Numerical Investigation of Natural Convection Heat Transfer in a Square Cavity

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***Abstract* - Natural convection heat transfer in enclosures find many applications such as heating and cooling of building spaces, solar energy utilization, thermal energy storage, cooling of electrical and electronic components etc. In the present study, Numerical Investigation is conducted in a square cavity with one vertical wall maintained at a high temperature and with the opposing vertical wall at a low temperature. The influence of Grashof numbers ranging from 20000 to 200000 for Prandtl number 0.7 (air) is studied. The governing vorticity and energy equations are solved by finite difference methods including Alternating Direction Implicit (ADI) and Successive Over Relaxation (SOR) techniques with C coding. Steady state isothermal lines and streamlines are obtained for all the Grashof numbers considered. In addition, the average Nusselt number, over the hot wall for the range of Grashof numbers is calculated. The contours of streamlines and isothermal lines are presented for all the parameters investigated. Changes in the streamline and isothermal line patterns are observed with the change in Grashof numbers. The results obtained in this study are useful for the design of devices with enclosures subjected to high temperature differences.**

Keywords: Natural convection, ADI, SOR, Prandtl number, Grashof number, Nusselt number.

1. INTRODUCTION

Natural convection heat transfer in enclosures has been extensively studied by researchers, because of its practical significance in science and technology. Applications include heating and cooling of building spaces, solar energy collectors, heat exchangers and effective cooling of electronic components and machinery. The fluid flow and heat transfer behavior of such systems are analysed numerically and experimentally by a number of researchers with different boundary conditions.

An effective algorithm for the analysis of unsteady thermo capillary convection in a rectangular cavity was developed by Hamed and Folryan [1].
Kazmierezak and Chinoda [2] investigated numerically the problem of laminar buoyancy driven flow of a fluid in a square cavity driven by a warm vertical wall having a uniform surface temperature whose magnitude is periodically changing. The transient behavior of an enclosure when the temperature of only a single wall was suddenly changed, while other walls were adiabatic, was studied by Hall et al [3]. Schaldow et al [4] performed an additional run in which they ramped the driving wall temperature in a linear fashion over a five second interval equal in magnitude to the step change. Vasseur and Robillard [5] investigated the case of transient convective coding of a rectangular enclosure with end walls that continually decreased in temperature at a constant rate. Kumar and Kandaswamy [6] have studied convection purely driven by buoyancy force in a square cavity with two different thermal boundary conditions; isothermal and linearly varying hot wall temperature in the presence of a uniform transverse magnetic field. This result shows the suppression of convection by an increase in the magnetic field strength. Wilkes and Churchill [7] used an implicit-alternating direction finite difference method to study numerically the natural convection of a fluid contained in a long horizontal rectangular enclosure with vertical wall temperature for different Grashof number and aspect ratios. Hellums and Churchill [8] developed an explicit finite difference methods for generating the transient solution for free convection at a vertical plate. Sundaravadivelu and kandaswamy [9] have taken a fourth order polynomial approximation of the temperature-density relation for water and studied the buoyancy driven nonlinear convection in a square cavity. The natural convection in the presence of a magnetic field in a rectangular enclosure is studied by Rudhraiah et al [10] who established that the magnetic field dampens the rate of heat transfer and velocity profiles. Also the influence of magnetic field on the combined mechanism in a low Prandtl number fluid was studied. Saikrishnan and Roy [11] investigated numerically the effect of temperature dependent viscosity on forced convection flow over a rotating sphere. Their results show that the heat transfer rate is found to depend strongly on viscous dissipation. Kandaswamy et al [12] studied the natural convection heat transfer in a cavity with a variable viscosity fluid applying ADI method coupled with SOR technique in which the heat transfer rate is found to increase with an increase in viscosity of the fluid, where in

they have chosen various Prandtl numbers like 0.05 (liquid metal), 0.7 (air) and 10(water). The objective of the present study is to numerically investigate in detail the natural convection in a two dimensional square cavity in which momentum transfer is significant.

2. MATHEMATICAL FORMULATION

A square cavity with different wall boundary conditions within which the fluid enclosed is considered for analysis. The geometry and temperature boundary conditions are shown in figure 1. Two of the opposing vertical walls are maintained at different temperatures. The horizontal walls are insulated from the surroundings. When a fluid is enclosed within the cavity, it starts to circulate within the cavity and the heat transfers by natural convection from the hot wall to the cold wall.

