Nonlinear Vibration Responses Analysis of Suspended-cable Structure based on Three Degrees of Freedom Model

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***Abstract*—According to its symmetry and mechanical characteristics of suspended-cable structure, the continuous suspended-cable structure is simplified to three degrees of freedom model. Meanwhile, considering that the geometric nonlinear of structure under loading, the expressions of kinetic energy, potential energy and the work done by damping forces and conservative forces of the system are obtained through the geometric relationships of the simplified model and parameter hypothesis. With the selected generalized coordinates, the dynamic and static equilibrium equations can be derived based on the Lagrange formulation. Using the state space formulation method and Runge-Kutta algorithm, the numerical solution of the generalized coordinates under the static and dynamic equilibrium can be calculated by the programming with MATLAB, and then the pictures for the static equilibrium path, the dynamic time-history curve and the phase plan can be plotted. The results show that when the suspended-cable structure vibrates, the vibration amplitude decreases with the increase of time, and the vibration of the suspended-cable structure is always in a stable equilibrium state.**

Keywords—suspended-cable structure; three degrees of freedom model ; nonlinear vibration ; Lagrange formulation ; numerical solution

I. INTRODUCTION

Suspension and suspended-cable structure in modern engineering construction are of highly important significance, because of the advantage of reasonable stress characteristic, economic use of materials, convenient construction, simple facilities, strong adaptability and attractive appearance, etc[1], they are widely used in large span buildings, bridges and other construction of facilities. However, under the service load, the suspended-cable structure will produce large displacement, so the geometric nonlinearity must be considered in the calculation[2]. At present, the researches on the dynamic response of single span suspension structure are most based on the continuous method and finite element method. In the aspect of simplified model research, nonlinear stability of the simplified model of the double suspension roofs have been studied by Sophianopoulos D. S.[3] based on the energy method. The continuous suspended-cable structure is simplified to three degrees of freedom model is this paper, considering that the geometric nonlinear of structure under loading, the dynamic and static equilibrium equations can be derived based on the Lagrange formulation[4]. Static characteristics and nonlinear vibration response of the symmetric vibration of suspended-cable structure based on three degrees of freedom model are analyzed.

II. SESIMPLIFIED MODEL OF THREE DEGREES OF FREEDOM

A. *Simplified model*

First, single-span suspended-cable structure with continuous distributed mass can be translated to three concentrated mass particles by using the lumped mass method[5], as shown in Fig.1, thus the infinite degrees of freedom problem is simplified to limited degree of freedom. Due to its symmetry, take half-span structure shown in Fig.2 as the research object. Further, three degrees of freedom of springs and masses model is shown in Fig.3. This model contains two linear springs k_i with corresponding damping c_i (i=1,2) and two concentrated mass m_1 and m_2, which interconnected by spring. Support A is an immovable hinge and support B is a movable hinge. The system will reach a new equilibrium position under loads P_1 and P_2. The values of the design parameters are shown in Table 1 of Section 4. Assuming that the parameters are scaled such as $l_{1,0}=k_1=m_1=1$, that is these quantities are expressed by using appropriate units rather than a standard MKS system of units.

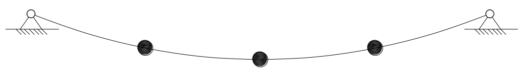


Fig. 1. Particle model for single-span suspended-cable structure

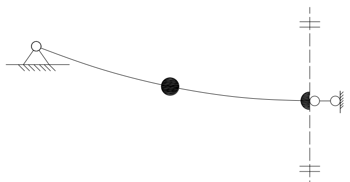


Fig. 2. Particle model for half-span suspended-cable structure

B. *Geometric relations*

In the ΔABH and $\triangle$AEG in Fig.3., according to trigonometric relations, we will get

