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Numerical Modelling of Transportation Cost Minimization in Surface Mining

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***Abstract*:- This study aims to minimize ore transportation cost in surface mining by numerical modelling. Minimization of ore transportation cost is very important in case of multiple locations of ore deposit such as phosphate ore of Aljalamid, in Saudi Arabia, the ore is found in three locations. The present numerical model aims to determine the best position of processing plant. The suggested mathematical based on the factors such as number of main locations, reserves in each location, transportation costs in different locations, and transportation distance from each ore deposit location to required processing plant position. Computer programming was used to solve the considered mathematical model. The main results of this study are determination of processing plant location, Contour map of additional cost in the area around the optimum location, and the Presented model is very useful in similar deposits.**

Keywords:- Numerical Modelling, Simulation, Transportation Cost, Cost Minimization, Optimum Processing Plant position.

1. INTRODUCTİON

Cost of transportation represents one of the main items of mining projects. The factors affecting the cost of transportation in mining are transportation distance, ore tonnage, and transportation method. Many transportation models were suggested such as Larwood and Benson Model [1], Anderson Model [2], Bechtel Model [3], and Zimmerman Model [4]. These models tried to minimize the cost transportation [5-10]. Fjellstrom [11] presented a model to estimate the transportation cost of ore to crusher and waste to backfilling rooms in Renstrom mine. Wegener [12] presented many contributions for development of transportation models. Brazil et al. [13] suggested a model to optimize the transportation costs of underground mining roads. This model focused on the underground mine networks. Two models were investigated by Dharma and Ahmad [14]. These models presented for application in two iron ore mines. These models were applied to optimize transportation cost. The numerical results were agreed with real-world situation. Shephard [15] suggested a transportation model based on transported quantity, distance, shipment delay, transport technology, and route. Mahran et al. [16]. Suggested two mathematical models to obtain the optimum location of the processing plant in Bahariya iron ore mines. These models based on ore reserves and transportation distance but it did not take into consideration the transportation costs of different locations in El-Gedida, Ghorabi, Nasser and El-Harra iron ore deposits. Also the gravity center calculations are not accurate due to uniformity ore thickness assumption and method of calculation. Inwood and Keay [17] investigated relationship between trade costs and trade volumes by using modern different tools such as evidence on effective transport costs of iron ore trade. linear programming was used by [18, 19] to minimize cost of transportation in mining. Chen et al. [20] proposed a model based on classical transportation problem and transport path.

Ahmed et al. [21] presented a study to minimize cost of transportation by Linear Programming Problem. Joshi [22] suggested a method to decrease cost of transportation. The method based on linear programming and was solved by using four different methods. Ahmad [23] presented a method that named Best Candidates Method (BCM) to solve optimization cases. The objective of his method to find optimal solution. Novikov et al. [24] considered an integer model to minimize cost of production of iron ore raw materials and its transportation during mining and ore processing. Saderova et al [25] presented a mathematical model of the production of raw material from a mining area to started point of next technological process. They suggested two methods for modelling. The first methods was based on traffic modelling with mathematical equations. The second method based on computer simulation using ExtendSim8. The previous transportation models were based on ore reserves and transportation distance but it did not take into consideration the transportation costs of different locations. Also the gravity center calculations are not accurate due to uniformity ore thickness assumption and method of calculation. The present model will avoid the shortcomings of the previous models to obtain optimum location of processing plant. The current model applied in Aljalamid area of Saudi Arabia, the ore located in three locations that are Fish, Southern and Western. In this case, selection of optimum location of processing plant represents a real challenge. The suggested model studied factors such as: number of locations, reserves in each location, transportation costs of one ton for one unit distance in different locations, and the distance from the gravity center of each location to optimum location of the processing plant.

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2. THEORETICAL CONSIDERATION

2.1. General

Choice of optimum location of ore processing factory depends upon the total transportation cost. Transportation cost parameters is dependent mainly upon the reserves of each ore deposit area, the distances of ore transportation from each ore deposit area to the optimum location, and transportation costs per ton in different locations.