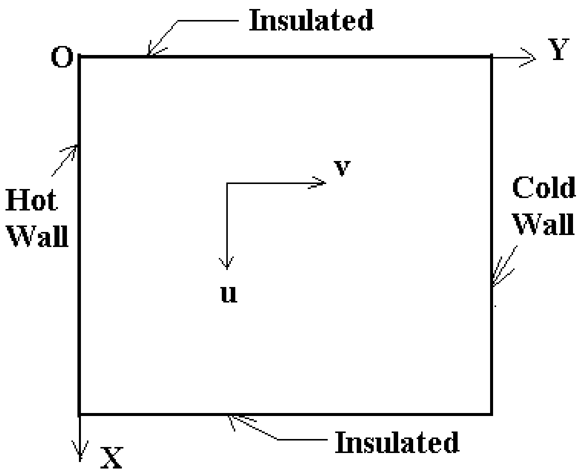

Figure 1 – Schematic of the square enclosure

2.1 Governing Equations

The two-dimensional governing equations are

Continuity: $$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (1)$$

Momentum: $$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}-g\beta(\theta-\theta_c)+\nu\nabla^2 u \quad (2)$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu\nabla^2 v \quad (3)$$

Energy: $$\frac{\partial \theta}{\partial t}+u\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial y}=\alpha\nabla^2\theta \quad (4)$$

2.2 Boundary Conditions

The initial and boundary conditions are,

$$\begin{aligned}
&t=0;\; u=v=0,\quad \theta=\theta_0,\; 0\le x\le L, 0\le y\le L\\
&t>0;\; u=v=0,\quad \frac{\partial\theta}{\partial x}=0,\quad x=0\\
&\qquad u=v=0,\quad \frac{\partial\theta}{\partial x}=0,\quad x=L\\
&\qquad u=v=0,\quad \theta=\theta_h,\quad y=0\\
&\qquad u=v=0,\quad \theta=\theta_c,\quad y=L
\end{aligned} \quad (5)$$

3. DISCRETIZATION OF THE GOVERNING EQUATIONS

At any grid point the term $\dfrac{\partial T}{\partial Y}$ in the energy equation, after nondimensionalising and the co-efficient velocities U & V are treated as constants over a time step. All space derivatives are given centered difference representations. The relevant finite-difference approximations to the energy equations, to be used consecutively over two half time steps, each of duration $\Delta\tau/2$ is,

$$\frac{\partial T}{\partial \tau}+U\frac{\partial T}{\partial X}+V\frac{\partial T}{\partial Y}=\frac{1}{pr}\nabla^2 T$$

For Y-direction

$$T_{i,j-1}{}^{*}\left[\frac{-V_{i,j}{}^{n}}{2\Delta y}-\frac{1}{pr(\Delta y)^2}\right]+T_{i,j}{}^{*}\left[\frac{1}{\Delta\tau/2}+\frac{2}{pr(\Delta y)^2}\right]+T_{i,j+1}{}^{*}\left[\frac{V_{i,j}{}^{n}}{2\Delta y}-\frac{1}{pr(\Delta y)^2}\right]=$$
$$T_{i-1,j}{}^{n}\left[\frac{U_{i,j}{}^{n}}{2\Delta x}+\frac{1}{pr(\Delta x)^2}\right]+T_{i,j}{}^{n}\left[\frac{1}{\Delta\tau/2}-\frac{2}{pr(\Delta x)^2}\right]+T_{i+1,j}{}^{n}\left[\frac{-U_{i,j}{}^{n}}{2\Delta x}+\frac{1}{pr(\Delta x)^2}\right]$$

