Non-Linear Finite Element Analysis of Deep Beam

Abstract—There are several analytical tools available for analyzing deep beams. Among all the available analytical methods, finite element analysis (FEA) offers a better option. In this paper, the attempt has been made to study the behavior of deep beam of various span to depth ratio by ANSYS 13.0 under two point loading of 50KN. The detailed analysis has been carried out by using non-linear finite element method and design of deep beam by using IS 456-2000. The objectives of this study are to observe deflection, cracking of deep beams subjected to two point loading of 50KN. To study non-linear finite element analysis of deep beam by using ANSYS having different L/D ratio (1.5, 1.6, 1.71) and To study stress distribution of deep beam.

Keywords—deep beam, non-linear finite element, ANSYS 13.0, deflection.

I. INTRODUCTION

Reinforced concrete deep beam are very useful members widely used in buildings, bridges and infrastructures. Deep beams are the beam with a depth comparable with their span length. To consider a beam as a deep beam, depth to span length should be less than a certain value. This ratio is the most frequently used parameter by researchers and engineers. Generally, the simply supported beam with span to depth ratio less than 2 is classified as deep beam and beam with span to depth ratio exceeding 2 as shallow beams.

Reinforced concrete deep beams, in particular, show much complicated response to any sort of loads subjected to such members than conventional beams. It is well-known that deep beams behave very differently from shallow beams as arch action rather than flexure dominates the behavior, after diagonal cracking has occurred. However, causes of size effect in deep beams remain unresolved. This is mainly because experimental data are relatively scarce on deep beams with geometrically-varied beam sizes. A common practice in experimental investigation of size effect of deep beams is to keep the beam width constant while increasing the beam height. It was implicitly assumed by researchers that the beam width has a negligible effect on the structural behavior of deep beams. However, this conclusion is largely drawn from test results of beams having small height-to-thickness ratios. Size effect in shallow reinforced concrete beams is represented by reduction in ultimate shear strength due to an increase in beam size.

Beams with comparable large depths in relation to spans are called deep beams. According to IS: 456-2000, a simply supported beam is classified as deep beam when the ratio of its effective span L to overall depth D is less than 2.0 and continuous beams are considered as deep beam when the ratio L/D is less than 2.5. The effective span is defined as the centre-to-centre distance between the supports or 1.15 times the clear span whichever is less. The bending stress distribution across any transverse section in deep beams deviates appreciably from the straight line distribution assumed in the elementary beam theory. Consequently section which is plane before bending does not remain plain after bending.

II. OBJECTIVE OF STUDY

The main objective of study is to analysis a deep beam of various Length to span ratio by ANSYS 13.0 under two point loading. The detailed analysis has been carried out using non-linear finite element method & design using IS 456-2000. The objective of the investigation are listed below

1. To observe deflection, cracking of deep beams subjected to two point loading.
2. To study non-linear finite element analysis of deep beam by using ANSYS of beam having different L/D ratio (1.5, 1.6, 1.71)
3. To study stress distribution (flexural, shear) of deep beam.

III. FINITE ELEMENT METHOD

The FEM is a numerical method for analyzing structures and continua. Usually the problem addressed is too complicated to be solved satisfactorily by classical analytical methods. The problem may concern stress analysis, heat conduction, or any of several other areas. The finite element procedure produces many simultaneous algebraic equations which are generated and solved on digital computer. Results are rarely exact. However, errors are decreased by processing more equations, and results accurate enough for engineering purposes are obtainable at reasonable cost.
The finite element analysis typically involves the following steps:

1. Divide the structure or continuum into finite elements. Mesh generation programs called preprocessors help the user in doing this work.
2. Formulate the properties of each element. In stress analysis, this means determining nodal loads associated with all elements deformation states.
3. Assemble elements to obtain the finite element model of the structure.
4. Apply the known loads; nodal forces, and/or moment in stress analysis.
5. In stress analysis, specify how the structure is supported. This step involves setting several nodal displacements into known values.
6. Solve the simultaneous linear algebraic equations to determine nodal degrees of freedom (d.o.f.).
7. In stress analysis, calculate the element strains from the nodal d.o.f. and the element displacement field interpolation, and finally stress from strains.

