Numerical Investigation to Predict the Performance of A GM Pulse Tube Refrigerator using of Pressure Wave Profile

GM Pulse tube Refrigerator

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***Abstract*— The development of GM pulse tube over Inertance pulse tube is to come across the requirement of high heat capacity. The performance of the GM pulse tube refrigerator was greatly improved by introducing different types of pressure wave profiles. Here, 3D model of DIPTR is created using ANSYS Design Modeler and Computational Fluid Dynamics (CFD) solution approach is chosen for numerical simulation purpose. The detailed study on cool down behavior at CHX of DIPTR, pressure variation at inlet and also heat transfer has been performed using CFD package Fluent®17 software. Four numbers of cases have been chosen in which the pressure User Defined Functions (UDFs) are different and all other parameters remain unchanged. Pressure UDFs are applied at inlet as a boundary condition to define the oscillating motion of piston inside the piston-cylinder arrangement. The four pressure UDFs are of different wave forms such as Sinusoidal, Rectangular and Trapezoidal. The operating frequency for all cases is 2 Hz. After simulation using four pressure UDFs, it has been found that pressure UDF generating trapezoidal pressure wave is more efficient than other pressure UDFs and a temperature of 110 K is obtained using Trapezoidal pressure wave profile.**

Keywords*— Computational Fluid Dynamics (CFD); Cryogenic Temperature; Double Inlet Pulse Tube Refrigerator (DIPTR); Pressure wave profile; Numerical simulation.

I. INTRODUCTION

The initial observation on one end of a hole tube called pulse tube with a pressure wave at low frequency reported in early 1960’s [1], and known as pulse tube refrigeration. Numerous developments in such field have been reported, however, Gifford McMahan (GM) type pulse tube refrigerator is one of them which has been used in case of high cooling capacity requirement [2, 3, 4].

The argument, need of analyzing a pulse tube is still a question or requirement, is still debatable. Because one hand is to understand the thermoacoustic phenomena and in other hand is establishing the technique to design a pulse tube for a desired requirement. Such intentions provoke us to concentrate the research on establishing a methodology to analyze a GM pulse tube using commercially available CFD software [5]. The fact, “Why”, is acknowledged by the audience, however, how with trustworthiness is still a prerequisite.

From aforementioned literature review it can be concluded that 3D analysis of pulse tube is a basic requirement which has not been addressed in detail till now.

In present extensive numerical investigation, the performance of different wave forms such as Sinusoidal, Rectangular and Trapezoidal, has been optimized to attain the lowest cooling temperature at CHX. To start with, the benchmark GMPTR has been examined by applying Sinusoidal wave and a robust CFD technique has been established. Various issues in numerical investigations have been addressed in detail. Moreover, the recommendations have been made to achieve steady state solution with low computational overhead.

II. SIMULATION MODEL AND BOUNDARY CONDITIONS

In order to justify the stability and reliability of the proposed numerical technique, a GM pulse tube from literature has been taken which has been experimentally investigated [1].

TABLE 1. DIMENSION AND THERMAL BOUNDARY CONDITIONS OF GM PULSE TUBE (KASTHURIRENGAN S.ET AL.,2001 [2])

Sl. No.	Component	Radius (m)	Length (m)	Thermal Condition
1.	Transfer Line	5.30E-03	11.50-02	Isothermal
2.	After Cooler (AFT)	22.00E-03	20.00E-03	Isothermal
3.	Regenerator	20.00E-03	21.00E-02	Adiabatic
4.	Cold Heat Exchanger (CHX)	15.00E-03	18.0E-02	Adiabatic
5.	Pulse tube	15.00E-03	25.00E-02	Adiabatic
6.	Hot Heat Exchanger (HHX)	22.00E-03	20.00E-03	Isothermal
7.	Surge Volume	10.0E-02	30.00E-02	Isothermal

The dimension of GM pulse tube under investigation has been mentioned in Table.1. The detailed investigation has been carried out to demonstrate the complexities such as the requirement of desired meshing, appropriate contacts, boundary conditions, solver configurations, and necessity of User Defined Functions (UDF) involved in simulation to achieve desired result. The Fig.1 represents the geometry of the GM pulse tube under investigation. To save the computational time, the symmetric sectional geometry has been simulated.