2.2. Mathematical model

The suggested model depends on the minimum sum of transportation cost from ore deposit locations to optimum location of processing plant. Ore deposit scattered into different locations (n). Each location has an ore reserves of (Q_i) which is considered to be concentrated at a defined point that is the center of gravity having coordinates (x_i, y_i, z_i) the cost of transported 1 ton for 1 kilometer distance from each ore deposit location to optimum location (C_i). All of the ore reserves are to be transported to a location where the mineral processing plant is to be construct so that the transportation cost should be minimized. Hence, this issue can be mathematically expressed as in equations from 1 to 6. The suggested model based on the minimum sum of transportation cost of different ore deposits to optimum location of processing plant as shown in equation (1)

$$\sum_{i=1}^{n} Q_i C_i D_i = Minimum \qquad (1)$$

Where:
n: Number of ore deposit locations.
i: to denote the i[th] ore deposit location.
Q_i: Reserves of ore deposit location of location number (*i*) in (tons).
C_i : Transportation cost of one ton for one unit distance for each path or transportation method in (U.S$/ton. m).
D_i: Distance between different ore locations and processing plant optimum location in (m)
x, y, z: The coordinates of processing plant optimum location.
x_i, y_i, z_i : The coordinates of the gravity center of different ore deposit locations.
The total transportation cost (US$) of ore deposit is:

$$S = \sum_{i=1}^{n} Q_i C_i D_i \qquad (2)$$

The distance (D_i) between two points by using its coordinates can be mathematically $= \sqrt{(x-x_i)^2 + (y-y_i)^2 + (z-z_i)^2}$

Hence, *S* from equation (2) may be mathematically expressed as follows:

$$S = \sum_{i=1}^{n} Q_i C_i \sqrt{(x-x_i)^2 + (y-y_i)^2 + (z-z_i)^2} \qquad (3)$$

For this sum to be minimum, the partial differentiation in regards to x, y and z should equal zero, which means that the following conditions have to be satisfied:

$$\frac{\partial s}{\partial x} = \sum_{i=1}^{n} Q_i C_i (x-x_i) / \sqrt{(x-x_i)^2 + (y-y_i)^2 + (z-z_i)^2} = 0 \qquad (4)$$

$$\frac{\partial s}{\partial y} = \sum_{i=1}^{n} Q_i C_i (y-y_i) / \sqrt{(x-x_i)^2 + (y-y_i)^2 + (z-z_i)^2} = 0 \qquad (5)$$

$$\frac{\partial s}{\partial z} = \sum_{i=1}^{n} Q_i C_i (z-z_i) / \sqrt{(x-x_i)^2 + (y-y_i)^2 + (z-z_i)^2} = 0 \qquad (6)$$

Where:
S : Total cost of transportation for all ore deposit locations (US$).
$\frac{\partial s}{\partial x}, \frac{\partial s}{\partial y}, \frac{\partial s}{\partial z}$: The partial differentials of equation (3)

2.3. Calculation of ore reserves and center of gravity of ore deposit

The objective of gravity center determination is to accumulate the quantity of ore (reserves) of each area or location at a point that has coordinates (x_i, y_i, z_i). Gravity center means an imaginary point in a body of matter where, for convenience in certain calculations, the total weight of the body may be thought to be concentrated [26]. The center of gravity of ore deposit can be determined as follows.

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- Firstly, the ore deposit map or area is divided into suitable number of triangles. Each triangle consists of three boreholes as shown in figure (1).

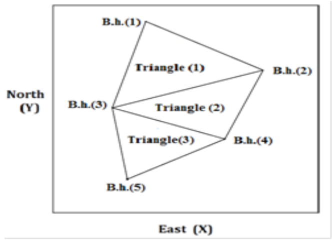

Figure 1. Ore deposit area is divided into triangles.

