

# Numerical and Analytical Study on Curved Beams

Shiv Pratap Singh Yadav, Sandeep G M, Avinash L  
Assistant Professors  
Nitte Meenakshi Institute of Technology  
Bengaluru, India

Kishan Reddy R, Akshay Bhat, Aaron George Biju,  
Akash Doijode, Jayanth J.  
UG Students  
Nitte Meenakshi Institute of Technology  
Bengaluru, India

Dr. Sudheer Reddy  
Professor  
Nitte Meenakshi Institute of Technology  
Bengaluru, India

**Abstract**—A Curved Beam is an element in which the neutral axis in the unloaded condition is curved as an alternative of straight. Curved beams eliminate the use of vertical columns in buildings. The horizontally Curved beam has a variety of uses such as in the design of buildings for good aesthetic looks, in the mechanical elements like crane hooks to lift huge loads. For the required load to withstand, it is tedious to design the beam for stresses at inner and outer layers manually. The parameters which are involved in the curved beams were analyzed. C Program for these parameters is designed and these results are compared with the outputs of FEA based software. Curved beam is modelled in the design software for the required dimensions.

**Keywords**—Curved Beams, Ring, Link.

## I. INTRODUCTION

Curved beams finds lot of applications in the fields of automotive and structural mechanics as the use of them reduces many linkages which are attached to the structures, even capable of withstanding diametral loads as in chain links, crane hooks etc. It is necessary to obtain quick solutions to the problems as they are of different cross sections like circular, rectangular and even trapezoidal, analysis of which is most necessary to know their failure conditions and the maximum stress conditions.

In this project we chose circular cross section as it is the most used, we designed chain link and circular rings using CATIA V5 which accepts the dimensions, these dimensions, where also given as inputs to C program coded in CODE BLOCKS (16.01) and the results obtained were checked theoretically and also with the outputs of ANSYS 14.5.

ANSYS being FEA approach approximate results were obtained but the regions of stress conditions was identified clearly. The outputs of C code were exact and quick. The stress conditions for different materials can be determined easily instead of solving theoretically which is tiring. The CATIA model displayed the exact model lookalike.

## II. METHODOLOGY

### A. Theoretical method

#### Steps and equations

Step 1: Consider the force acting on the link, length of the link, centroidal diameter and diameter of the cross section.

Step 2: Obtain inner radius, centroidal radius and outer radius.

$$\bullet \text{ Inner radius } R_i = \frac{(R-d)}{2} \quad (1)$$

$$\bullet \text{ Outer radius } R_o = \frac{(R+d)}{2} \quad (2)$$

Step 3: Obtain values of  $R_n$ ,  $e$ ,  $C_i$ ,  $C_o$ .

$$\bullet \text{ Radius of Neutral fiber } R_N = \frac{(\sqrt{R_o} + \sqrt{R_i})^2}{4} \quad (3)$$

$$\bullet \text{ Eccentricity } e = R_c - R_N \quad (4)$$

$$\bullet \text{ Distance of N.A. to inner fiber } C_i = R_N - R_i \quad (5)$$

$$\bullet \text{ Distance of N.A. to outer fiber } C_o = R_o - R_N \quad (6)$$

$$\bullet \text{ Area of cross section } A = \frac{\pi d^2}{4} \quad (7)$$

Step 4: calculate the axial and shear stresses across the cross section.

$$\bullet \text{ Bending moment at section A } M_{bA} = \frac{F \times R(2R+l)}{2(\pi R+l)} \quad (8)$$

$$\bullet \text{ Inner fiber stress } \sigma_{biA} = \frac{M_{bA} \times C_i}{A e R_i} \quad (9)$$

$$\bullet \text{ Outer fiber stress } \sigma_{boA} = \frac{M_{bA} \times C_o}{A e R_o} \quad (10)$$

$$\bullet \text{ Bending moment at section B } M_{bB} = \frac{F \times R(2R - \pi R)}{2(\pi R+l)} \quad (11)$$

- Inner fiber stress  $\sigma_{biB} = \frac{F}{2A} + \frac{M_{bB} \times C_i}{As R_i}$
- Outer fiber stress  $\sigma_{boB} = \frac{F}{2A} - \frac{M_{bB} \times C_o}{As R_o}$
- Maximum shear stress  $\tau = 0.5 \times \sigma_{max}$

**B. C algorithm**

Step 1: Enter the type of curved beam, i.e S-Link, or Chain Link, or Ring.