$$h_1 = \sin \alpha_{1,0} \tag{1a}$$

$$x_1 = \cos \alpha_{1,0} \tag{1b}$$

$$l_1 = \sqrt{\left(x_1 - v_1\right)^2 + \left(h_1 + w_1\right)^2} \tag{1c}$$

In the same way, in △BCJ and △EFK

$$h_2 - h_1 = l_{2,0} \sin \alpha_{2,0} \tag{2a}$$

$$x_2 = l_{2,0} \cos \alpha_{2,0} \tag{2b}$$

$$l_2 = \sqrt{(x_2 + v_1)^2 + (h_2 - h_1 + w_2 - w_1)^2} \tag{2c}$$

The initial length of the springs AB and BC are respectively

$$l_{1,0} = \sqrt{x_1^2 + h_1^2} \tag{3}$$

$$l_{2,0} = \sqrt{x_2^2 + (h_2 - h_1)^2} \tag{4}$$

And the time derivatives of l_1 and l_2 are

$$\dot{l}_1 = \frac{(v_1 - x_1)\dot{v}_1 + (h_1 + w_1)\dot{w}_1}{l_1} \tag{5}$$

$$\dot{l}_2 = \frac{(x_2 + v_1)\dot{v}_1 + (h_2 - h_1 + w_2 - w_1)(\dot{w}_2 - \dot{w}_1)}{l_2} \tag{6}$$

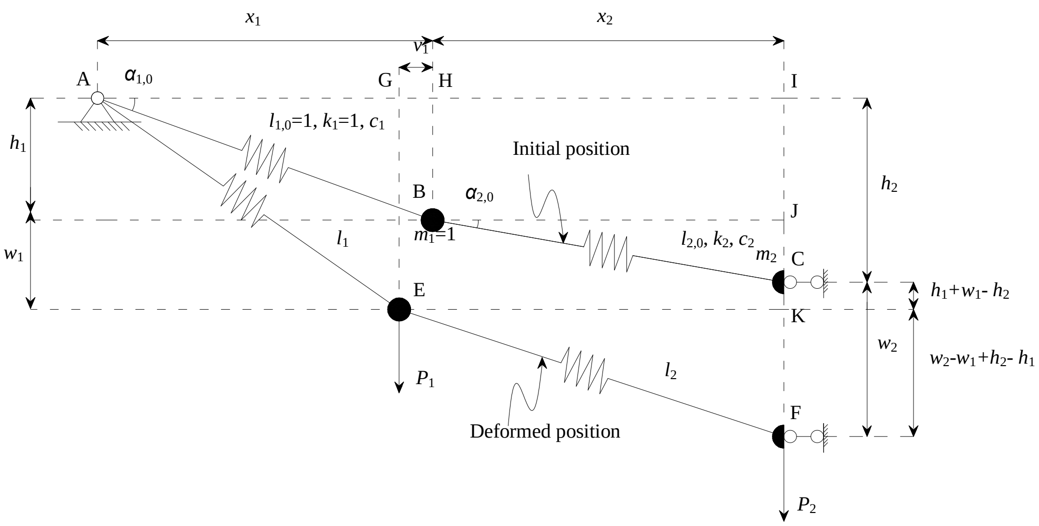


Fig. 3. Three degrees of freedom simplified model of suspended-cable structure

III. LAGRANGE FORMULATION

A. *Kinetic and potential energies, Rayleigh dissipation function and work of conservative forces*

Setting $\lambda=P_1$, $\gamma=P_2/P_1$, $\xi=m_2$[3], the kinetic energy is given by

$$K = \frac{1}{2}\left(\dot{v}_1^2 + \dot{w}_1^2\right) + \frac{1}{2}\xi \dot{w}_2^2 \tag{7}$$

Assuming that the springs are linear and unstressed in initial position. The potential energy of system can be written as

$$U = \frac{1}{2}\delta_1^2 + \frac{1}{2}k_2\delta_2^2 \tag{8}$$

Where $\delta_1 = l_1 - l_{1,0}$, $\delta_2 = l_2 - l_{2,0}$

Assuming conservative forces are vertically applied on the masses m_1 and m_2, the corresponding work W calculating from the initial position becomes

$$\begin{aligned} W &= -P_1 w_1 - P_2 w_2 \\ &= -\lambda w_1 - \gamma\lambda w_2 \end{aligned} \tag{9}$$

Due to friction in the springs, the linear dissipation has occurred, and the corresponding Rayleigh function is given by

$$D = \frac{1}{2}c_1\dot{\delta}_1^2 + \frac{1}{2}c_2\dot{\delta}_2^2 \tag{10}$$

B. *Lagrange formulation*

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{z}}\right) - \frac{\partial L}{\partial z} = -\frac{\partial D}{\partial \dot{z}} \tag{11}$$

Where $L=K-U-W$, z is a generalized coordinate of the system, take $z=v_1$, $z=w_1$, $z=w_2$ respectively. In this problem U and W only change with z, thus the Eq.(11) can be simplified to

$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{z}}\right) - \frac{\partial K}{\partial z} + \frac{\partial U}{\partial z} + \frac{\partial D}{\partial \dot{z}} + \frac{\partial W}{\partial z} = 0 \tag{12}$$

