

# Non-Linear Static Buckling of Reinforced Concrete Frames

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**Abstract:-** The purpose of this paper is, to study nonlinear static buckling of the reinforced concrete frames, through studying geometric and material nonlinearity. Both physical and mathematic models of eigenvalue and nonlinear buckling analysis were established. The main objective of the study is the determination of the critical buckling loads and generation of the complete force deformation behavior of such structures within a specified load range, based on the use of a computer program developed for this purpose by using MATLAB program. The general approach to the solution of the problem is based on the finite element method and incremental numerical solution techniques, also the finite element analysis software ANSYS was used to the determination of the critical buckling loads and generation of the force deformation behavior of such structures. The results obtained from the analysis above were compared with experimental published results, the American Standard (ACI 318) and the European Standard (CEB) showed reasonable and acceptable with the computer program and ANSYS software results.

**Keywords:** *Nonlinear buckling analysis, eigenvalue buckling analysis, reinforced concrete frames, finite element*

## 1. INTRODUCTION

Calculating the stability of structures has always been an important engineering discipline. Especially the calculation of the critical buckling load of a structure has been a subject for study since Leonard Euler in 1744 calculated the critical buckling load for a simply supported column. Buckling is a phenomenon, where a structure suddenly changes from one equilibrium configuration to another equilibrium configuration. The calculation of buckling loads of a structure is of great importance, due to the possibility of sudden failure of the structure, if the critical buckling load is reached. Some structures might lose all stability, when the buckling load is reached, which could put people at risk, if a roof or other similar structures loses all stability. Many researchers have studied finite element method and nonlinear analysis of structures. Some of these studies will be cited here. Wood and Zienkiewicz (1977), presented a continuum approach for the geometrically nonlinear analysis of oriented bodies e.g

beams, frames, and arches in total lagrangian coordinate system using finite element method. The nonlinear equilibrium equations are solved using the Newton-Raphson method for which a number of examples were solved. The derivations were extended to include ax symmetric structures. W.A.M.Alwis and T.usami (1978), presented a finite element formulation to determine the elastic lateral torsional buckling load of braced and un braced planar structures through a series of equilibrium states at incremental steps of in plane loadings. Chajes and Churchill (1987), presented the basic concepts involved in the construction of load deflection curves for geometrically nonlinear system by the finite element method. Kam (1988), presented a general method for the large deflection analysis of inelastic plane frames. Chan (1988), presented a geometric and material nonlinear analysis procedure for framed structures, using a solution algorithm of minimized residual displacements. Kassimali and Abbasnia (1991), suggested a method for large deformation and stability analysis of elastic space frames. Kw wong and Rf warner (1997), presented an efficient approach for the nonlinear analysis of reinforced concrete frame using line elements. Zdenek p. Bazant and Yuyin xiang (1997), proposed an improved method of analysis of reinforced concrete columns in braced (no sway) frames, which was suitable as a simple computer solution for design practice and was more realistic than the existing ACI and CEB methods. Endre Dulacska (2007), discussed the approximate determination of the elastic plastic load parameter of structure and he also established a lower and an upper bound for the critical failure load parameter, with the aid of which the results of computer calculations can be checked.

Rifat Sezer (2010), studied the geometrical nonlinear analysis of the prismatic plane frames with stiffness matrix method by using the stability functions. K. A. Tzaros, E. S. Mistakidis (2011), presented a method for calculating the buckling loads and the buckling shapes of continuous beams with unilateral intermediate constraints based on the theory of elastic stability using the Euler

equilibrium method. The purpose of the study was to find the critical load or buckling load of the reinforced concrete frames.

## 2. METHODOLOGY

To predict buckling loads and load-deformation curves of reinforced concrete frames the methodology will be prepared through the following steps:

Different finite element approaches have been proposed for the analysis of different types of structures. The theoretical study depends on nonlinear static buckling of plane frames. The Newton-Raphson approach will be utilized to trace the equilibrium path during the load deformation response. About three models (one single reinforced concrete column and two

reinforced concrete frames) will be prepared. Loading should be applied to the plane frames until the failure occurred. The mode of failure characterized by flexural failure of concrete. Load deflection curves will be plotted for each model and compared to predicted ultimate loads. The experimental results of plane frame models subjected to transverse load will be utilized for calibration of the finite element models

using structural program (ANSYS). The analysis will be carried for nonlinear static buckling and different buckling modes will be resulted. The comparison of results will be done for the experimental models and mathematical models using software program (MATLAB) and two different code of practice (ACI, CEB).

Examples:

Reinforced concrete column model

Table (1): Experimental Data

Specimen	Slenderness	Height	Width	Depth	ECC.Ratio	I (mm <sup>4</sup> )	F <sub>cu</sub>
G1	H/h	H (mm)	b (mm)	h (mm)	e2/h	(mm <sup>4</sup> )	(N/mm <sup>2</sup> )
	20	2040	152	102	0.30	1.3E+07	25.60

Reinforced concrete square frame model

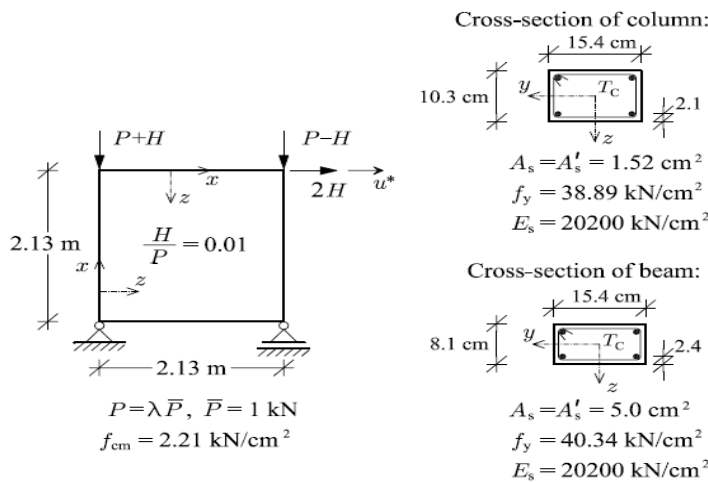


Fig. 1: Geometry and Load condition

Reinforced concrete portal frame model (Multi stories)

Table (2): Assumed Data  
 Concrete strength is 42.7 MPa

Beam Section		Column Section		E(MPa)
Width (mm)	Depth(mm)	Width (mm)	Depth(mm)	
700	350	350	350	37600