- Secondly, Determination of ore reserves and center of gravity of each triangle. Gravity center of a triangle is the intersection point of its medians [27] as shown in figure (2). The center of gravity divides each of the medians in the ratio 2:1, which is to say it is located ⅓ of the perpendicular distance between each side and the opposing point. Its Cartesian coordinates are the means of the coordinates of the three vertices. That is, if the three vertices are $a=(x_a, y_a)$, $b=(x_b, y_b)$, and $c=(x_c, y_c)$, then the x and y coordinates of the center of gravity $\bar{x}, \bar{y}$ and can be calculated by equation (7).

$$C.G.=(\bar{x},\bar{y})=\frac{a+b+c}{3}=[\frac{x_a+x_b+x_c}{3},\frac{y_a+y_b+y_c}{3}] \qquad (7)$$

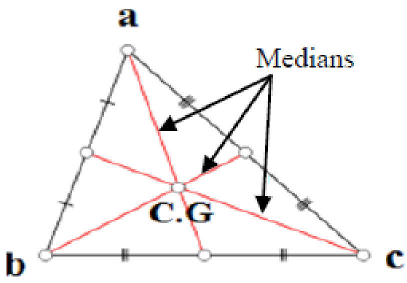

Figure 2. Gravity center determination of a triangle.

- Thirdly, Determination of Z coordinate of gravity center of a triangle. Z coordinate of gravity center of a triangle can be determined for the point, which was determined in the second step. Z coordinate of gravity center of the triangle can be calculated by equation (8).

$$\bar{z}=z-t_{o.b}-\frac{t_b}{2} \qquad (8)$$

Where:
z- : z coordinate of the gravity center of the triangle.
z : z coordinate of ground surface at ($\bar{x}, \bar{y}$) coordinates of C.G.
$t_{o.b}$: thickness of overburden at ($\bar{x}, \bar{y}$) coordinates of C.G.
t_b : Average thickness of the bed of required triangle.

- Fourthly, Calculation of gravity center of ore deposit that consists of a number of triangles as shown in figure (3) by using the following equations (9,10,11).

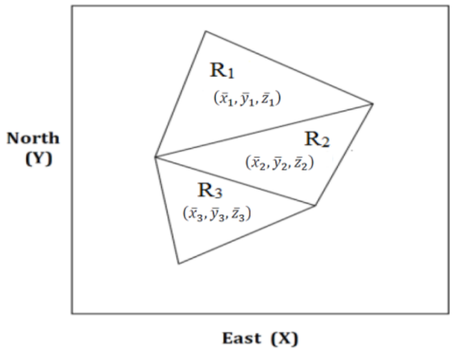

Figure 3. Ore deposit consists of a number of triangles.

$$x = \sum_{i=1}^{n} R_i \bar{x}_i \Big/ \sum_{i=1}^{n} R_i \tag{9}$$

$$y = \sum_{i=1}^{n} R_i \bar{y}_i \Big/ \sum_{i=1}^{n} R_i \tag{10}$$

$$z = \sum_{i=1}^{n} R_i \bar{z}_i \Big/ \sum_{i=1}^{n} R_i \tag{11}$$

Where:
x = x coordinate of the center of gravity of the ore deposit.
y = y coordinate of the center of gravity of the ore deposit.
z = z coordinate of the center of gravity of the ore deposit.
$\bar{x}_i$ = x coordinate of the center of gravity of the i th triangle of ore deposit.
$\bar{y}_i$ = y coordinate of the center of gravity of the i th triangle of ore deposit.
$\bar{z}_i$ = x coordinate of the center of gravity of the i th triangle of ore deposit.
R_i = The reserves of the i th triangle of ore deposit.
n = Number of triangles of ore deposit.

3. RESULTS AND DISCUSSIONS

The suggested mathematical model was applied in Aljalamid phosphate ore deposit, in the northern part of Saudi Arabia, the ore is spread on three different locations that are Fish area, Southern area and Western area.

3.1. The center of gravity of ore deposit.

Data of boreholes were obtained from Maaden Company for Aljalamid deposit. These data represent three different locations. Determination of gravity center coordinates of ore deposit consists of two steps as follows: First, calculation of the gravity center of each triangle of ore deposit using equations (7, 8). Second, application of the moment method (Equations 9,10,11). The ore reserves and gravity center of different locations of Aljalamid phosphate ore deposits are shown in **Table 1**.