1) The generalized coordinates $z=v_1$

$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{v}_1}\right) - \frac{\partial K}{\partial v_1} + \frac{\partial U}{\partial v_1} + \frac{\partial D}{\partial \dot{v}_1} + \frac{\partial W}{\partial v_1} = 0 \tag{13a}$$

$$\ddot{v}_1+\left(\delta_1+c_1\dot{\delta}_1\right)\frac{\left(v_1-x_1\right)}{l_1}+\left(k_2\delta_2+c_2\dot{\delta}_2\right)\frac{\left(x_2+v_1\right)}{l_2}=0 \tag{13b}$$

2) The generalized coordinates z=w_1

$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{w}_1}\right)-\frac{\partial K}{\partial w_1}+\frac{\partial U}{\partial w_1}+\frac{\partial D}{\partial \dot{w}_1}+\frac{\partial W}{\partial w_1}=0 \tag{14a}$$

$$\ddot{w}_1+\left(\delta_1+c_1\dot{\delta}_1\right)\frac{\left(h_1+w_1\right)}{l_1}+ \left(k_2\delta_2+c_2\dot{\delta}_2\right)\frac{\left(h_1-h_2+w_1-w_2\right)}{l_2}-\lambda=0 \tag{14b}$$

3) The generalized coordinates z=w_2

$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{w}_2}\right)-\frac{\partial K}{\partial w_2}+\frac{\partial U}{\partial w_2}+\frac{\partial D}{\partial \dot{w}_2}+\frac{\partial W}{\partial w_2}=0 \tag{15a}$$

$$\xi\ddot{w}_2+\left(k_2\delta_2+c_2\dot{\delta}_2\right)\frac{\left(h_2-h_1+w_2-w_1\right)}{l_2}-\lambda\gamma=0 \tag{15b}$$

The resulting equations of motion can be obtained, namely

$$\begin{cases}\ddot{v}_1+\left(\delta_1+c_1\dot{\delta}_1\right)\dfrac{\left(v_1-x_1\right)}{l_1}+ \left(k_2\delta_2+c_2\dot{\delta}_2\right)\dfrac{\left(x_2+v_1\right)}{l_2}=0\\ \ddot{w}_1+\left(\delta_1+c_1\dot{\delta}_1\right)\dfrac{\left(h_1+w_1\right)}{l_1}+ \left(k_2\delta_2+c_2\dot{\delta}_2\right)\dfrac{\left(h_1-h_2+w_1-w_2\right)}{l_2}-\lambda=0\\ \xi\ddot{w}_2+\left(k_2\delta_2+c_2\dot{\delta}_2\right)\dfrac{\left(h_2-h_1+w_2-w_1\right)}{l_2}-\lambda\gamma=0\end{cases} \tag{16}$$

Rewritten in a matrix form as

$$\left[M_L\right]\left[\ddot{z}_L\right]=\left[F_L\left(z_L,\dot{z}_L\right)\right] \tag{17}$$

Where $\left[M_L\right]=\begin{bmatrix}1 & & \\ & 1 & \\ & & \xi\end{bmatrix}$, $\left[\ddot{z}_L\right]=\begin{bmatrix}\ddot{v}_1\\ \ddot{w}_1\\ \ddot{w}_2\end{bmatrix}$

$$\left[F_L\left(z_L,\dot{z}_L\right)\right]=$$

$$\lambda\begin{bmatrix}0\\1\\\gamma\end{bmatrix}-\begin{bmatrix}\dfrac{\left(v_1-x_1\right)}{l_1} & \dfrac{\left(x_2+v_1\right)}{l_2} & 0\\ \dfrac{\left(h_1+w_1\right)}{l_1} & \dfrac{\left(h_1-h_2+w_1-w_2\right)}{l_2} & 0\\ 0 & \dfrac{\left(h_2-h_1+w_2-w_1\right)}{l_2} & 0\end{bmatrix}\begin{bmatrix}\left(\delta_1+c_1\dot{\delta}_1\right)\\ \left(k_2\delta_2+c_2\dot{\delta}_2\right)\\ 0\end{bmatrix}$$

For any given value of λ, by setting $\dot{z}_L=\ddot{z}_L=0$ in the Eq.(17), we can obtain

$$\left[F_L\left(z_L,0\right)\right]=0 \tag{18}$$

IV. NUMERICAL RESULTS

A. Calculation Parameters

Selecting the shape of suspended-cable as a quadratic parabola, namely

$$y=4f\left[\left(\frac{x}{l}\right)-\left(\frac{x}{l}\right)^2\right] \tag{19}$$

Where f is the span sag, l is the calculation span.