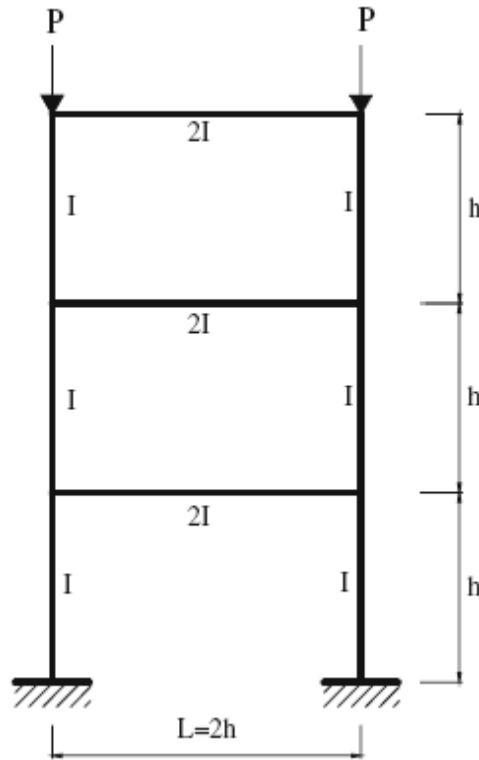


Fig. 2 Geometry and Load condition

### 3. RESULTS AND DISCUSSION

The results obtained for linear and nonlinear buckling analysis for reinforced concrete models will be presented according to the following:

1. Buckling Load Factors and Buckling Mode Shapes.
2. Load and deformation relationship.
3. Elastic modulus change with different codes and experiments.
4. Buckling loads calculated with elastic modulus obtained from different design codes, experiment, MATLAB and ANSYS program.

#### 3.1 Buckling Load Factors and Buckling Mode Shapes

Table (3): Buckling mode & Buckling load factor

Buckling mode No.	Buckling load factor (N)
1	0.19696E+06
2	0.17858E+07
3	0.51561E+07

Tables (3, 4, 5) show the buckling load values and figures (3, 4, 5, 6, 7, 8, 9, 10, 11) show the buckling modes which represent the shape a structure assumed under buckling load (elastic formulation).



Fig. 3: Buckling mode (1) - Model (1)



Fig.4: Buckling mode (2) - Model (1)

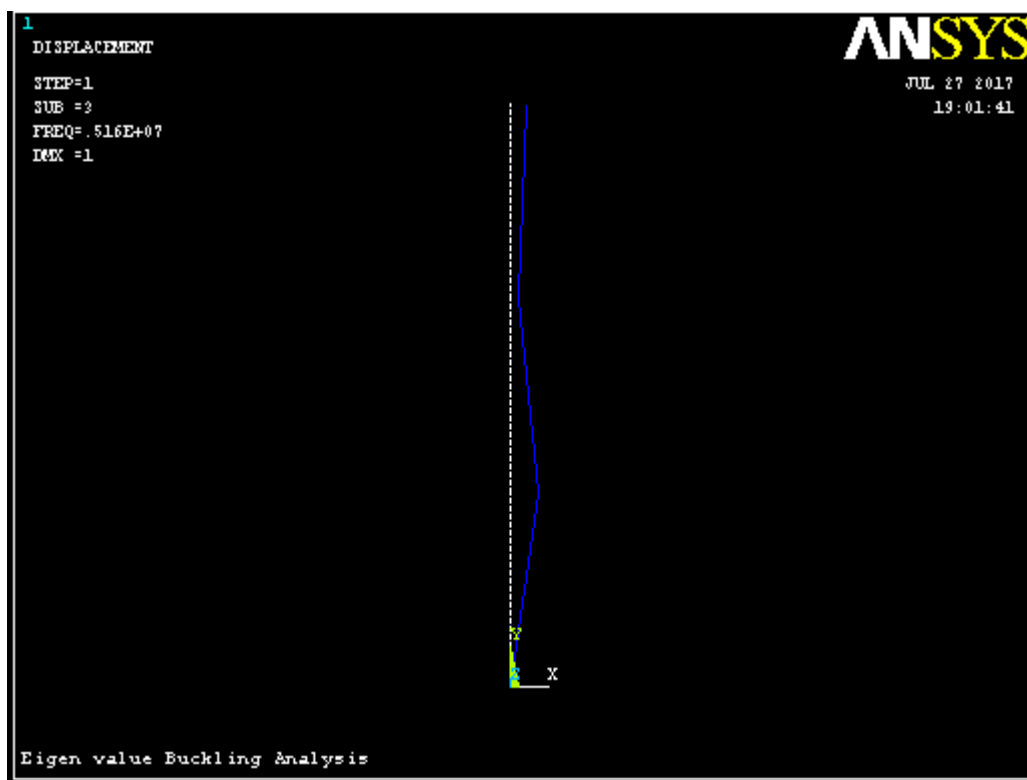


Fig. 5: Buckling mode (3) - Model(1)

Table (4): Buckling mode & Buckling load factor

Buckling mode No.	Buckling load factor (N)
1	0.33223E+06
2	0.11688E+07
3	0.16293E+07