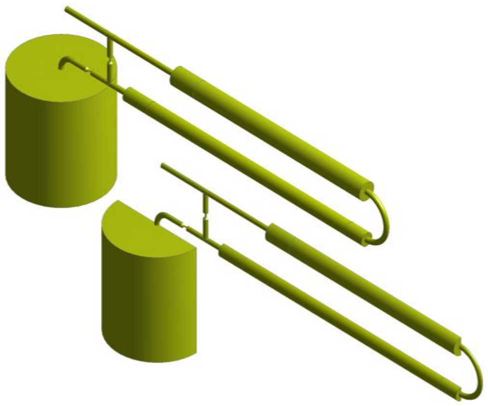

FIG 1. SCHEMATIC GEOMETRY OF THE GM PULSE TUBE UNDER INVESTIGATION; (A) 3D GEOMETRY, AND (B) SECTIONAL GEOMETRY.

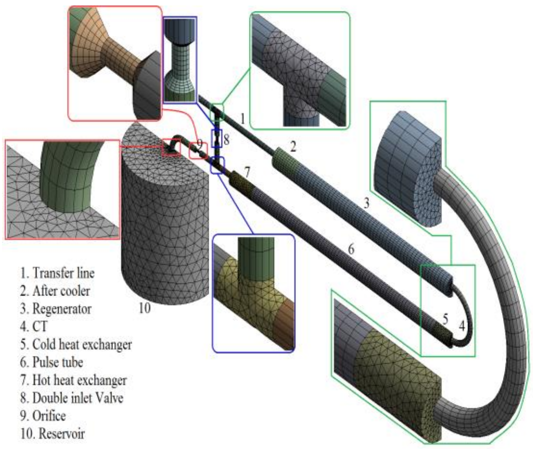

FIG 2. FINITE ELEMENT DISCRETIZATION OF 3D-AXISYMMETRIC MODEL OF AN GMPTR.

III. GOVERNING EQUATIONS

Continuum-based conservation equations can be applied everywhere in the system. The mass, momentum and energy equations solved by fluent are as follows [6]:

$$\frac{\partial \rho_f}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\rho_f V_r\right)+\frac{\partial}{\partial x}\left(\rho_f V_x\right)=0 \text{ ------- (1)}$$

$$\frac{\partial}{\partial t}\left(\rho_f \vec{v}\right)+\nabla\left(\rho_f \vec{v}\vec{v}\right)=-\nabla p+\nabla.\left[\bar{\bar{\tau}}\right]$$

$$\frac{\partial}{\partial t}\left(\rho_f E\right)+\nabla.\left[\vec{v}\left(\rho_f E+p\right)\right]=\nabla.\left[k_f\nabla T+\left(\bar{\bar{\tau}}.\vec{v}\right)\right]$$

Where,

$$E=h-\frac{p}{\rho}+\frac{v^2}{2}$$

One thing is very important i.e. all the properties is applicable only for helium as a working fluid. All the above equation is not applying for after cooler, cold heat exchanger (CHX), hot heat exchanger (HHX) and regenerator components. These components are modelled as porous media.

The resistance of gas flow through the porous media is modeled by an additional momentum source term to the standard fluid flow equations. The source term is nothing but it is an additional pressure drop term due to porous matrix. This source term consists of two parts: a viscous loss terms and an inertial loss term. For a simple homogeneous porous media, the source term is [5]

$$S_i=-\left(\frac{\mu}{\alpha}\bar{v}+C_2\frac{1}{2}\rho_f\left|\bar{v}\right|\bar{v}\right)$$

Where, α and C2 are the permeability and the inertial resistance factor, respectively. Thus, the mass, momentum and energy equations for the porous media can be expressed as [7]

$$\frac{\partial}{\partial t}\left(\phi\rho_f\right)+\frac{1}{r}\left(\phi r\rho_f v_r\right)+\frac{\partial}{\partial r}\left(\phi\rho_f v_x\right)=0$$

$$\frac{\partial}{\partial t}\left(\phi\rho_f\vec{v}\right)+\nabla\left(\phi r\rho_f\vec{v}\vec{v}\right)=-\phi\nabla p+\nabla.\left[\phi\bar{\bar{\tau}}\right]+S_i$$