IV. PARAMETRIC INVESTIGATION

A. Data For Design Of Deep Beam:

Following are the data chosen for design purpose:
- Length \(L\) = 700 mm, Effective depth \(d\) = 320 mm, Depth \(D\) = 350 mm, Width \(B\) = 150 mm, two point loading of 50 KN
- Shear span = 200 mm, Clear Cover = 30 mm.

Use M20 and Fe 415.

B. Design Procedure Of Deep Beam:

Following are the procedure of design of deep beam:

Step 1: Calculation of effective span
This shall be taken as lesser of
1) Center to center distance between supports = 600 mm
2) 1.15 times the clear distance between supports = 1.15 x 540 = 621 mm ...............cl.29.2, IS 456-2000, pn 51

Hence effective span \(l\) = 600 mm.

Step 2: Check for deep beam action

\[ l / D = 600 / 350 = 1.71 < 2.00 \]  hence beam behaves as a deep beam. ........cl.29.1, IS 456-2000, pn 51

Step 3: Calculation of lever arm

\[ z = 0.2 (1 + 2D) - l/D \]  is between 1 to 2. cl.29.2, IS 456-2000

\[ z = 260 \] mm.

Step 4: Thickness of beam

1. It is controlled by two conditions to avoid buckling failure
   i) D/T ratio should be less than 25.
   ii) L/T ratio should be less than 50.

2. Thickness of beam = 150 mm.

Step 5: Check for bearing stresses

Per. bearing stress = 0.45 fck ..., cl.34.4, IS 456-2000, pn 66

\[ = 0.45 \times 20 = 9 \text{ N/mm}^2 \]

Bearing stress at supports and at loading point

\[ \sigma_{\text{Be}} = \frac{V_u}{Sc} \]

Where

\[ V_u = \text{factored S.F.} \]

\[ S = \text{width of support 60 mm} \]

\[ t = \text{thickness of beam} \]

\[ = 8.33 \text{ N/mm}^2 \times 0.45 \text{ fck...hence safe} \]

Provide bearing plate of 60 x 150 mm at support as well as loading point.

Step 6: Calculation of bending moment

a) Load due to self weight = 1.5 (b x D x 25)

\[ = 1.5 (0.150 \times 0.35 \times 25) = 1.968 \text{ KN/m} \]

b) Maximum Factored B.M. due to self weight = \(wl^2/8\)

\[ = 1.968 \times 0.6^2 / 8 = 0.0885 \text{ KNm} \]

C) Maximum Factored B.M. due to two point loads

\[ = 50 \times 0.20 \times 1.5 = 15 \text{ KNm} \]

Total Factored B.M. (Mu) = 15 + 0.0085 = 15.0085 KNm

Step 7: Calculation of flexural reinforcement

A_{ut} = \frac{Mu}{(0.87 \times fyx \times z)}

\[ = \frac{15.0085 \times 10^6}{(0.87 \times 415 \times 260)} \]

A_{ut} = 133.408 mm^2

Minimum reinforcement

A_{ut} = (0.85x b x d) / fy

A_{ut} = (0.85 \times 100 \times 864) / 415

A_{ut} = 176.96 mm^2 \geq A_{ut,reqd.}

Hence provide A_{ut} min
Provide 2-10 mm \(\Phi\) at bottom after a clear cover of 30 mm

Zone (depth) of placement = 0.25D-0.05L

\[ = 0.25 \times 350 - 0.05 \times 600 = 57.5 \text{ mm} \]

Tension steel can be placed in the zone of 57.5 mm above cover.