Table 1. Reserves and coordinates of gravity center of different locations of Aljalamid phos-phate ore deposit

Ore deposit locations	Reserves (Tons)	C.G. Coordinates		
		x (m)	y(m)	z (m)
Fish Area location (L1)	146,130,862	291095	245985	756
Southern Area location (L2)	117,299,884	290119	191354	712
Western Area location (L3)	109,819,355	242008	216954	723

3.2. Optimum Processing Plant Location

The optimum location of processing plant requires the number of ore deposit locations(n), reserves (Q_i) and center of gravity for

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each location (x_i,y_i,z_i). The Equations (4, 5&6) of model can be used in the form of 12, 13 and 14 as follows.

$$\frac{\partial s}{\partial x} = (Q_1C_1(x-x_1)\Big/\sqrt{(x-x_1)^2+(y-y_1)^2+(z-z_1)^2}\,) + (Q_2C_2(x-x_2)\Big/\sqrt{(x-x_1)^2+(y-y_1)^2+(z-z_1)^2}\,) +$$

$$............+ (Q_nC_n(x-x_n)\Big/\sqrt{(x-x_n)^2+(y-y_n)^2+(z-z_n)^2}\,) = 0 \quad (12)$$

$$\frac{\partial s}{\partial y} = (Q_1C_1(y-y_1)\Big/\sqrt{(x-x_1)^2+(y-y_1)^2+(z-z_1)^2}\,) + (Q_2C_2(y-y_2)\Big/\sqrt{(x-x_1)^2+(y-y_1)^2+(z-z_1)^2}\,) +$$

$$............+ (Q_nC_n(y-y_n)\Big/\sqrt{(x-x_n)^2+(y-y_n)^2+(z-z_n)^2}\,) = 0 \quad (13)$$

$$\frac{\partial s}{\partial z} = (Q_1C_1(z-z_1)\Big/\sqrt{(x-x_1)^2+(y-y_1)^2+(z-z_1)^2}\,) + (Q_2C_2(z-z_2)\Big/\sqrt{(x-x_1)^2+(y-y_1)^2+(z-z_1)^2}\,) +$$

$$...........+ (Q_nC_n(z-z_n)\Big/\sqrt{(x-x_n)^2+(y-y_n)^2+(z-z_n)^2}\,) = 0 \quad (14)$$

The unknowns in equations 12, 13 and 14 are x, y and z coordinates of the optimum location of the processing plant. These equations represent the mathematical model. Computer program was developed to solve these equations.

3.3. Model validation

Validation step of model is one of main important steps of modelling. Validation based on the values of equations 12, 13 and 14 equal zero by using the input data of different locations and the calculated result of optimum location. Model v alidation can be summarized as follows:

1. Specify number of main locations of ore deposit (n)
2. Specify ore reserves of each location Q_i
3. Specify gravity center coordinates for each location (x_i,y_i,z_i)
4. Specify cost of transportation in different locations or paths to optimum location.
5. Input the above data in steps from 1 to 4.
6. Run the developed program to obtain optimum location coordinates of processing plant.
7. Calculate the values of the equations.12, 13 & 14 using the coordinates of optimum location resulted from step 6, and input values of steps from 1 to 4.
8. If the calculated values of equations (12, 13, and 14) in step 7 were zero or within the permissible errors it means the model is valid.
9. The above steps were repeated for 10 varied cases with different program inputs (variables), n, Q, C and center of gravity coordinates.

Table 2 shows the obtained results for 10 validation cases together with their corresponding errors. As an example, the output result of case no. 1 is shown in **Figure 4** and **Figure 5**. Where **Figure 4** shows a snapshot of the program with the input variables and **Figure 5** shows a snapshot of program final results of the optimum location for case No.1.