The above equation reduces to the tridiagonal form $AT_{i,j-1}{}^{*}+BT_{i,j}{}^{*}+CT_{i,j+1}{}^{*}=D$

where,

$$A=\left[\frac{-V_{i,j}{}^{n}}{2\Delta y}-\frac{1}{pr(\Delta y)^2}\right]$$

$$B=\left[\frac{2}{\Delta\tau}+\frac{2}{pr(\Delta y)^2}\right]$$

$$C=\left[\frac{V_{i,j}{}^{n}}{2\Delta y}-\frac{1}{pr(\Delta y)^2}\right]$$

$$D=T_{i-1,j}{}^{n}\left[\frac{U_{i,j}{}^{n}}{2\Delta x}+\frac{1}{pr(\Delta x)^2}\right]+T_{i,j}{}^{n}\left[\frac{2}{\Delta\tau}-\frac{2}{pr(\Delta x)^2}\right]+T_{i+1,j}{}^{n}\left[\frac{-U_{i,j}{}^{n}}{2\Delta x}+\frac{1}{pr(\Delta x)^2}\right]$$

For X-direction ,

$$T_{i-1,j}{}^{n+1}\left[\frac{-U_{i,j}{}^{n}}{2\Delta x}-\frac{1}{pr(\Delta x)^2}\right]+T_{i,j}{}^{n+1}\left[\frac{1}{\Delta\tau/2}+\frac{2}{pr(\Delta x)^2}\right]+T_{i+1,j}{}^{n+1}\left[\frac{U_{i,j}{}^{n}}{2\Delta x}-\frac{1}{pr(\Delta x)^2}\right]=$$
$$T_{1,j-1}{}^{*}\left[\frac{V_{i,j}{}^{n}}{2\Delta y}+\frac{1}{pr(\Delta y)^2}\right]+T_{i,j}{}^{*}\left[\frac{1}{\Delta\tau/2}-\frac{2}{pr(\Delta y)^2}\right]+T_{i,j+1}{}^{*}\left[\frac{-V_{i,j}{}^{n}}{2\Delta y}+\frac{1}{pr(\Delta y)^2}\right]$$

The above equation reduces to the tridiagonal form:

$$AT_{i-1,j}{}^{n+1}+BT_{i,j}{}^{n+1}+CT_{i+1,j}{}^{n+1}=D$$

where,

$$A = \left[\frac{-U_{i,j}{}^{n}}{2\Delta x} - \frac{1}{pr(\Delta x)^2} \right]$$

$$B = \left[\frac{2}{\Delta \tau} + \frac{2}{pr(\Delta x)^2} \right]$$

$$C = \left[\frac{U_{i,j}{}^{n}}{2\Delta x} - \frac{1}{pr(\Delta x)^2} \right]$$

$$D = T_{1,j-1}{}^{*} \left[\frac{V_{i,j}{}^{n}}{2\Delta y} + \frac{1}{pr(\Delta y)^2} \right] + T_{i,j}{}^{*} \left[\frac{2}{\Delta \tau} - \frac{2}{pr(\Delta y)^2} \right] + T_{i,j+1}{}^{*} \left[\frac{-V_{i,j}{}^{n}}{2\Delta y} + \frac{1}{pr(\Delta y)^2} \right]$$

Similar approximations also hold for the vorticity equation which precedes the stream function across a time step.

4. METHOD OF SOLUTION

The governing equations- energy, vorticity and stream function are solved via a finite difference technique consisting of Alternating Direction Implicit (ADI) and Successive Over Relaxation (SOR) methods. The added advantage of using this unconditionally stable numerical scheme is that larger time increments may be used without loss of stability. The vorticity and temperature equations are parabolic, while the stream function equation is elliptic. The resulting stream function values are then used to determine the velocity components and the boundary values of the vorticity. Thus the sequence beginning with the solution of the energy equation is applied repeatedly until the desired results are obtained. The convergence criterion used for the field variables ϕ is

$$\frac{\phi_{n+1}(i,j) - \phi_n(i,j)}{\phi_{n+1}(i,j)} \leq 10^{-5}$$

In the above expression the subscript n refers to appropriate time level and ϕ represents $T, \varsigma \; and \psi$. The mesh 51 x 51 was opted as the ideal one with a suitable time increment. In this study a computational code, using ‘C’ language is developed to obtain the finite difference solution implementing Alternating Direct Implicit (ADI) and Successive Over Relaxation (SOR) methods. The contour plots for various quantities are plotted using MATLAB.