Step 2: Based on the type of Curved Beam, Run respective function.

Step 3: Create a Function named Circular(). In circular(), Enter the diameter of circular cross section, and save it to variable d. Calculate  $x=d/2$ . Select the type of radius being entered, i.e. inner radius, outer radius, or centroidal radius. Based on the value entered, calculate the other radii, the neutral axis radius, the distance of inner fiber from neutral axis, the distance of the outer fiber from neutral axis.

Step 4: Create functions slink(), chain-link(), ring(). In each of these functions, the moment is calculated by multiplying force and centroidal axis radius. The stresses at inner and outer fiber, at both cross sections, are calculated using formulae. The stresses are then converted to positive values, and the maximum value is then saved. The maximum shear stress is the half of the stress obtained by calculations

**C. ANSYS Method**

- Create a new file under the structural menu.
- Import the .CAT file into the ANSYS.
- Create element type of solid tetrahedron 4 node 285.
- Assume model material to be mild steel ( $E=2.1e11$ ) and poissons ratio as 0.3.
- Free mesh the link/ring in YZ plane and refine mesh to the accuracy of 3.
- Constrain the bottom part of the link/ring in all degrees of freedom and apply the given load to the upper part of the link/ring in ‘-Z’ direction.
- Solve the problem.
- Obtain the contour plot of “VONMISES stresses” for deformed and un-deformed shape.

**III. RESULTS**

**A. C output**

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ENTER THE MAGNITUDE OF FORCE ACTING ON THE RING IN NEWTON :      12000
ENTER THE DIAMETER OF THE CIRCULAR CROSS SECTION IN mm :      30
WHICH RADIUS IS TO BE ENTERED
1. INNER RADIUS RI
2. OUTER RADIUS RO
3. CENTROIDAL AXIS RADIUS RC                                     2
ENTER THE RADIUS in mm:                                       90
THE INNER RADIUS IS, RI :60.000000
THE OUTER RADIUS IS, RO :90.000000
THE RADIUS OF CENTROIDAL AXIS IS, RC :75.000000
THE RADIUS OF NEUTRAL AXIS IS, RN :74.242348
THE ECCENTRICITY IS, e :0.757652
DISTANCE OF INNER FIBER FROM NEUTRAL AXIS, CI : 14.242348
DISTANCE OF OUTER FIBER FROM NEUTRAL AXIS, CO : 15.757652
direct stress is is 8.492569
THE STRESSES ARE
-127.104555 N/mm^2,
93.751837 N/mm^2,
-63.957024 N/mm^2,
61.931113 N/mm^2
THE MAXIMUM SHEAR STRESS THAT THE RING CAN WITHSTAND IS :
63.552278 N/mm^2
    
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Fig. 1: Result obtained when F=12000N, d=30mm, Ro=90mm