According to the common rise-span ratio of suspended-cable structure, $f/l = 1/10,\ 1/9,\ 1/8$ are used respectively, when the numerical results are calculated. Concentrated massed m_1, m_2 are both located in a quadratic parabola as Eq.(19), and their horizontal distance from left end bearing are $l/4$ and $l/2$ respectively, that is x_1=x_2 in Fig.3. Parameters are obtained by simple calculations which are shown in Table 1.

TABLE I. PARAMETER TABLE

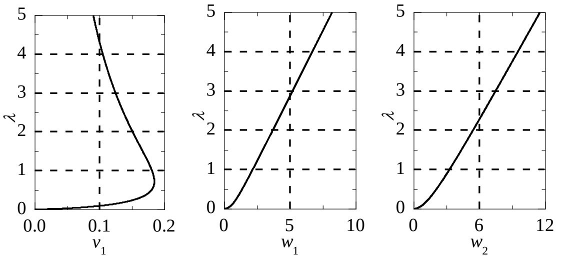
Parameter	**Case**		
	1	2	3
rise-span ratio f/l	1/10	1/9	1/8
$\alpha_{1,r}$(°)	16.70	18.43	20.56
$\alpha_{2,r}$(°)	5.71	6.34	7.13
$l_{2,0}$	0.9626	0.9545	0.9436
λ	1	1	1
γ	0.5	0.5	0.5
ξ	0.5	0.5	0.5
k_2	1	1	1
c_1	0.05	0.05	0.05
c_2	0.05	0.05	0.05

B. Numerical results of static equilibrium

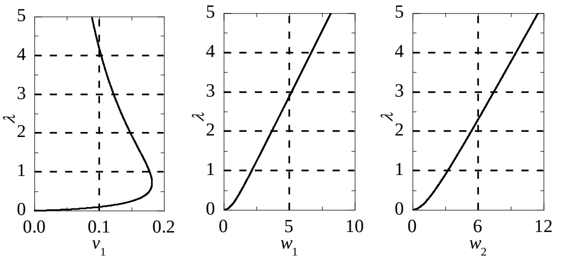
The static equilibrium position can be obtained by solving Eq.(19), that is

$$\lambda\begin{bmatrix}0\\1\\\gamma\end{bmatrix}=\begin{bmatrix}\dfrac{\left(v_1-x_1\right)}{l_1} & \dfrac{\left(x_1+v_2\right)}{l_2} & 0\\ \dfrac{\left(h_1+w_1\right)}{l_1} & \dfrac{\left(h_1-h_2+w_1-w_2\right)}{l_2} & 0\\ 0 & \dfrac{\left(h_2-h_1+w_2-w_1\right)}{l_2} & 0\end{bmatrix}\begin{bmatrix}\delta_1\\ k_2\delta_2\\ 0\end{bmatrix} \tag{20}$$

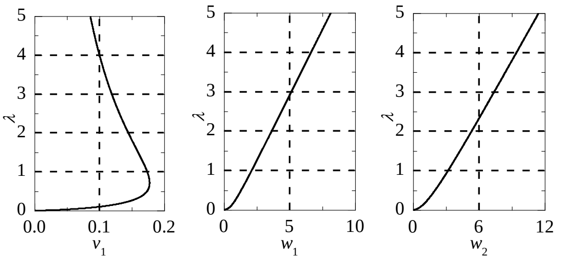
For nonzero λ values, the result of equation v_1, w_1, w_2 can be solved depending on a continuation procedure based on the algorithm described in reference [6] and using MATLAB[7] soft to program. The initial value of λ is 0, other parameter values are shown in Table 1. With the change of λ, the curve of equilibrium for three cases can be obtained as shown in Fig.4.