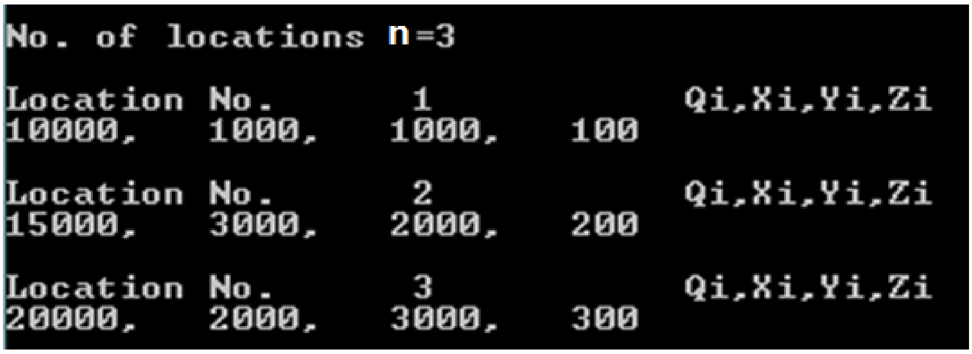

Figure 4 A screenshot of the program after completion of ariables enteredfor case No. 1.

Figure 5 A screenshot of the program after the completion of program run for case No.1

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Table 2. Validation Table

Case No.	No. of Locatio	First Location				Second Location				Third Location				Fourth Location if any				Processing Plant Optimum Location			$\frac{\partial s}{\partial x}$	$\frac{\partial s}{\partial y}$	$\frac{\partial s}{\partial z}$
		Q ton	X m	Y m	Z m	Q ton	X m	Y m	Z m	Q ton	X m	Y m	Z m	Q ton	X m	Y m	Z m	X m	Y m	Z m	Eq.12	Eq.13	Eq.14
1	3	10000	1000	1000	100	15000	3000	2000	200	20000	2000	3000	300	_	_	_	_	2034.29257	2895.39495	289.53949	0.0	0.0	0.0
2	4	5000	500	500	50	8000	1000	1500	70	7000	2000	1000	60	9000	2500	500	80	1997.62097	998.13330	60.12598	0.0	0.0	0.0
3	3	60000	800	700	90	75000	1100	950	110	90000	1300	800	115	_	_	_	_	1126.63943	885.40152	109.49180	0.0	0.0	0.0
4	4	75000	1200	1000	150	80000	1500	1800	140	9000	2000	1600	175	68000	2500	900	130	1580.09806	1368.02748	143.46178	0.0	0.0	0.0
5	3	150000	29400	25000	1100	165000	33500	39000	1140	173000	38600	28200	1210	_	_	_	_	35624.57185	29567.31558	1172.70062	0.0	0.0	0.0
6	4	258000	15300	11230	276	195800	17200	13394	245	188600	18100	12850	264	210000	19100	10960	239	17591.21462	12530.08223	256.78175	0.0	0.0	0.0
7	3	310700	36970	11230	320	344270	30900	15610	339	442870	41830	17330	361	_	_	_	_	37298.58255	14357.96841	338.66142	0.0	0.0	0.0
6	4	570280	55370	61700	1230	390900	68300	81370	1270	410380	54800	73230	1290	87100	83120	95419	1320	56407.47650	71864.64699	1276.39557	0.0	0.0	0.0
9	3	51200	52300	41100	360	63400	59420	70900	345	69500	65310	45600	355	_	_	_	_	63703.61335	46721.67314	354.86492	0.0	0.0	0.0
10	4	95800	53120	71225	317	83400	57340	85340	335	76800	77600	86460	350	79200	81300	70250	345	63945.42062	79182.78477	334.47552	0.0	0.0	0.0

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3.4. Optimum processing plant location of Aljalamid phosphate ore.

The mathematical model was used to calculate the optimum location of the processing plant. The required data to apply the mathematical model are the reserves, coordinates of the gravity centers and transportation cost of one ton for unit distance of the different Aljalamid phosphate ore deposits. These data are given in Table 1. According to the output of the computer program as shown in figure 6, the coordinates of the optimum location of the processing plant are approximately (278842, 223149, 735) in the east, north, and elevation directions, respectively. The obtained optimum location of processing plant related to the different ore deposit locations of Aljalamid phosphate ore is shown in figure 7.

```
iteration    290    of     2999
             --X--                --DF--
          0.27884E+06          0.31250E+01
          0.22315E+06          0.00000E+00
          0.73526E+03          0.00000E+00

                 The error reached           0.31250E+01

          Calling Function Number is        2331
 EX.   3   N =        3
       Optimum Location              Error
X=    278842.44368                 0.00000E+00
Y=    223148.98519                 0.00000E+00
Z=       735.26214                 0.00000E+00
```

Figure 6 A Screanshot of the optimum location results achieved after the completion of program run.