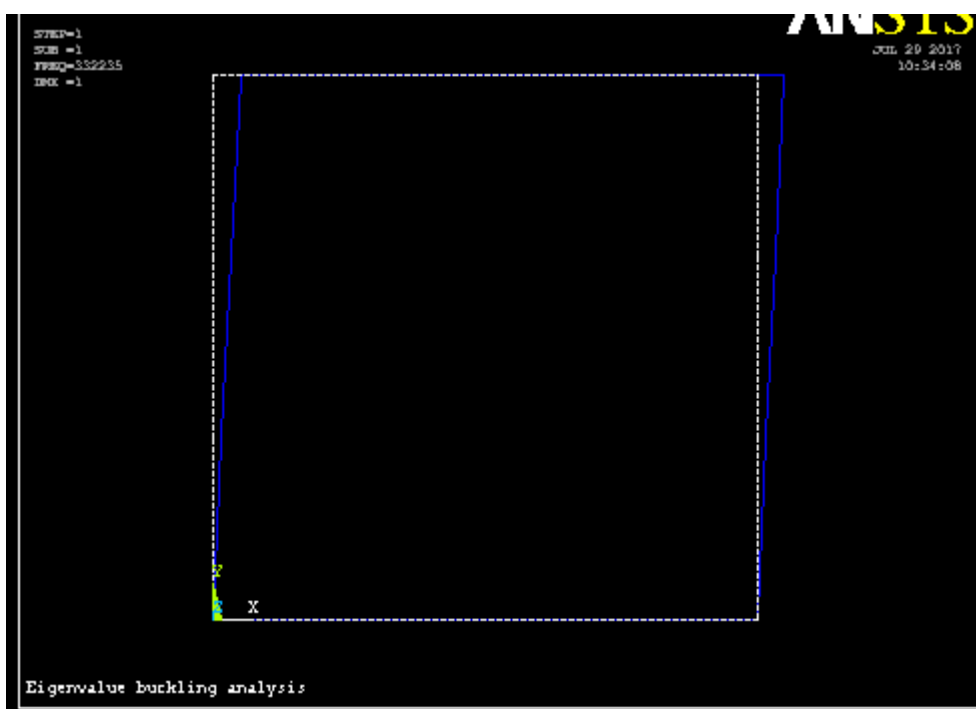


Fig. 6: Buckling mode (1) - Model (2)

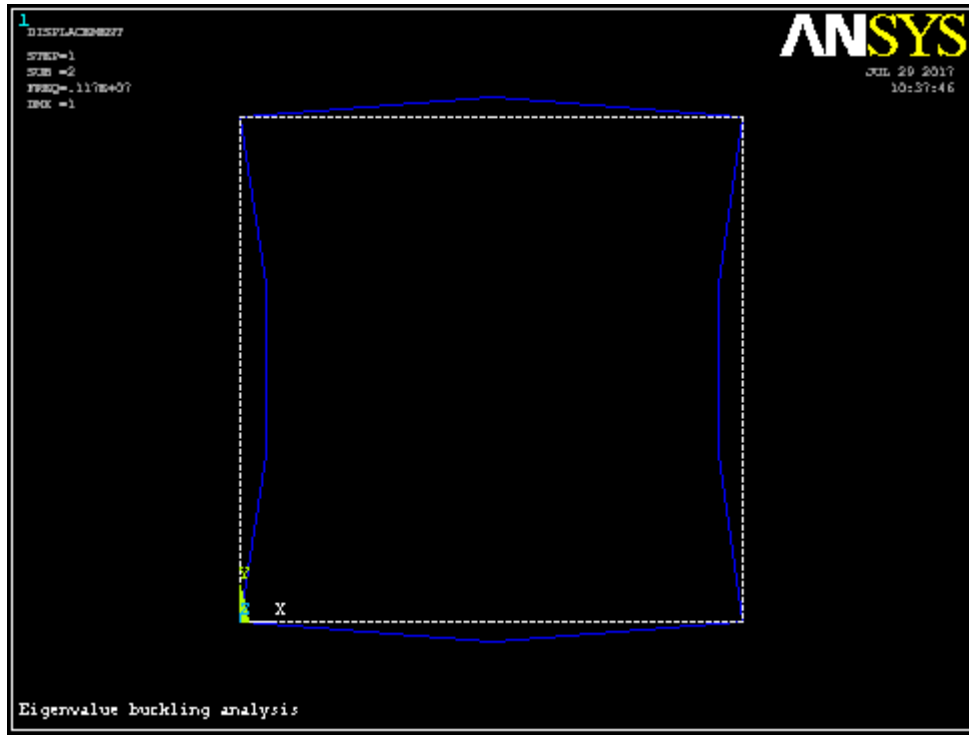


Fig. 7: Buckling mode (2) - Model (2)

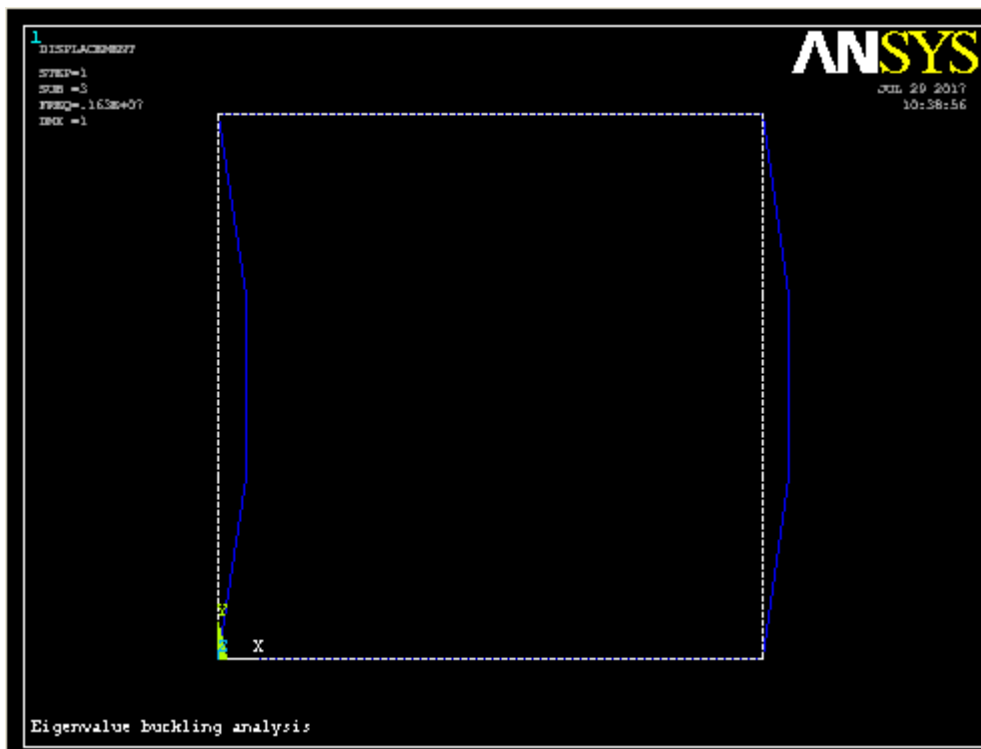


Fig. 8: Buckling mode (3) - Model (2)

Table (5): Buckling mode & Buckling load factor

Buckling mode No.	Buckling load factor (N)
1	0.23442E+08
2	0.34177E+08
3	0.48471E+08