(a) *Rise-span ratio is 1/10*



(b) *Rise-span ratio is 1/9*



(c) *Rise-span ratio is 1/8*

Fig. 4. Equilibrium paths of model rise-span ratio with different rise-span ratio

C. *Numerical results of dynamic equilibrium*

Dynamic equilibrium equations have been obtained in Section Ⅲ, namely Eq.(16) or (17), this is a second order nonlinear ordinary differential equations. Using the method of state space formulation to select state variables, the first order differential equations can be obtained. If set

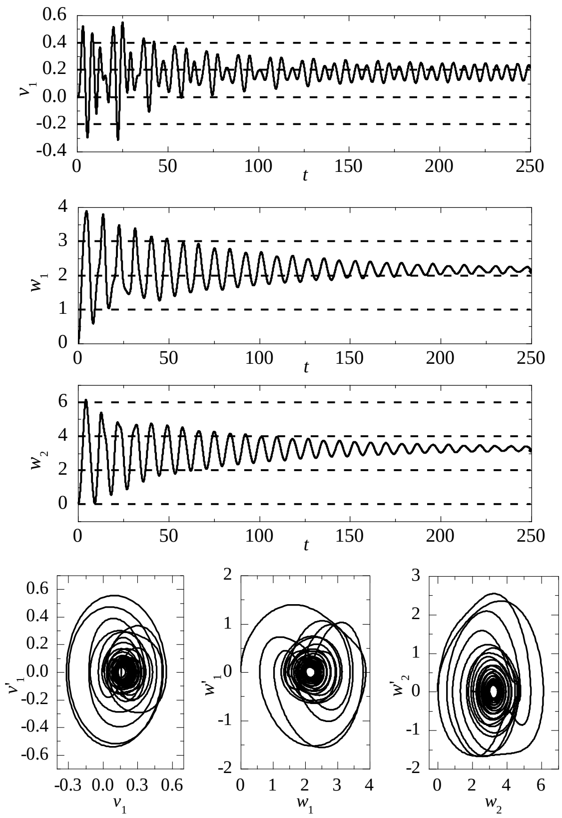
$$\begin{cases} y_1 = v_1 \ ; \ y_4 = \dot{v}_1 \\ y_3 = w_1 \ ; \ y_5 = \dot{w}_1 \\ y_4 = w_2 \ ; \ y_6 = \dot{w}_2 \end{cases} \tag{21}$$

Then Eq.(16) can be rewritten as

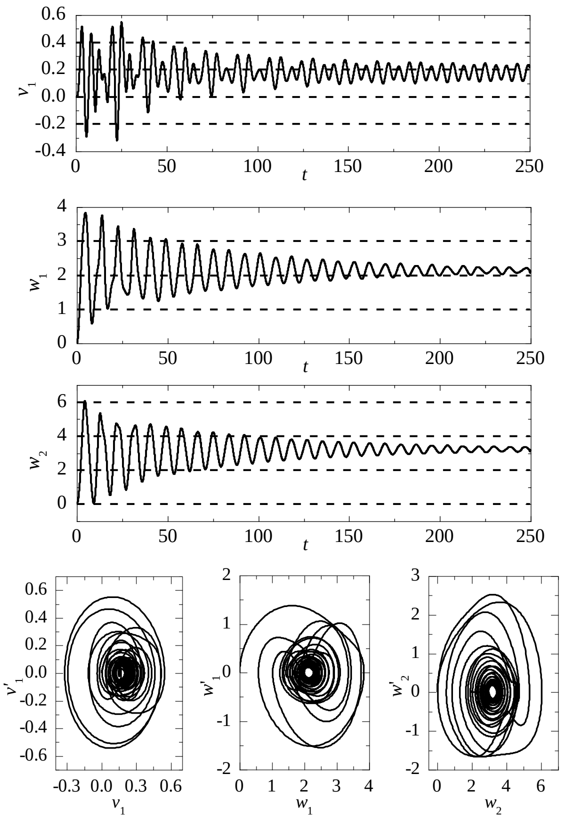
$$\begin{cases} y_1' = \dot{v}_1 \\ y_2' = \dot{w}_1 \\ y_3' = \dot{w}_2 \\ y_4' = \ddot{v}_1 \\ \quad = -\left(\delta_1 + c_1 \dot{\delta}_1\right)\dfrac{(v_1 - x_1)}{l_1} - \left(k_2\delta_2 + c_2 \dot{\delta}_2\right)\dfrac{(x_2 + v_1)}{l_2} \\ y_5' = \ddot{w}_1 \\ \quad = -\left(\delta_1 + c_1 \dot{\delta}_1\right)\dfrac{(h_1 + w_1)}{l_1} - \\ \quad \left(k_2\delta_2 + c_2 \dot{\delta}_2\right)\dfrac{(h_1 - h_2 + w_1 - w_2)}{l_2} + \lambda \\ y_6' = \ddot{w}_2 \\ \quad = \left(-\left(k_2\delta_2 + c_2 \dot{\delta}_2\right)\dfrac{(h_2 - h_1 + w_2 - w_1)}{l_2} + \lambda\gamma\right)\Big/\xi \end{cases}$$