$$\frac{\partial}{\partial t}\left(\phi\rho_f E_f+\left(1-\phi\right)\rho_s E_s\right)+\nabla.\left[\vec{v}\left(\rho_f E_f+p\right)\right]=$$
$$\nabla.\left[\phi k_f+\left(1-\phi\right)k_s\nabla T+\left(\phi\bar{\bar{\tau}}.\vec{v}\right)\right]$$

where ϕ is porosity of the porous media; k_f and k_s are thermal conductivity of fluid and solid structure, respectively. The axial thermal conductivity of the solid structure is set to be about 10% of the screen material due to the contact thermal resistance between the screen layers [8].

The following one-dimensional empirical equations are considered for the pressure gradient in the porous media [7]:

$$\frac{\partial p}{\partial x}=-f_F\frac{2\rho_f v^2}{d_h}$$

where f_F is fanning friction factor based on steady flow correlation and expressed as:

$$f_F=\frac{n\phi d_h^{\,2}}{4l\beta}\frac{33.6}{R_{eh}}+0.337\frac{n\phi d_h}{4\beta^2},R_{eh}=\frac{\rho_f v d_h}{\mu}$$

Substituting Eq. (10) in Eq. (9) the pressure gradient in porous media becomes

$$\frac{\partial p}{\partial x}=-\frac{33.6n\phi v\mu}{2l\beta}-0.337\frac{n\phi^2\rho_f v^2}{2\beta^2}$$

where l is the mesh distance; β is the opening area ratio of screen; dh is the hydraulic diameter of screen, n is the number of packed screens per length; μ is the viscosity. Comparing Eq. (11) with Eq. (5) leads to

$$D=\frac{1}{\alpha}=\frac{33.6n\phi}{2l\beta};C_2=\frac{0.337n\phi^2}{\beta^2}$$

For a perfect stacking of square mesh screens in which the weaving causes no inclination of the wires and screen layers are not separated, ϕ and β can be calculated as [9]

$$\phi = 1 - \frac{\pi}{4x_t}, x_t = \frac{0.0254}{d_w m}, \beta = (x_t - 1)^2 / x_t^2$$

where m is mesh per inch; dw is wire diameter of screen in meter. The geometry of porous medium for regenerator is 304SS woven wire mesh of 200 mesh size and for other heat exchangers it is copper wire mesh of 100 mesh size is used for the simulation.

IV. ASSUMPTIONS AND BOUNDARY CONDITIONS

From literature, it can be noticed that the CFD analysis of GM pulse tube still persists as a challenge for researchers due it its computational overhead. The present extensive numerical investigation tries to address such issue in detail. The computational fluid dynamics of the GM pulse tube refrigerator has been investigated in ANSYS/Fluent platform for a given dimension [10]. The finite element discretization has been carried out in ANSYS meshing module. The mid-side node has been dropped to reduce the number of nodes. In general, the boundary layer meshing used to be carried out to solve the fluid flow problem. However, in present situation the compressor motion generates a pressure wave of very high amplitude (approximately ± 5 bar on the operating pressure 16 bar). So, the effect of boundary layer may be neglected to reduce the computational overhead. At the start of the simulation, the temperature in the system has been assumed to be at 293 K. Table.1 shows the required thermal boundary conditions on all components, corresponding to executed experiment [11]. The simulation has been carried out using below said assumptions.

- The computational overhead is due to number of elements and nodes. Moreover, the piston simulation using dynamic meshing adds significant more computational overhead.
- In order to reduce the computational time, the piston is simulated using pressure user defined function (UDF). The symmetric sectional analysis has been carried out to reduce the computational overload for parametric optimization.
- The computational overhead is high due to time duration required in transient analysis to attain steady state, demonstrated in first section. In order to reduce the computational time, the transient analysis has been proposed to initiate from a steady state solution using a known temperature profile, discussed in subsequent section.
- The simulation has been carried out using ideal gas model (Helium) by solving Naiver-Strokes continuity, momentum and energy equations including enabling k-ϵ turbulent model to simulate flow in porous media.
- The operating pressure across GM pulse has been taken to be 16 bar and the operating frequency of the compressor piston motion has been taken to be 2 Hz.
- In real-time investigation [11], the AFT, regenerator, CHX, and HHX have been prepared using fine wire meshes (Copper wire of 100 mesh size). In simulation, those components have been modelled with porous media using equivalent viscous resistance (1.085e+09 m-2), inertial resistance (5750 m-1) and porosity (0.697), following empirical models [12].
- The conduction equations corresponding to porous media model have been solved by using steel as material apart from CHX, where copper has been taken.