5. RESULTS AND DISCUSSION

The analysis is performed for five different Grashof numbers viz. (a) 20000 (b) 50000 (c) 100000 (d) 150000 and (e) 200000, maintaining Prandtl number equal to 0.7 for all the cases. In this study, the configuration consists of one vertical cold wall, one vertical hot wall and with two adiabatic top and bottom wall. Program iterations are performed for each case with time increment $\Delta\tau$ equal to 0.0002, until a steady state solution is arrived. Figures 2 to 6 show the streamline and isothermal contours and also the average Nusselt number over the hot wall for the range of Grashof numbers considered. Observation of stream line pattern in Figure 2, for Grashof number equal to 20000, show that the stream line spacing is low close to the top boundary and are high in the region close to the bottom boundary. This indicates higher velocity regions close to the top boundary and lower velocity region over the bottom boundary. The streamline pattern is almost symmetrical about the centroidal X-axis (Y=0.5). The temperature contour line shows the uniform temperature variation within the domain i.e. T = 1.0 at the left wall and gradually changes to T = -1.0 at the right boundary. No significant temperature variation is observed along the X direction at any location. During the iteration the average Nusselt number is initially high and it decreases continuously, when the steady state solution is obtained and beyond which it remains constant. Figure 3 shows the streamlines, isothermals and average Nusselt number for Pr = 0.7, Gr =50000. The streamlines and isotherms were found to be of minimal variation to the above case. In Figure 4 for Pr = 0.7, Gr = 100000, the isotherms bends slightly towards the hot wall. In Figures 5 and 6, for Pr = 0.7, Gr = 150000 and 200000 respectively, the streamlines are crowded more near the hot wall and the isotherms are found to bend slightly towards the hot wall. The average Nusselt number over the hot wall estimated for the range of Grashof number is analysed and they are given in Table 1.

Table 1.   Average Nusselt numbers for different Grashof numbers

Prandtl number	Grashof number	Average Nusselt number
0.7	20000	1.461311
0.7	50000	1.590601
0.7	100000	1.894743
0.7	150000	2.222238
0.7	200000	2.540704

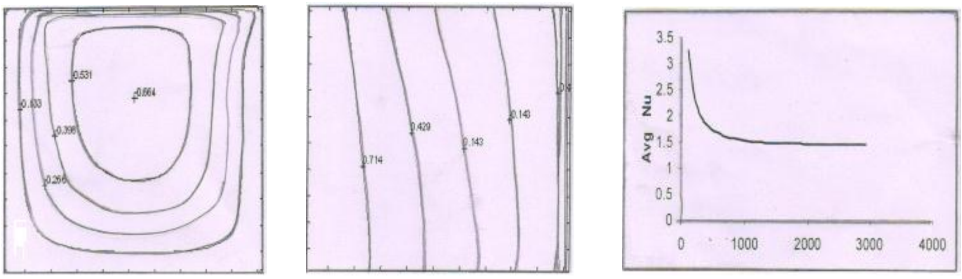

Figure 2. Streamlines, Isothermal lines and Average Nusselt Number for Gr = 20000

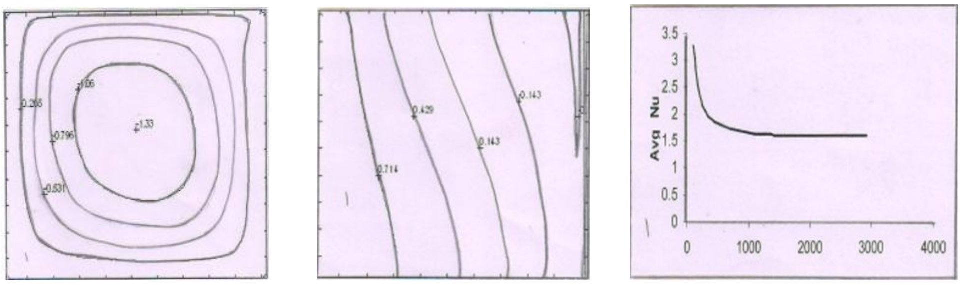

Figure 3. Streamlines, Isothermal lines and Average Nusselt Number for Gr = 50000