(22)

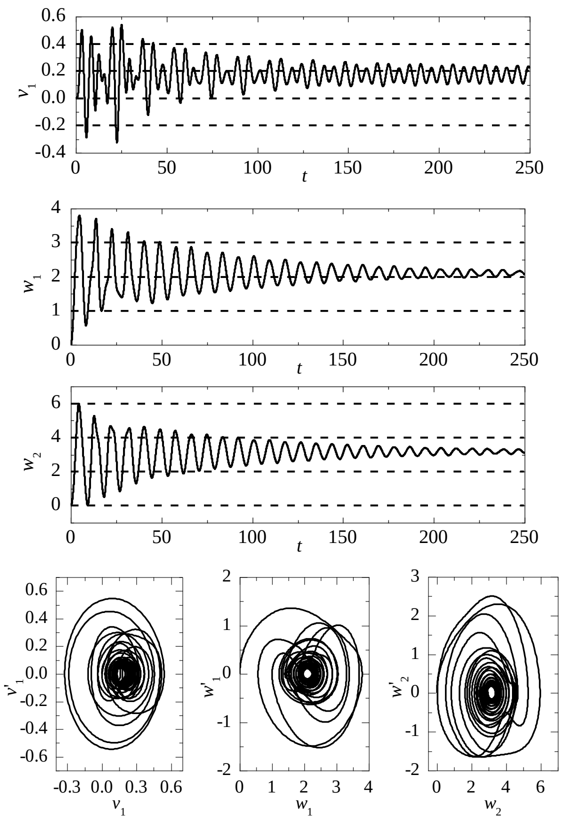
According to Runge-Kutta algorithm[8], the solver ode45[9] of ordinary differential equations in MATLAB software is used to program and solve these equations. Specific parameters refer to Table 1. Fig.5 shows the time-history curve of v_1, w_1, w_2, and phase plane $(v_1, \dot{v}_1)$, $(w_1, \dot{w}_1)$, $(w_2, \dot{w}_2)$ about three cases.



(a) Rise-span ratio is 1/10



(b) *Rise-span ratio is 1/9*



(c) *Rise-span ratio is 1/8*

Fig. 5. Time-history curve and phase plane behavoir of model with different rise-span ratio

V. CONCLUSIONS

In this article the continuous suspended-cable structure is simplified to three degrees of freedom nonlinear model, through the analysis of the simplified model, the expression of kinetic energy, potential energy, damping and conservative forces work of the system are obtained, and then the dynamic and static equilibrium equations are presented by using Lagrange formulation. The solver ode45 of ordinary differential equations is used to get the time-history curve and phase plane behavior. It can be concluded that when the suspended-cable structure vibrates, the vibration amplitude decreases with the increase of time, and the vibration of the suspended-cable structure is always in a stable equilibrium state.

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REFERENCES

[1] W. F. Zhang, "Spatial structure", Beijing, The Science Press, 2005, pp. 143-144.

[2] S. Z. Shen Shizhao, C. B. Xu, C. Zhao, "Suspended-cable structure design", Beijing, China Architecture & Building Press, 1997, pp. 3-4.

[3] D. S. Sophianopoulos, G. T. Michaltsos, "Nonlinear stability of a simplified model for the simulation of double suspension roofs", Engineering Structures, vol. 23, 2001, pp. 705-714.

[4] R. W. Clough, J. Penzien, "Dynamics of structures", California, Computers and Structures, 1995, pp. 344-347.

[5] J. B. Liu, X. L. Du, "Suspended-cable structure", Beijing, China Architecture and Building Press, 2005, pp. 226-227.

[6] E. L. Allgower, K. Georg, "Numerical Continuation Methods", Berlin, Springer-Verlag, 1990, pp. 288-295.

[7] W.G. Liu, "MATLAB programming tutorial ", Beijing, China Water & Power Press, 2005, pp. 162-165.

[8] D. F. Li, S. X. Li, F. S. Zhu, "Calculation method", Wuhan, Wuhan University Press, 2012, pp.177-178.

[9] J. F. Jiang, L. J. Hu, J. Tang. "Numerical analysis and MATLAB experiment", Beijing, The Science Press, 2004, pp.158-163.