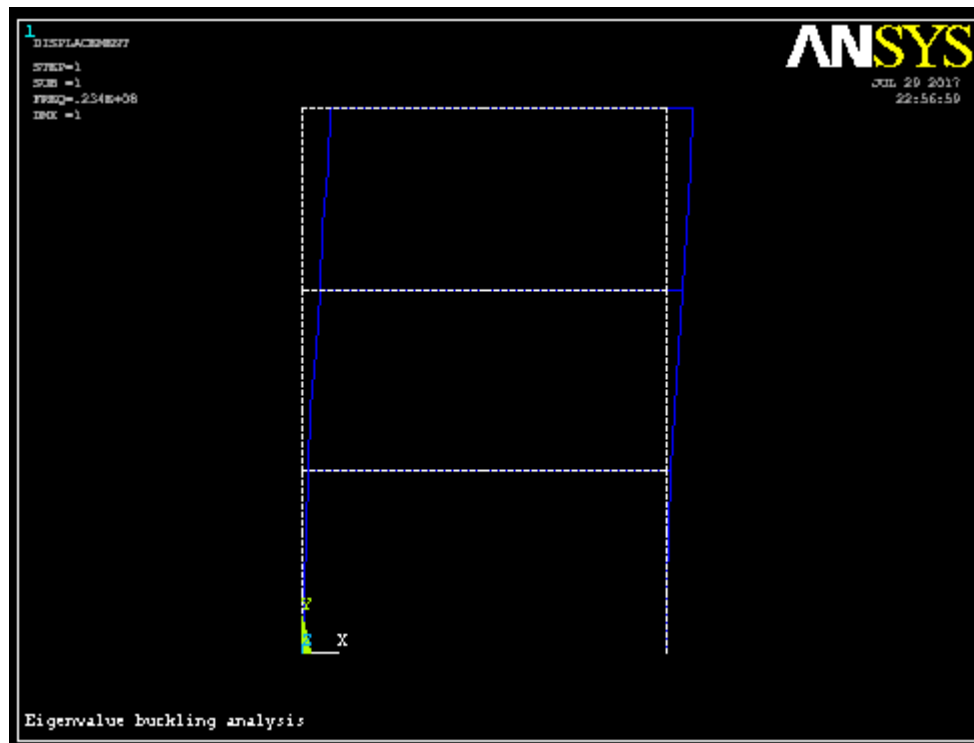


Fig. 9: Buckling mode (1) - Model (3)

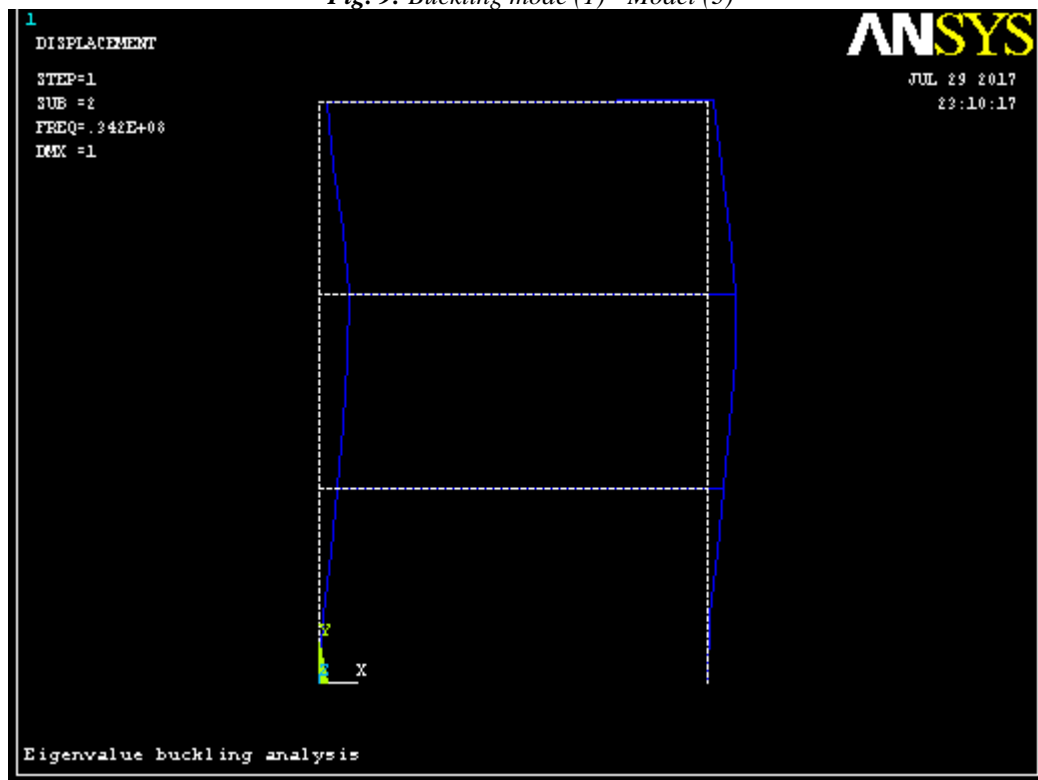


Fig. 10: Buckling mode (2) - Model (3)