V. GRID INDEPENDENCE TEST

The grid testing has been carried out in HP Compaq Elite LE1902x computer having 16GB of RAM. The testing has been carried out for different numbers of element and nodes for 2D-axisymmetric model and 3D model separately. The observations have been tabulated below. In order to ensure the accuracy of the computational grid, a details verification of grid independence test for the cold end temperature in the IPTR with five different grid number are checked.

TABLE 2: GRID TESTING OF IPTR (3D ANALYSIS)

Sl. No.	Parameter	Case-a	Case-b	Case-c	Case-d
(a)	No. of Elements	39878	40148	48306	55248
(b)	No. of Nodes	34958	35423	37687	44978
(c)	Lowest Temperature	140	125	110	108.5
(d)	Computational Time	150	175	200	232

From above analyses, it can be inferred that adequate meshing plays a vital role in achieving the desired result. From observations, it can be recommended that the CHX, and HHX should be meshed using triangular or tetrahedron elements for the requirement of adequate heat transfer. The pulse tube and regenerator including transfer lines should be modeled using rectangular or hexahedron elements using sweep method in case of 3D simulation, correspondingly.

VI. RESULTS AND ANALYSIS

The present paper aims for developing a procedure to generate such predictions of the PTR performance for different pressure waveforms. This will help to determine an optimum pressure waveform for a given PTR configuration and will be useful in a significant way for the design of the rotary valve. The performance of the GM PTR system, various wave forms are studied. The performance of the GM PTR system, various wave forms are studied.

Case-1. Pressure UDF (Sinusoidal wave form)

Pressure UDF having sinusoidal wave form is as below:

#include "udf.h" DEFINE_PROFILE (unsteady_pressure, thread, position)

```
{
face_t f;
real t = CURRENT_TIME;
begin_f_loop (f, thread)
{
F_PROFILE (f, thread, position) = 101325.0 × (10 + 5:0 × sin (2 × 3:1416 × 2000 × t));
}
end_f_loop (f, thread)
}
```

This UDF is compiled in the FLUENT software and applied as boundary condition at inlet and then simulation is carried out. The pressure wave generated at inlet of DIPTR which is shown in fig 3.

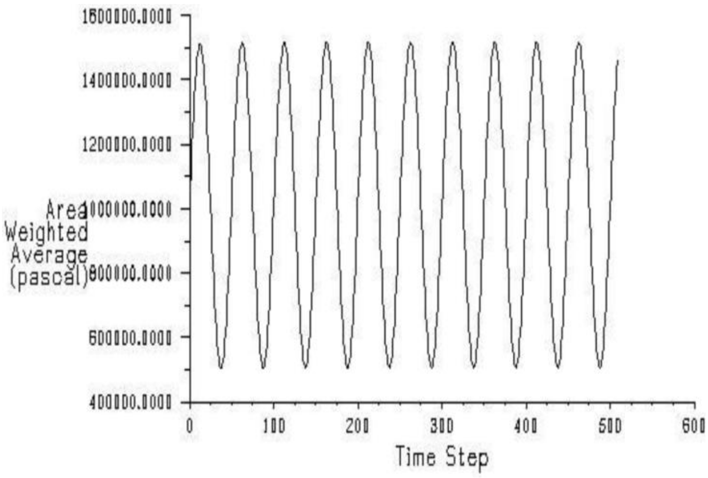

FIG 3. PRESSURE WAVE GENERATED AT INLET OF DIPTR FOR CASE - 1.