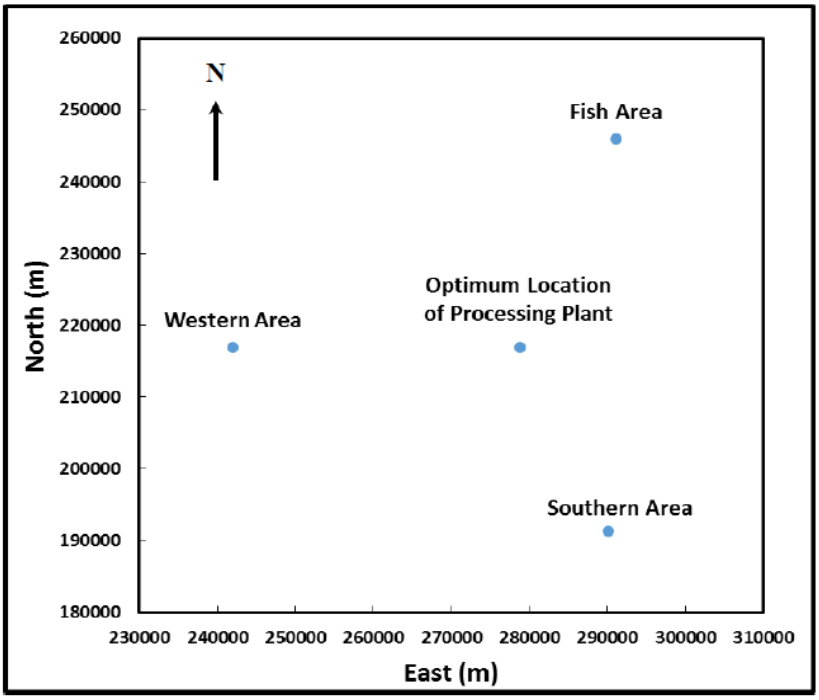

Figure 7 Map shows the optimum location of processing plant related to the different ore deposit locations of Aljalamid phosphate ore.

3.3. Effect of processing plant location deviation from optimum

The current presented results showed the optimum location of mineral processing plant in an ideally theoretical case. Due to any reason whatsoever, it may be impossible to construct the processing plant at the determined optimum location. Now, it is of importance to investigate the transportation of ore to any location somehow around the optimum location. Off course this deviation from the optimum plant location will increase transportation cost. Total transportation cost of the ore can be determined by the equation (3) as follows:

$$S = \sum_{i=1}^{n} Q_i * C_i * \sqrt{(x-x_i)^2 + (y-y_i)^2 + (z-z_i)^2}$$

The additional total transportation cost percentage due to deviation of processing plant location from the optimum plant location can be calculated from equation (15).

$$Additional\ Cost\ \% = (\frac{S - S_O}{S_O}) * 100 \qquad (15)$$

S: is total transportation cost of the ore for the different locations to the processing plant location. (Equation 3).

$S_{o:}$ is the total ore transportation cost to the optimum processing plant location, It can be calculated from Equation 3, when x, y and z coordinates refer to optimum processing plant location i.e S_o is a special case of S when transportation of ore is going to be to the optimum mineral processing location.

Figure 8 shows a contour map of the additional cost percentage compared to the minimum for different selected mineral processing plant locations. The figure shows that choosing one of three locations at Aljalamid (Fish area L1, Southern area L2 & Western area L3) may be increase the transportation cost to 8 %, 22% and 28 % respectively compared with the optimum location. Also, the figure shows that there may be more than 28 % increase in the ore transportation cost due to an incorrect selection of the processing plant location.