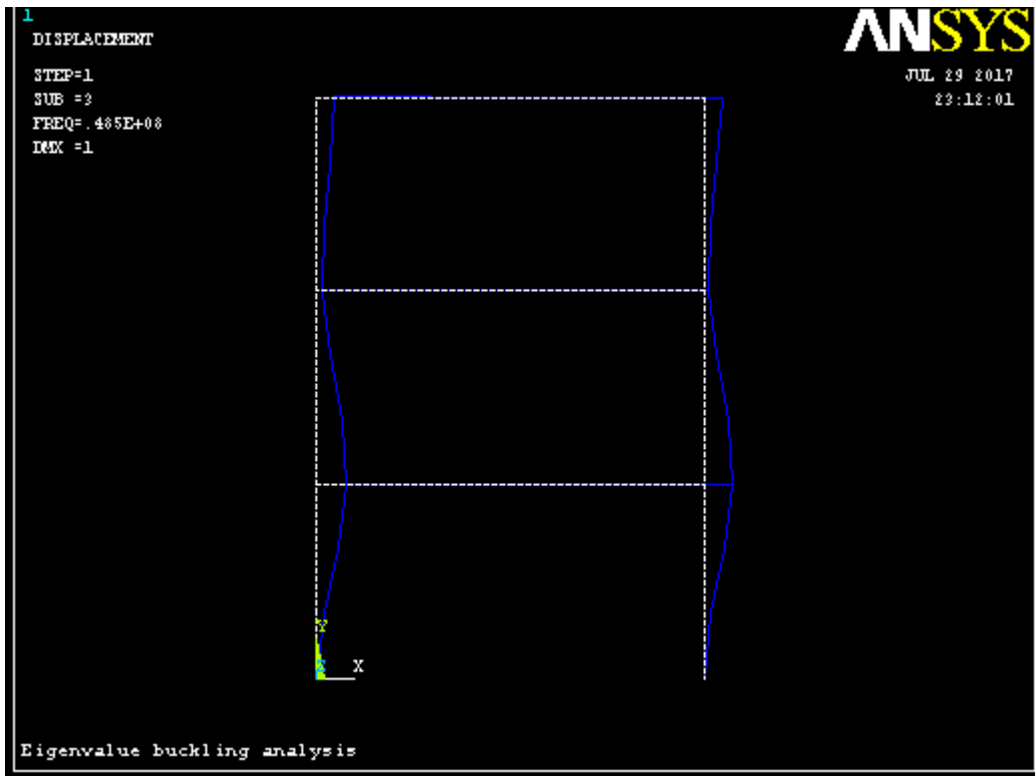


Fig. 11: Buckling mode (3) - Model (3)

### 3.2 Load and Deformation Relationship

Figures (12, 13, 14,) show the actual displacements and reactions obtained from nonlinear buckling analysis.

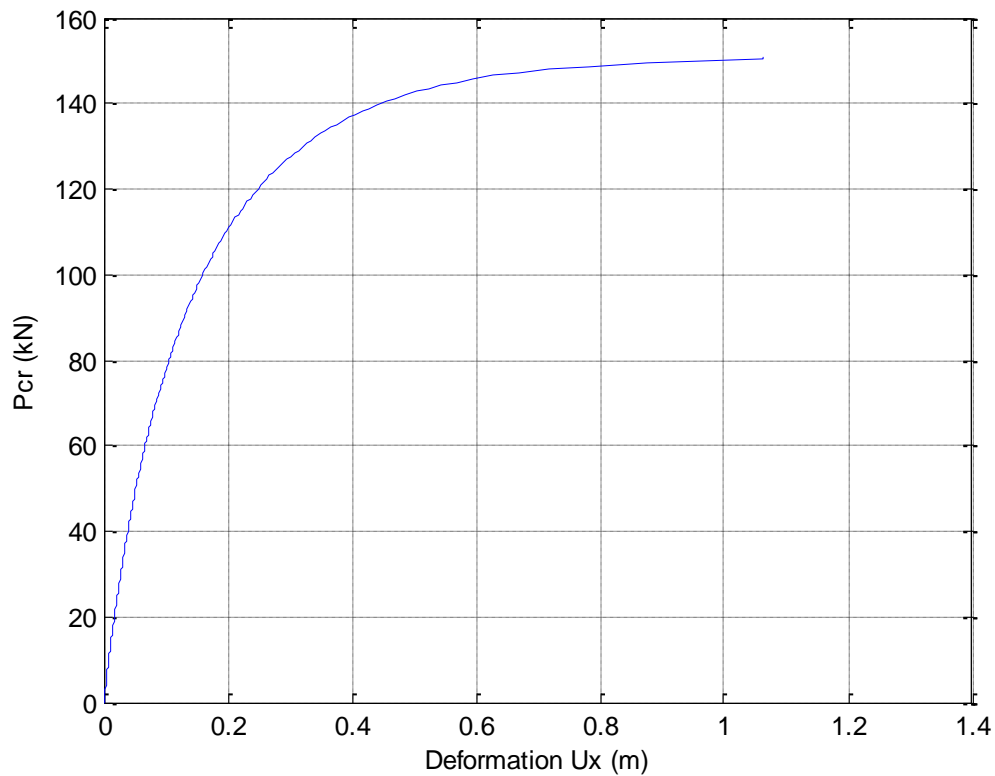


Fig. 12: Load - deformation Model (1)



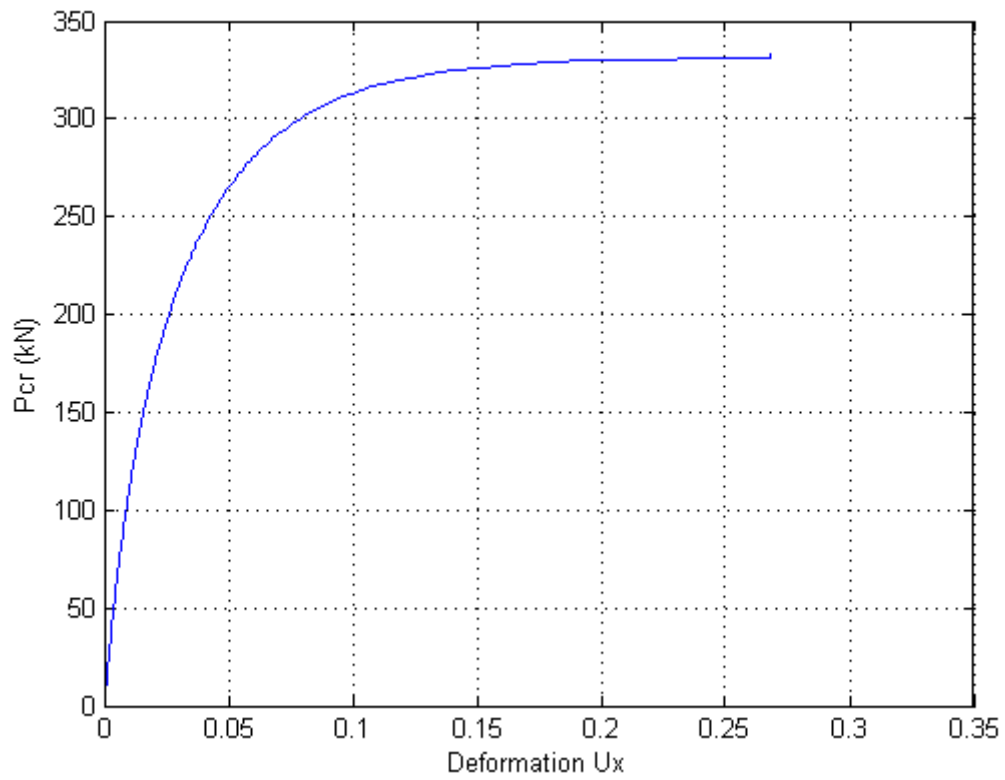


Fig. 13: Load - deformation Model (2)