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ENTER THE MAGNITUDE OF FORCE ACTING ON THE CHAIN LINK IN NEWTON : 9000
ENTER LENGTH OF CHAIN LINK :      60
ENTER THE DIAMETER OF THE CIRCULAR CROSS SECTION IN mm :      30
WHICH RADIUS IS TO BE ENTERED
1. INNER RADIUS RI
2. OUTER RADIUS RO
3. CENTROIDAL AXIS RADIUS RC                                     3
ENTER THE RADIUS in mm:                                       45
THE INNER RADIUS IS, RI :30.000000
THE OUTER RADIUS IS, RO :60.000000
THE RADIUS OF CENTROIDAL AXIS IS, RC :45.000000
THE RADIUS OF NEUTRAL AXIS IS, RN :43.713203
THE ECCENTRICITY IS, e :1.286797
DISTANCE OF INNER FIBER FROM NEUTRAL AXIS, CI : 13.713203
DISTANCE OF OUTER FIBER FROM NEUTRAL AXIS, CO : 16.286797
THE STRESSES ARE
-75.869656 N/mm^2,
45.054157 N/mm^2,
-19.577996 N/mm^2,
21.777949 N/mm^2
THE MAXIMUM SHEAR STRESS THAT THE CHAIN LINK CAN WITHSTAND IS :
37.934828 N/mm^2
    