Step 8: Calculation of development length

For flexural steel

\[ L_d = \frac{9 \sigma_s}{4 \tau_{t,c}} \]

\[ = (10 \times 0.87 \times 230) / (4 \times 1.2 \times 1.6) \]

= 566.40 mm

\[ 0.8 \times 0.8 = 0.64 \]

= 53.125 mm

Provide development length of 450 mm for 10 mm \(\Phi\) bar.

Step 7: Check for shear:
No separate checking for shear is specified in I.S.456. It is assumed that the arching action of the main tension steel & the web steel together with concrete will carry the shear. A deep beam complying with the requirements of Cl.29.2 and Cl.29.3 shall be deemed to satisfy the provisions for shear.
Step 8: Side Face reinforcement:

As per Clause 32.5 of I.S. 456: 2000, Page 62 and 63, all specifications require minimum amount of vertical steel & horizontal steel in the form of U bars to be placed on both faces of deep beams. They not only overcome the effects of shrinkage & temperature but also act as shear reinforcement. These specifications are:

1. Vertical steel shall be 0.15 % for Fe 250 & 0.12 % for Fe 415. The bars shall not be more than 14 mm diameter & spaced not more than 3 times the thickness of the beams 450 mm.
2. diameter & spaced not more than 3 times the thickness of the beams or 450 mm.

1. Vertical Steel \( Av = \left( \frac{0.12}{100} \right) \times 150 \times 350 \) 
\[ = 63\text{mm}^2 \]
i.e. 63mm² on each face.
Hence provide 9 two legged vertical stirrups of 6 mm diameter bar at 150 mm c/c spacing.

2. Horizontal Steel \( Ah = \left( \frac{0.20}{100} \right) \times 150 \times 350 \) 
\[ = 105\text{mm}^2 \]
i.e. 105 mm² on each face.
Provide 2- 8 mm dia. horizontal stirrups at 200mm c/c. on each face of the beam.

**TABLE I. REINFORCEMENT OF BEAM DEPTH 350MM &375MM**

<table>
<thead>
<tr>
<th>Sr. No</th>
<th>Type of reinforcement</th>
<th>Details of reinforcement</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Main Steel</td>
<td>2-10 mm Φ (Fe 415)</td>
</tr>
<tr>
<td>2</td>
<td>Side face reinforcement</td>
<td></td>
</tr>
<tr>
<td></td>
<td>I. Vertical Steel</td>
<td>Two legged 8 mm dia.</td>
</tr>
<tr>
<td></td>
<td>II. Horizontal Steel</td>
<td>stirrups @ 190 mm c/c</td>
</tr>
<tr>
<td></td>
<td></td>
<td>spacing (8 stirrups)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2-8 mm dia. @ 200 mm</td>
</tr>
<tr>
<td></td>
<td></td>
<td>c/c spacing.</td>
</tr>
</tbody>
</table>

**TABLE II. REINFORCEMENT OF BEAM DEPTH 400MM**

<table>
<thead>
<tr>
<th>Sr. No</th>
<th>Type of reinforcement</th>
<th>Details of reinforcement</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Main Steel</td>
<td>2-10 mm Φ (Fe 415)</td>
</tr>
<tr>
<td>2</td>
<td>Side face reinforcement</td>
<td></td>
</tr>
<tr>
<td></td>
<td>I. Vertical Steel</td>
<td>Two legged 8 mm dia.</td>
</tr>
<tr>
<td></td>
<td>II. Horizontal Steel</td>
<td>stirrups @ 190 mm c/c</td>
</tr>
<tr>
<td></td>
<td></td>
<td>spacing (8 stirrups)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3-8 mm dia. @ 200 mm</td>
</tr>
<tr>
<td></td>
<td></td>
<td>c/c spacing.</td>
</tr>
</tbody>
</table>

**IV NON LINEAR FINITE ELEMENT ANALYSIS.**

**A. Element type**

**TABLE III. ELEMENT TYPE FOR MODEL**

<table>
<thead>
<tr>
<th>Sr. No</th>
<th>Material type</th>
<th>ANSYS13.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Concrete</td>
<td>Solid65</td>
</tr>
<tr>
<td>2</td>
<td>Steel Reinforcement</td>
<td>Link180</td>
</tr>
</tbody>
</table>

A Solid65 element was used to model the concrete. This element has eight nodes with three degrees of freedom at each node translations in the nodal x, y, and z directions. A schematic of the element was shown in Fig.1.