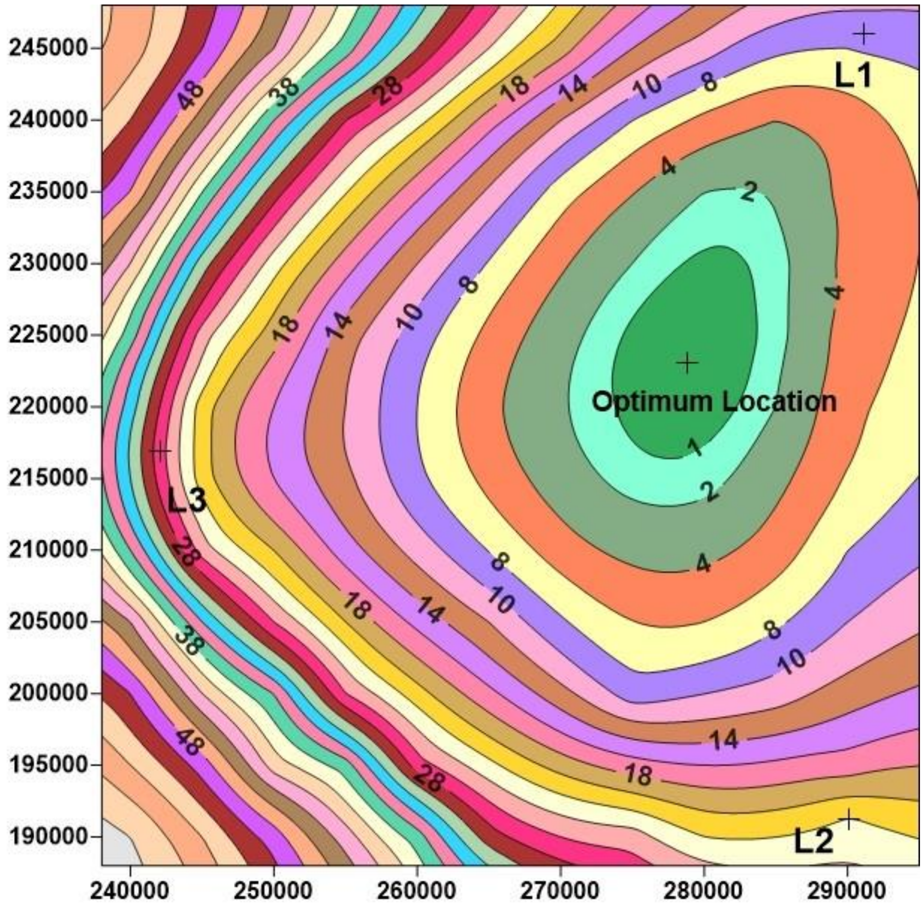

Figure 8 Contour map of additional cost % related deviation of processing plant location from optimum.
(L1= Fish Area, L2 =Southern Area, and L3 refers for Western Area)

4. CONCLUSIONS

From the obtained results, the following conclusions can be made:

- The phosphate ore in Aljalamid is found in 3 main locations having ore reserves of 146, 117, and 109 million tons respectively.
- Centers of gravity for three locations of ore deposit were determined.
- The presented model satisfies minimum total cost of ore transportation from all ore deposit locations to processing plant location.
- Both of the mathematical model and Computer program to solve the model were validated.
- The processing plant optimum location were obtained.
- Deviation of processing plant location from optimum increases the transportation cost of ore deposit.
- The presented model will be useful in similar ores.

REFERENCES

[1] Bernknopf, R. L. (1985). Domestic coal distribution: an interregional programming model for the US coal industry.

[2] United States. Department of Transportation. Office of University Research. (1976). Summary of Awards, Program of University Research. US Department of Transportation, Research and Special Programs Administration, Transportation Programs Bureau, Office of University Research.

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
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[3] Nagarvala, P. J., Ferrell, G. C., & Olver, L. A. (1975). Clean coal energy: source-to-use economics, phase II. Annual report for the period July 1974--July 1975 (No. FE-1552-1 (Vol. 1)). Bechtel Corp., San Francisco, Calif.(USA).

[4] Zimmerman, M. B. (1977). Modeling depletion in a mineral industry: The case of coal. The Bell Journal of Economics, 41-65.

[5] Hartwick, J. M. (1972). The gravity hypothesis and transportation cost minimization. Regional and Urban Economics, 2(3), 297-308.