```

Fig. 2: Result obtained when F=9000N, d=30mm, Rc=45mm

**B. Theoretical method**

Chain Link is made up of 30mm diameter steel rod has a mean diameter of 90mm. the straight sides are each 60mm long, if the link carries a load of 9KN. Determine the Maximum fiber stress induced in the link.[1]

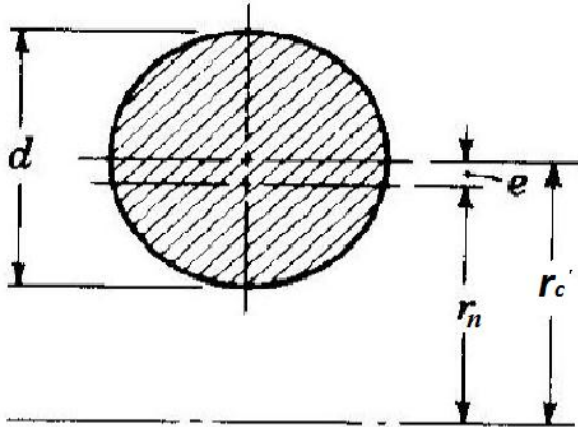


Fig. 3: cross sectional area

- d = 30mm (cross section diameter)
- D<sub>c</sub> = 90mm, R<sub>c</sub> = 45mm (radius to centroidal axis)
- l = length of the straight sides=60mm
- F = 9000N

$$R_i = \frac{R-d}{2} = \frac{45-30}{2} = 30\text{mm}$$

$$R_o = \frac{R+d}{2} = 60\text{mm}$$

$$\text{Step 1: } R_N = \frac{(\sqrt{R_o} + \sqrt{R_i})^2}{4}$$

$$R_N = \frac{(\sqrt{60} + \sqrt{30})^2}{4}$$

$$R_N = 43.713203 \text{ mm}$$

$$\text{Step 2: } e = R_c - R_N = 45 - 43.713203 = 1.286797 \text{ mm}$$

$$C_i = R_N - R_i = 43.713203 - 30 = 13.713203 \text{ mm}$$

$$C_o = R_o - R_N = 60 - 43.713203 = 16.286797 \text{ mm}$$

$$A = \frac{\pi d^2}{4} = \frac{\pi 30^2}{4} = 706.5 \text{ mm}^2$$

$$\text{Step 3: } M_{bA} = \frac{F \times R(2R+l)}{2(\pi R+l)}$$

$$M_{bA} = \frac{9000 \times 45(2 \times 45 + 60)}{2(\pi \times 45 + 60)}$$

$$M_{bA} = 150894.1878 \text{ Nmm}$$

(1) Bending stress at inner fiber

$$\sigma_{biA} = \frac{M_{bA} \times C_i}{A e R_i} = \frac{150894.1878 \times 13.713203}{706.5 \times 1.286797 \times 30}$$

$$\sigma_{biA} = -75.869628 \text{ N/mm}^2 \text{ (compression)}$$

(2) Bending stress at outer fiber

$$\sigma_{bo(A)} = \frac{M_{bA} \times C_o}{A e R_o}$$

$$\sigma_{boA} = 45.054432 \text{ N/mm}^2 \text{ (tensile)}$$

$$M_{Bb} = \frac{F \times R(2R - \pi R)}{2(\pi R + l)} = \frac{9000 \times 45(2 \times 45 - \pi \times 45)}{2(\pi \times 45 + 60)}$$

$$M_{Bb} = -51605.81222 \text{ N-mm}$$

(i) Combined stress at inner fiber

$$\sigma_{biB} = \frac{F}{2A} + \frac{M_{bB} \times C_i}{A e R_i} = \frac{9000}{2 \times 706.5} + \frac{-51605.81222 \times 13.713203}{706.5 \times 1.286797 \times 30}$$

$$\sigma_{biB} = -19.580378 \text{ N/mm}^2 \text{ (compression)}$$

(ii) Combined stress at outer fiber

$$\sigma_{boB} = \frac{F}{2A} - \frac{M_{bB} \times C_o}{A e R_o} = \frac{9000}{2 \times 706.5} - \frac{-51605.81222 \times 16.286797}{706.5 \times 1.286797 \times 60}$$

$$\sigma_{boB} = 21.7970 \text{ N/mm}^2 \text{ (tensile)}$$

$$\tau_{max} = 0.5 \times 75.86$$

$$\tau_{max} = 37.93 \text{ N/mm}^2$$

**C. ANSYS Output**

Results obtained from ANSYS for Ring and Chain Link

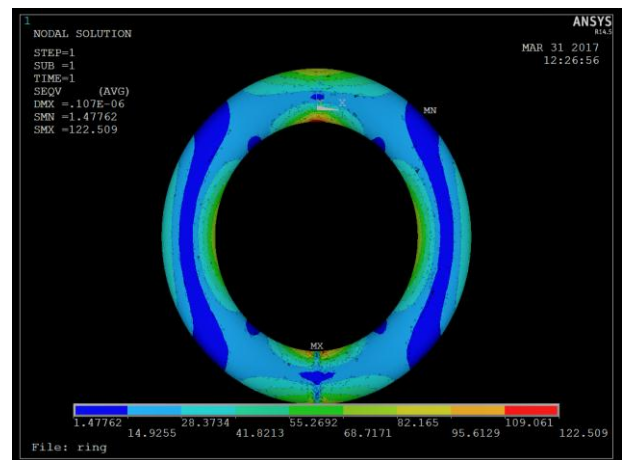


Fig.4: Ring Analysis

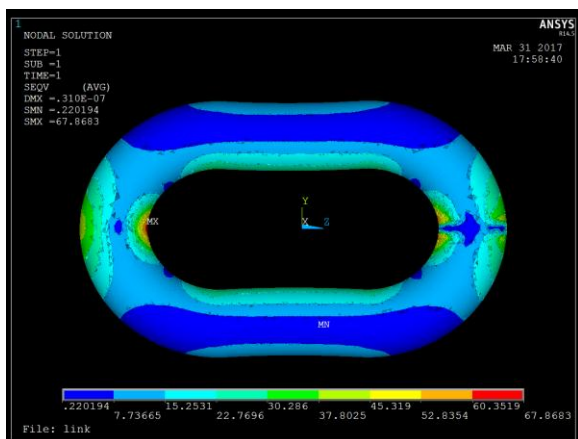


Fig.5: Chain link Analysis

#### D. Comparison of Results

TABLE I. COMPARISON OF RESULTS

LINK			
Method	Analytical Method	C	FEA
Parameters			
Max. stress (MPa)	75.869628	75.869656	67.8683
Min. stress (MPa)	19.580378	19.577996	23.007
RING			
Max. stress (MPa)	126.851493	127.10455	122.509
Min. stress (MPa)	64.112296	63.931113	62.825

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