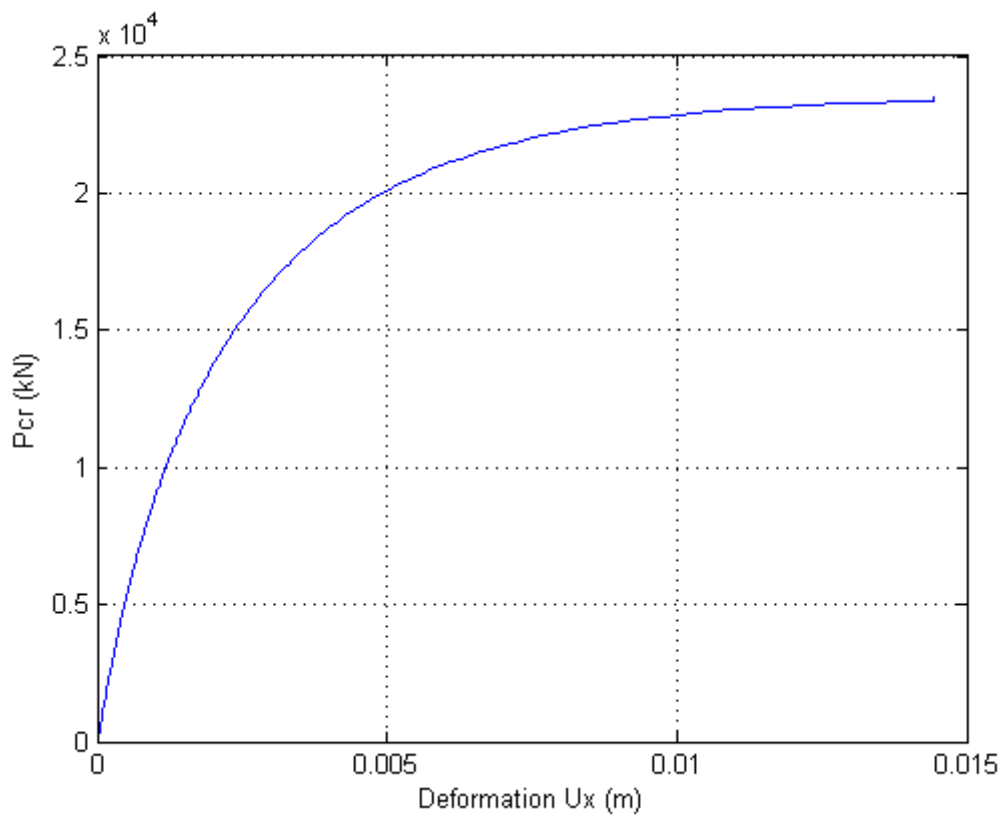


Fig. 14: Load - deformation Model (3)

3.3 Elastic Modulus Change with Different Codes and Experiments

Tables (6, 7, 8) and figures (15, 16, 17) show the differences in material properties (modulus of elasticity) cause differences in calculation of critical buckling load. Because critical buckling load directly relates to material properties.

Table (6): Elastic modulus values for a given concrete strength  $F_{cu}$  25.6 N/mm<sup>2</sup> (Model 1):

References	E (MPa)
Experimental	18,947
ACI	25,551
CEB	29,402
Lower bound	18,600
Upper pound	32,750

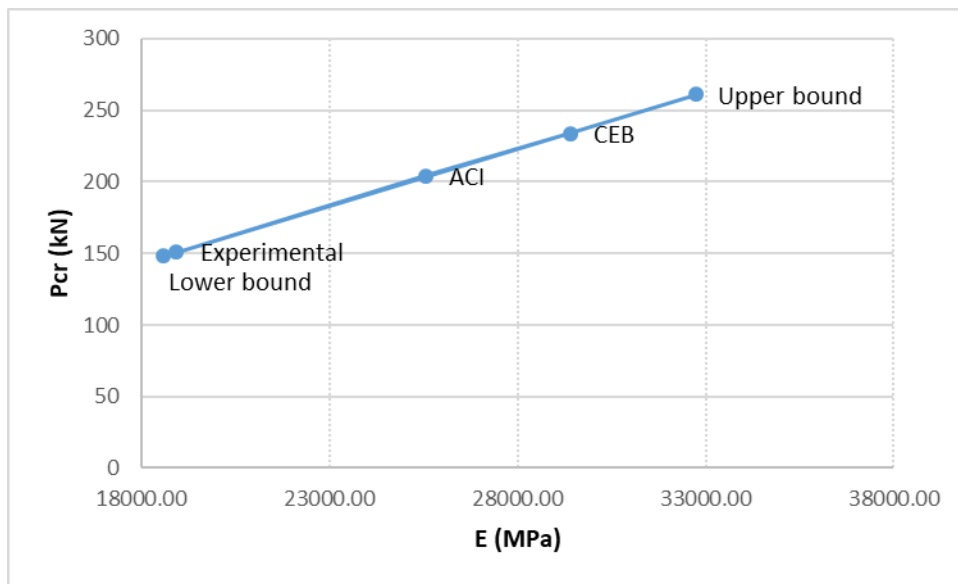


Fig. 15: Material property change with critical buckling load according to different codes and Experiment (Model 1)

Table (7): Elastic modulus values for a given concrete strength  $F_{cu}$  22.1 N/mm<sup>2</sup> (Model 2):

References	E (MPa)
Experimental	28,000
ACI	23,740
CEB	27,997
Lower bound	19,700
Upper pound	28,250