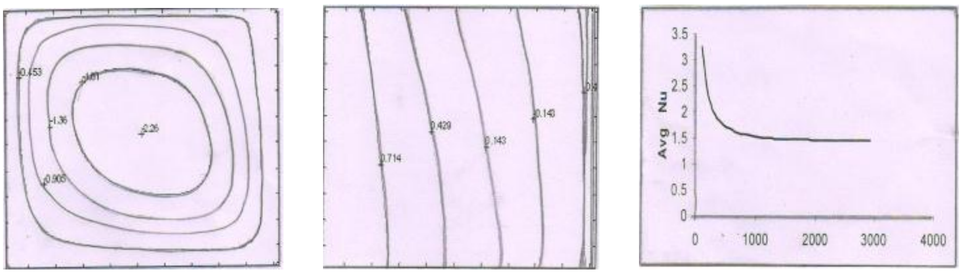

Figure 4. Streamlines, Isothermal lines and Average Nusselt Number for Gr = 100000

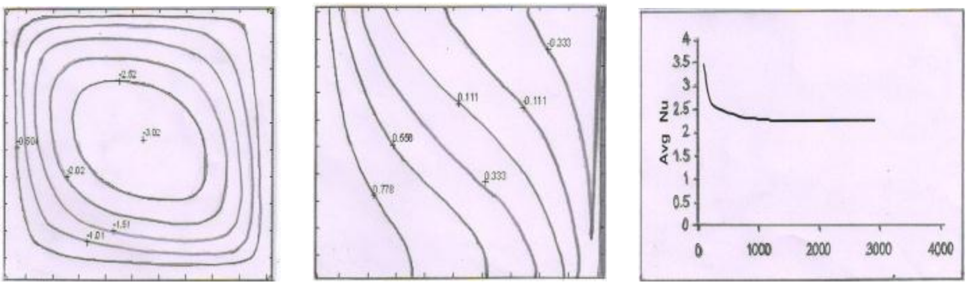

Figure 5. Streamlines, Isothermal lines and Average Nusselt Number for Gr = 150000

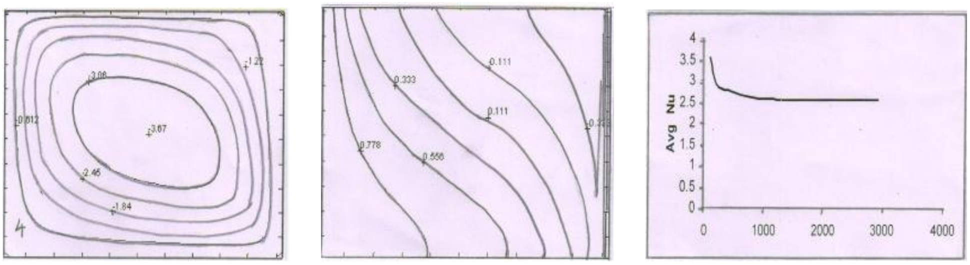

Figure 6. Streamlines, Isothermal lines and Average Nusselt Number for Gr = 200000

6. CONCLUSION

A finite difference technique has been developed using 'C' language to predict the natural convection heat transfer in an enclosed cavity consisting of a fluid, with different wall boundary temperatures. The numerical results are obtained for the range of Grashof number 20000 to 200000 and Pr=0.7. The influences of wall boundary temperature, Prandtl and Grashof number of fluid within the cavity on flow pattern and temperature distribution are analyzed in detail. It is observed that the increase in Grashof number changes the streamline pattern and temperature contours. The fluid velocities are increased in the regions where the streamline spacing is low. Increase in Grashof number, considerably gives an increase in average Nusselt number which in turn increases the heat transfer rate. The results obtained are presented in the form of contour plots of streamlines and isotherms drawn using MATLAB. The scope of the work involves the design of devices with enclosures subject to high temperature differences and the devices which include partition walls

Nomenclature

g = acceleration due to gravity
β = volume coefficient of thermal expansion
α = thermal conductivity
ρ = density
υ = kinematic viscosity
θ_h = hot wall temperature
θ_c = cold wall temperature
θ_0 = initial temperature
θ = temperature of fluid at any point
T= non-dimensional form of temperature
u,v = components of velocity along x and y directions respectively
U,V = non-dimensional forms of velocity components along X and Y directions respectively
$\Delta X, \Delta Y$ = grid spacing in the X and Y directions respectively
$\Delta\tau$ = time increment
ζ = dimensionless vorticity
ψ = dimensionless stream function
ω = relaxation parameter
Pr = Prandtl number
Gr = Grashof number
Nu = Nusselt number
L = ratio of cavity height to its width

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