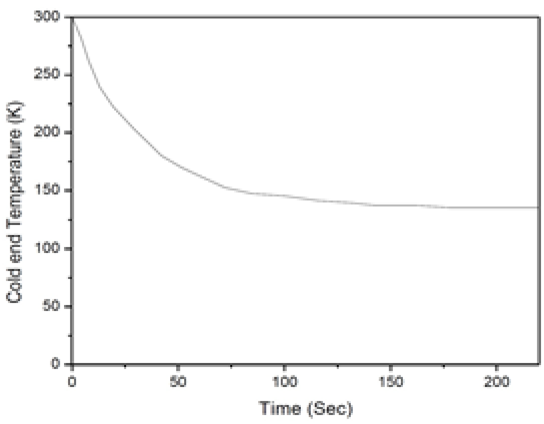

FIG 4. TEMPERATURE DECREMENT BEHAVIOUR OF DIPTR FOR CASE - 1.

It has been observed that by implementing pressure UDF to simulate piston and transient simulation using steady state solution by sinusoidal wave profile approach, the GM pulse tube in hand has been investigated. From simulation it has been observed that, the corresponding cooling curve for 510 seconds with a time step of 0.001 second has been shown in Fig.4. From Fig.4, it can be noticed that the lowest temperature CHX achieve is very closely to 134 K.

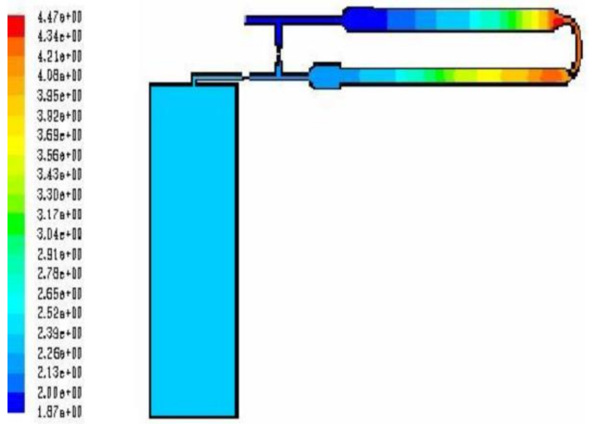

FIG 5. TEMPERATURE CONTOUR OF DIPTR FOR CASE-1.

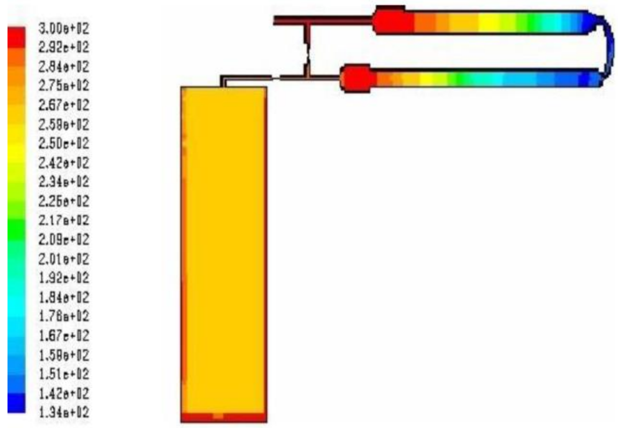

FIG 6. DENSITY CONTOUR OF DIPTR FOR CASE-1.

For detailed understanding the contour of temperature and density distribution have been extracted and have been shown in Fig.6. From contour it has been observed that the temperature distribution gradually decreases in regenerator however in pulse tube it is not that gradual.

Case-2. Pressure UDF (Rectangular wave form)

Pressure UDF having rectangular wave form is as below:

```
#include "udf.h" DEFINE_PROFILE (unsteady_pressure, thread, position)
{
face_t f;
real t = CURRENT_TIME;
begin_f_loop (f, thread)
{
```

```
for (i = 0; i <100; i++)
{
if (t < (p + 0.0025))
F_PROFILE (f, thread, position) = 20;
else if (t = = (p + 0.0025))
F_PROFILE (f, thread, position) = 15;
else if (t < (p + 0.005))
F_PROFILE (f, thread, position) = 10;
else if (t = = (p+ 0.005))
F_PROFILE (f, thread, position) = 15;
else
p = p + 0.005;
}
}
end_f_loop (f, thread)
}
```

The pressure wave generated at inlet of DIPTR, which is shown in Fig.7. When we implemented rectangular pressure wave profile by similar fashion in DIPTR then by simulation it has been observed that, the corresponding cooling curve for 240 seconds with a time step of 0.001 second has been shown in Fig.8. From Fig.8, it can be noticed that approximately after 210 sec. the lowest temperature at CHX is constant which is very closely to 128 K.