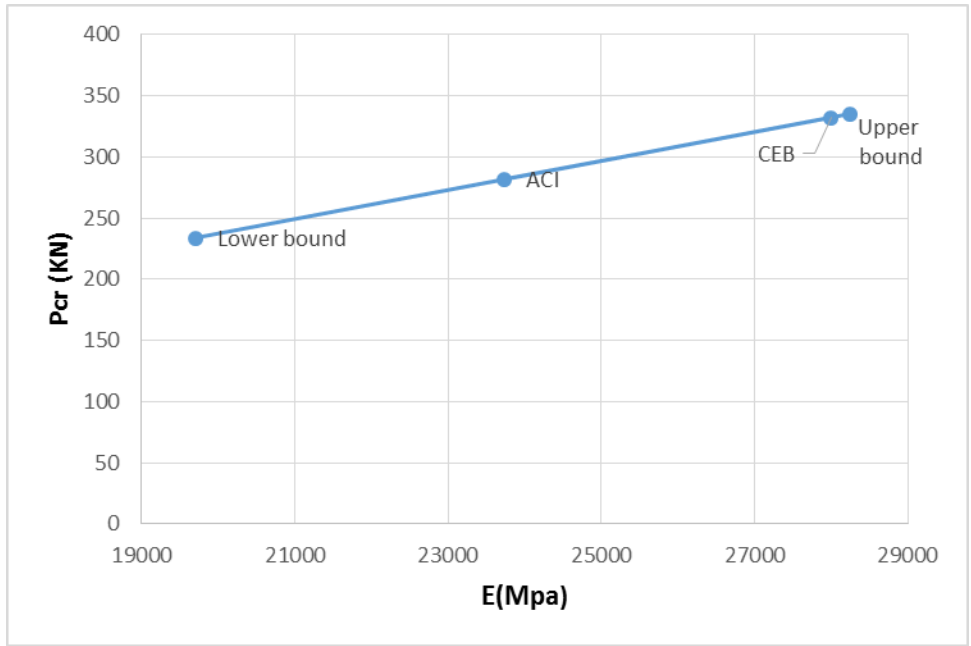


Fig. 16: Material property change with critical buckling load according to different codes (Model 2)

Table (8): Elastic modulus values for a given concrete strength  $F_{cu}$  42.7 N/mm<sup>2</sup> (Model 3):

References	E (MPa)
Experimental	37,600
ACI	30,900
CEB	37,010
Lower bound	30,500
Upper pound	40,200

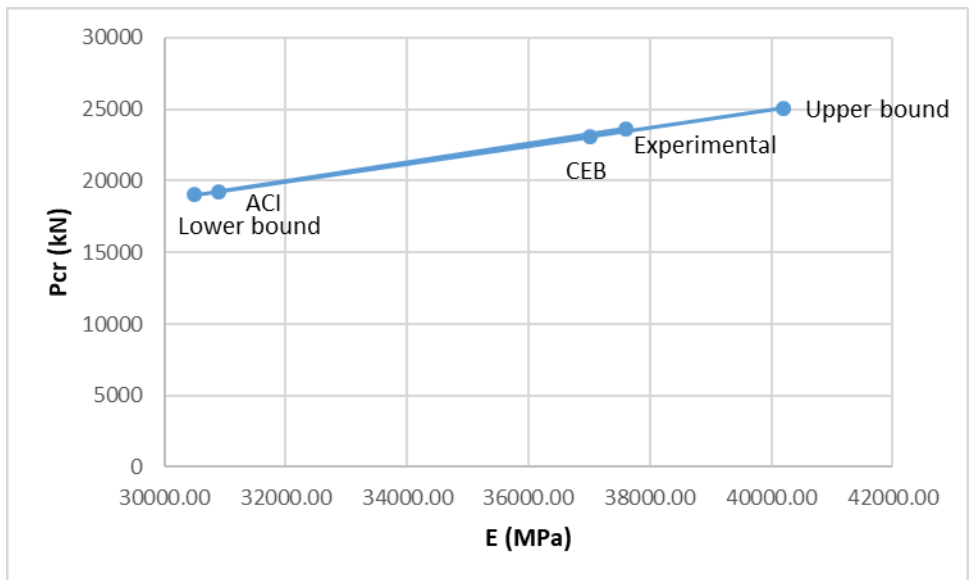


Fig. 17: Material property change with critical buckling load according to different codes and Experiment (Model 3)

3.4 Buckling Loads Calculated with Elastic Modulus Obtained From Different Design Codes, Experiment, MATLAB and ANSYS Program

Tables (9, 10, 11,) and figures (18, 19, 20) show the buckling load values are in between lower and upper bounds.

Table (9): Buckling Loads for Model (1)

Buckling Cal. Method	Buckling Loads $P_{cr}$ (kN)
Experiment	151.0
ACI 318 Code	204.0
CEB 90 Code	234.0
Lower bound	148.0
Upper bound	261.0
Matlab	197.0
ANSYS <sup>1</sup>	197.0
ANSYS <sup>2</sup>	197.4
ANSYS <sup>3</sup>	197.0

ANSYS<sup>1</sup>: Eigenvalue buckling analysis

ANSYS<sup>2</sup>: Nonlinear analysis (Geometrically nonlinearity)

ANSYS<sup>3</sup>: Nonlinear analysis (Geometrically& Material nonlinearity)

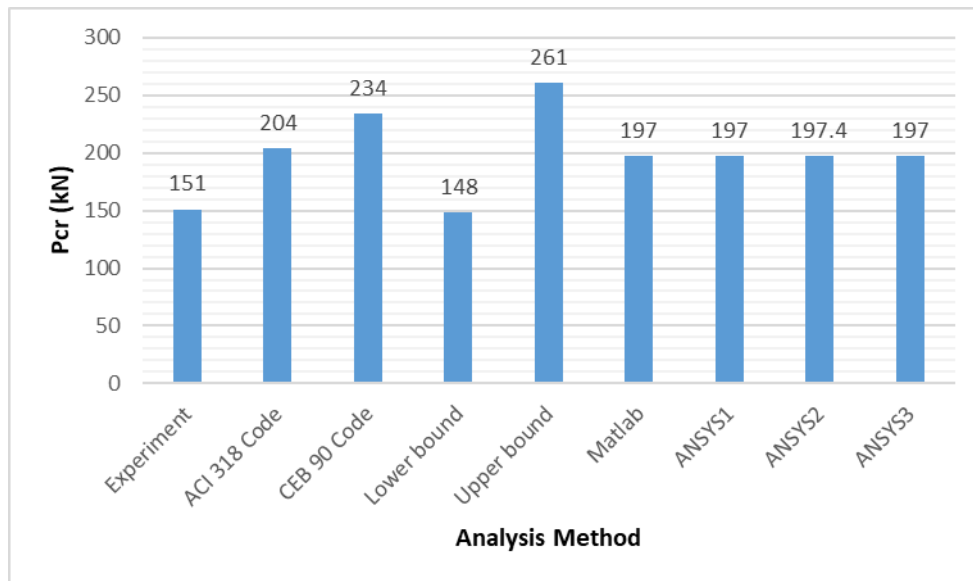


Fig. 18: Buckling loads for Model (1) calculated according to different design codes and experiment

Table (10): Buckling Loads for Model (2)