[6] Lee, S. M., & Moore, L. J. (1973). Optimizing transportation problems with multiple objectives. AIIE transactions, 5(4), 333-338.

[7] Akay, A. E. (2006). Minimizing total costs of forest roads with computer-aided design model. Sadhana, 31(5), 621-633.

[8] Nikolić, I. (2007). Total time minimizing transportation problem. Yugoslav Journal of Operations Research, 17(1), 125-133.

[9] Villani, C. (2009). Optimal transport: old and new (Vol. 338, p. 23). Berlin: Springer.

[10] Kennedy, B. A., & Kennedy, B. A. (Eds.). (1990). Surface mining. SME.

[11] Fjellström, N. (2011). Simulation of an underground haulage system, Renström Mine, Boliden Mineral.

[12] Wegener, M. (2004). Transport Geography and Spatial Systems. Handbook 5 of the Handbook in Transport.

[13] Brazil, M., Lee, D. H., Rubinstein, J. H., Thomas, D. A., Weng, J. F., & Wormald, N. C. (2002). A network model to optimise cost in underground mine design. Transactions-South African Institute of Electrical Engineers, 93(2), 97-103.

[14] Dharma, S., & Ahmad, A. (2005). Optimization of transportation problem with computer aided linear programming. In Proceedings of the Postgraduate Annual Research Seminar (p. 140).

[15] Shephard, R. W. (2015). Theory of cost and production functions. Princeton University Press.

[16] Mahran, G. M. A, Aboushook, M. I. & Yassien, M. A.(2001). Optimum Location of Processing Plant in Bahariya Iron Ore Mines 7th International Conference on Mining, Petroleum and Metallurgical Engineering (MPM) Assiut – Egypt, 10-12 February, 2001.

[17] Inwood, K., & Keay, I. (2013). Trade policy and industrial development: iron and steel in a small open economy, 1870− 1913. *Canadian Journal of Economics/Revue canadienne d'économique*, *46*(4), 1265-1294.

[18] Reeb, J. E., & Leavengood, S. A. (2002). Transportation problem: a special case for linear programming problems.

[19] Ali, M. A., & Sik, Y. H. (2012). Transportation problem: A special case for linear programing problems in mining engineering. *International Journal of Mining Science and Technology*, *22*(3), 371-377.

[20] Chen, Y., Liu, T., & Zhang, P. (2012). Research on Optimization of Imported Iron Ore Transportation Organization Mode in the Yangtze River Basin Based on Minimum Generalized Cost. *International journal of advancements in computing technology*, *4*, 193-199.

[21] Ahmed, M. M., Tanvir, A. S. M., Sultana, S., Mahmud, S., & Uddin, M. S. (2014). An effective modification to solve transportation problems: a cost minimization approach. *Annals of Pure and Applied Mathematics*, *6*(2), 199-206.

[22] Joshi, R. V. (2013). Optimization techniques for transportation problems of three variables. *IOSR Journal of Mathematics*, *9*(1), 46-50.

[23] Ahmad, H. A. (2012). The best candidates method for solving optimization problems. *Journal of computer science*, *8*(5), 711.

[24] Novikov, A. N., Novikov, I. A. and Zagorodnij,N. A.(2021) Reducing Production and Transportation Costs for the Transportation of Iron Ore Raw Materials from Mining and Processing Plants on the Basis of the Use of an Integer Model." *In IOP Conference Series: Earth and Environmental Science*, vol. 666, no. 5, p. 052038. IOP Publishing, 2021.

doi:10.1088/1755-1315/666/5/052038

[25] Saderova, J., Rosova, A., Kacmary, P., Sofranko, M., Bindzar, P., & Malkus, T. (2020). Modelling as a Tool for the Planning of the Transport System Performance in the Conditions of a Raw Material Mining. *Sustainability*, *12*(19), 8051.

[26] Cummins, A. B., & Given, I. A. (1973). Mining engineering handbook. *American Review of Respiratory Disease*, *2*, 203-206.

[27] http://en.wikipedia.org/wiki/Centroid#Of_triangle_and_tetrahedron