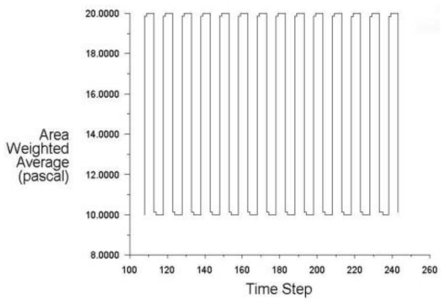

FIG 7. PRESSURE WAVE GENERATED AT INLET OF DIPTR FOR CASE – 2

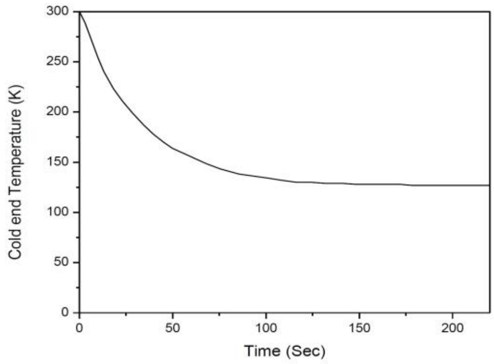

FIG 8. TEMPERATURE DECREMENT BEHAVIOUR OF DIPTR FOR CASE – 2

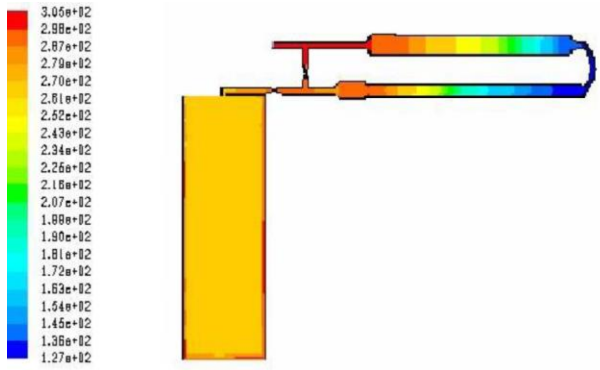

FIG 9. TEMPERATURE CONTOUR OF DIPTR FOR CASE-2.

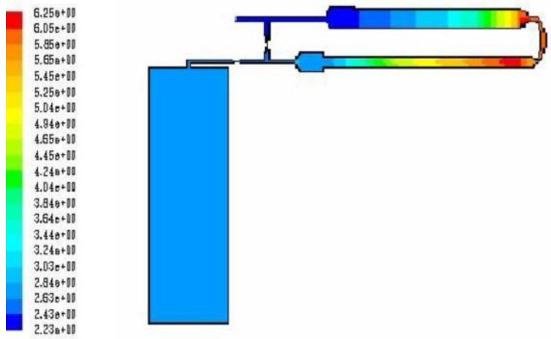

FIG 10. DENSITY CONTOUR OF DIPTR FOR CASE-2.

Afterward, various parameters over DIPTR, the contour plot of temperature and density have been extracted and are shown in Fig. 9 and Fig.10.

Case-3. Pressure UDF (Trapezoidal wave form)

Pressure UDF having trapezoidal wave form is as below:

```
#include "udf.h" DEFINE_PROFILE (unsteady_pressure, thread, position)
{
face_t f;
real t = CURRENT_TIME;
begin_f_loop (f, thread)
{
for (i = 0; i <100; i++)
{
if (t < (p + 0.001))
F_PROFILE (f, thread, position) = 15 + 5000* (t-p);
else if (t < (p + 0.0015))
F_PROFILE (f, thread, position) = 20;
else if (t < (p + 0.0035))
F_PROFILE (f, thread, position) = 20 - 5000*(t-0.0015-p);
else if (t < (p + 0.004))
```

```
F_PROFILE (f, thread, position) = 10;
else if (t < (p + 0.005))
F_PROFILE (f, thread, position) = 10 + 5000*(t-0.004-p);
else
p = p + 0.005;
}
}
end_f_loop (f, thread)
}
```

Last, simulation is carried out by trapezoidal pressure wave profile and it gives a better result as compare with other two pressure profile. Fig. 11 shows the trapezoidal wave profile.