Buckling Cal. Method	Buckling Loads $P_{cr}$ (kN)
Experiment	138.0
ACI 318 Code	282.0
CEB 90 Code	332.2
Lower bound	234.0

Upper bound	335.2
Matlab	333.0
ANSYS <sup>1</sup>	332.4
ANSYS <sup>2</sup>	210.3
ANSYS <sup>3</sup>	210.3

ANSYS1: Eigenvalue buckling analysis

ANSYS2: Nonlinear analysis (Geometrically nonlinearity)

ANSYS3: Nonlinear analysis (Geometrically& Material nonlinearity)

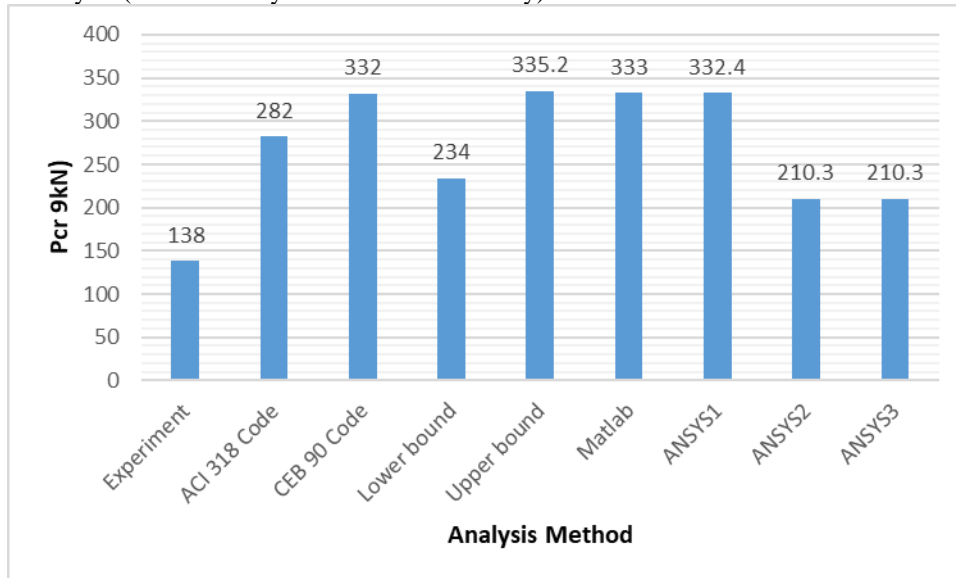


Fig. 19: Buckling loads for Model (2) calculated according to different design codes and experiment

Table (11): Buckling Loads Model (3)

Buckling Cal. Method	Buckling Loads Pcr (kN)
Experiment	23668.0
ACI 318 Code	19256.0
CEB 90 Code	23069.0
Lower bound	19015.0
Upper bound	25063.0
Matlab	23500.0
ANSYS <sup>1</sup>	23442.0
ANSYS <sup>2</sup>	23442.0
ANSYS <sup>3</sup>	23442.0

ANSYS1: Eigenvalue buckling analysis

ANSYS2: Nonlinear analysis (Geometrically nonlinearity)

ANSYS3: Nonlinear analysis (Geometrically& Material nonlinearity)

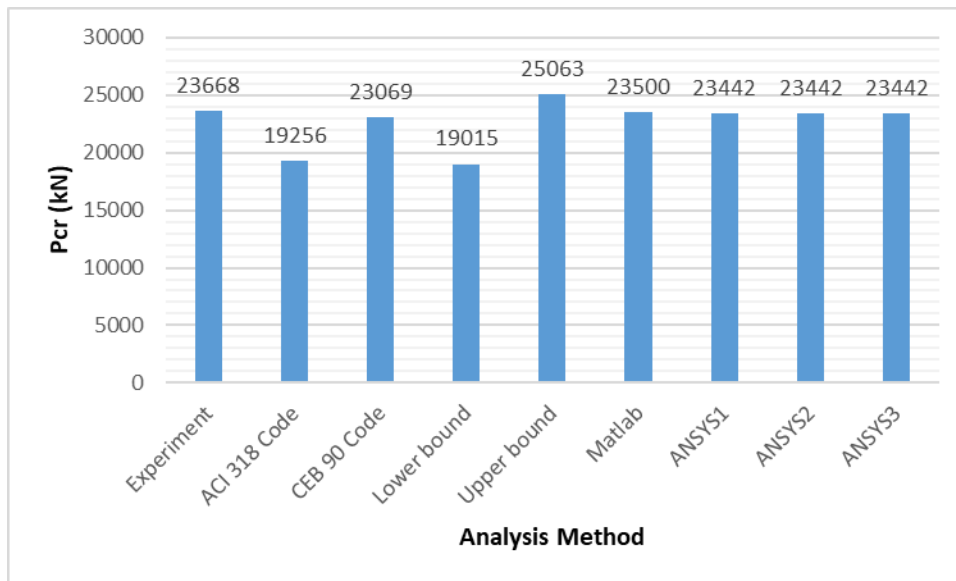


Fig. 20: Buckling loads for Model (3) calculated according to different design codes and experiment

Table (12) Buckling load (Pcr) comparison: (Nonlinear buckling analysis):

Model No.	Pcr Experiment. (kN)	Pcr ANSYS (kN)	Pcr MATLAB (kN)	Pcr Experiment./ Pcr ANSYS	Pcr Experiment./ Pcr MATLAB	Pcr ANSYS/ Pcr MATLAB
1	151	197.36	196.94	0.76	0.78	1.00
2	138	210.32	333	0.66	0.41	0.63
3	23668	23442	23500	1.00	1.00	1.00

#### 4. CONCLUSION

The discussion of the results of the analysis obtained in this study resulted in the following conclusions:

1. The eigenvalue buckling loads and the buckling modes for various reinforced concrete frames were presented using ANSYS computer program.
2. Nonlinear buckling analysis program by using MATLAB, was used to compute the buckling load utilizing the stability functions to determine the buckling load, comparison with ANSYS program results showed little differences in their values.
3. The buckling load value is strongly dependent on the choice of modulus of elasticity that is known material uncertainly. Nonlinearity of material (concrete) is assumed to be presented by concrete average strength and lower value of modulus of elasticity (see fig.6.26 – fig.6.31). Under this assumption the results obtained showed very little differences in the values between MATLAB, ANSYS, ACI, and CEB and published experimental works in buckling loads in structural members and frames of reinforced concrete.

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