1$  
when $n \equiv 1 \pmod{2}$

**Subpart S-2**: The labeling function $f$ in this subpart is defined for vertices $u_{n+2}$, $u_{n+3}$, ..., $u_{2n-1}$ and for vertices $v_{n+2}$, $v_{n+3}$, ...., $v_{2n-1}$.

$f(u_{2i}) = 4i - 2$ and $f(v_{2i}) = 8n - (4i - 1)$  
for $\frac{n}{2} + 1 \leq i \leq n - 1$  
when $n \equiv 0 \pmod{2}$  
and  
for $\left\lfloor \frac{n}{2} \right\rfloor + 2 \leq i \leq n - 1$  
when $n \equiv 1 \pmod{2}$

**Remark**: i) When $n \equiv 0 \pmod{2}$, in subpart S-2, for the validity of the range of the parameter $i$, we need $n \geq 4$. For $2 \leq n < 4$, we have only one such value of $n$ which is 2. When $n = 2$, the set of vertices belonging to S-2 is an empty set.

ii) When $n \equiv 1 \pmod{2}$, in subpart S-2, for the validity of the range of the parameter $i$, we need $n \geq 3$. For $3 \leq n < 5$ we have only one value of $n$ which is 3. When $n = 3$ the set of vertices belonging to S-2 are $u_5$ and $v_5$.

**Subpart S-3**: In this subpart there are only two vertices viz. $u_{2n}$ and $v_{2n}$ with labeling function as follows:
f (u_{2n}) = 4n - 2 and f(v_{2n}) = 1 if n \equiv 0 \pmod{2} and n \equiv 1 \pmod{2}

All odd edge weights in descending order from 8n-1 to 4n + 3 are covered in Part -I. Remaining odd edge weights from 4n +1 to 1 are covered in Part- II as follows :-
1. | f(u_1) - f(u_{2n}) | = 4n + 1
2. | f(v_{n+1}) - f(u_{n+1}) | = 4n - 1
3. | f(u_{2n}) - f(v_{2n}) | = 4n - 3
4. For \left\lceil \frac{n}{2} \right\rceil + 1 \leq i \leq n-1 edge weights covered are from 4n - 5 to 7 in descending order.
5. | f (u_{2n-1}) - f (u_{2n}) | = 5
6. | f (u_{n+2}) - f (u_{n+1}) | = 3.
7. | f (u_{n+1}) - f (u_n) | = 1

**Remark** : i)In case of n \equiv 0 \pmod{2}, when n = 2, edge weights covered in Part-I are from 8n-1 to 4n + 3 i.e. 15, 13 and 11, then in subpart S-1 edge weights covered are 1 and 7. Since subpart S-2 is empty for n = 2, we get
| f (u_{2n}) - f(u_{n+1}) | = | 4n - 2 - (2n-1) | = | 2n-1 | = 3.
Thus, edge weights covered in subpart S-3 are 5 & 9.

ii) In case of n \equiv 1 \pmod{2}, when n =3, the edge weights covered in Part -I are from 8n-1 to 4n +3 i.e. 23,21,19,17,15. In subpart S-1 edge weights covered are 3 & 11. In subpart S-2, edge weights covered are 1,7 and 5. In subpart S-3, edge weights covered are 9 and 13.

**Thus,** the labeling function f is injective and that each odd edge weight from 1 to 8n-1 is covered exactly once. Hence, the graph C_{2n}*K_1 is odd-graceful for n \geq 2.

**Illustration 1:**

**Figure 7:** Odd-gracefulness of C_8*K_1

**Illustration 2:**

**Figure 8:** Odd-gracefulness of C_{10}*K_1

**Conclusion**
The odd graceful labeling is presented to the corona graph C_{2n}*K_1. The corona graph C_{2n}*K_1 is thus a edge-odd graceful graph. It is interesting to apply this
labeling to certain classes of graph. It is also an area of interest to make computer programmes for the given labeling.

References