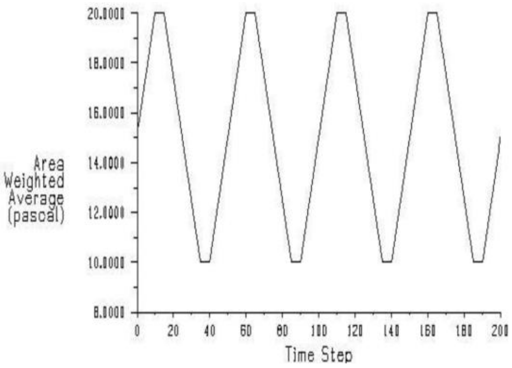

FIG 11. PRESSURE WAVE GENERATED AT INLET OF DIPTR for CASE - 3.

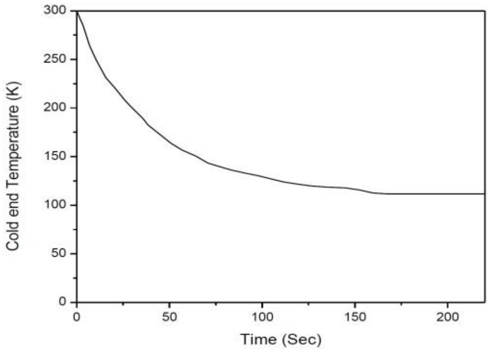

FIG 12. TEMPERATURE DECREMENT BEHAVIOUR OF DIPTR FOR CASE – 3.

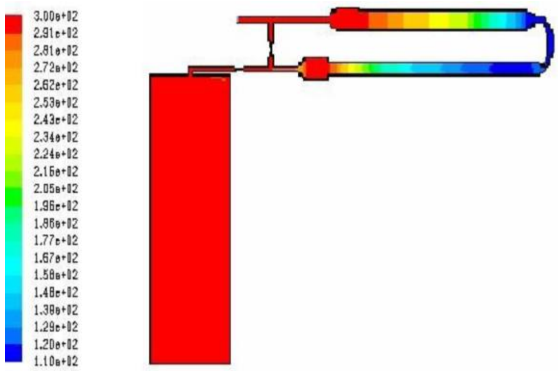

FIG 13. TEMPERATURE CONTOUR OF DIPTR FOR CASE - 3.

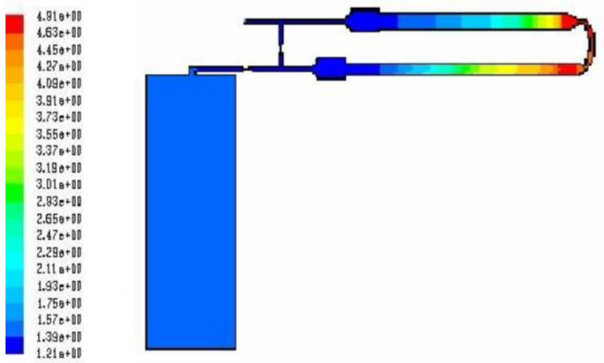

FIG 14. DENSITY CONTOUR OF DIPTR FOR CASE -3.

As discussed earlier the GM pulse tube has been investigated in a similar manner. From simulation it has been observed that, the computational overhead time reduce by applying trapezoidal wave profile. The corresponding cooling curve for 200 seconds with a time step of 0.001 second has been shown in Fig.12. From Fig.12, it can be noticed that the lowest temperature CHX achieve is very closely to 110 K.

VII. CONCLUSION

In order to observe the difference of refrigeration performance of a DIPTR at different pressure wave profile, a same pressure input has been applied for the simulation by user define function (UDF). Simulation is carried out for DIPTR using the ANSYS FLUENT 15.0 software. All the four cases having different pressure UDFs have been considered and the cold end temperature is determined for all the three cases. From simulation result, temperature obtained at cold end of DIPTR are 134 K, 128 K and 110 K for Sinusoidal, Rectangular and Trapezoidal wave forms respectively.

From the above simulation result, it can be concluded that the temperature obtained using trapezoidal pressure UDF i.e. 110 K is lower than that of the other cases. So trapezoidal wave form of pressure is much better than all other wave forms.

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