A Link180 element was used to model steel reinforcement [2]. This element is a 3D spar element and it has two nodes with three degrees of freedom translations in the nodal x, y, and z directions. This element is capable of plastic deformation and element was shown in the Fig.2.

**TABLE IV REAL CONSTANTS**

**D. Material properties**

The material properties defined in the model are given in Table 4. For the reinforcing bars, the yield stress was obtained from the experimental test as \( f_y = 432 \text{ MPa} \) and the tangent modulus as 847 MPa. The concrete cube compressive strength fck determined from the experimental result is 44.22 MPa, 80% of which is used as the cylinder strength. The multilinear isotropic material uses the Von Mises failure criterion along with the Willam and Warnke (1974) model to define the failure of the concrete. \( E_c \) is the modulus of elasticity of the concrete, and \( v \) is the Poisson’s ratio. The characteristic strength of the concrete considered was 25 N/mm² and the Poisson’s ratio was 0.3.
The multilinear isotropic stress-strain curve for the concrete under compressive uniaxial loading was obtained using equation (Macgregor 1992).

\[ f = E_c \varepsilon / (1 + (\varepsilon/\varepsilon_0)^2) \]

where,

- \( f \) = stress at any strain \( \varepsilon \),
- \( \varepsilon \) = strain at stress \( f \),
- \( \varepsilon_0 \) = strain at the ultimate compressive strength

### D. Material properties

<table>
<thead>
<tr>
<th>Material model no</th>
<th>Element type</th>
<th>Material Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Solid 65</td>
<td>Multi linear Isotropic</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Reference Point</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Point 1</td>
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<td>Point 2</td>
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<td>Point 4</td>
</tr>
<tr>
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<td>Point 5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Concrete</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Uniaxial Tensile cracking Stress</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Uniaxial crushing Stress</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Biaxial crushing Stress</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Ambient Hydrostatic stress state</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Biaxial crushing stress under ambient hydrostatic stress state</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Stiffness multiplier for cracked tensile condition</td>
</tr>
<tr>
<td>2</td>
<td>Link8</td>
<td>Steel</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Linear Isotropic</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Ex</td>
</tr>
<tr>
<td></td>
<td></td>
<td>PRXY</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Bilinear Isotropic</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Yield Stress</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Tang Modulus</td>
</tr>
</tbody>
</table>

### E. Modelling

The model was 700 mm long with a cross section of 150 mm X 350 mm. The Finite Element beam model was shown in Fig.3. The dimensions for the concrete volume were shown in Table.3.

### F. Meshing

To obtain good results from the Solid65 element, the use of a rectangular mesh was recommended. Therefore, the mesh was set up such that square or rectangular elements were created. The meshing of the reinforcement was a special case compared to the volumes. No mesh of the reinforcement was needed because individual elements were created in the modeling through the nodes created by the mesh of the concrete volume. The meshing and reinforcement configuration of the beam were shown in Fig.3 and Fig.4.

### G. Loading and Boundary Conditions

Displacement boundary conditions were needed to constraint the model to get a unique solution. To ensure that the model acts the same way as the experimental beam boundary conditions need to be applied at points of symmetry, and where the supports and loading exist. The support was modeled as a hinged support at both ends. Nodes on the plate were given constraint in all directions, applied as constant values of zero. The loading and boundary conditions of the beam were shown in Fig.5.

### H. Cracking Patterns

The model was 700 mm long with a cross section of 150 mm X 350 mm. The Finite Element beam model was shown in Fig.3. The dimensions for the concrete volume were shown in Table.3.

**TABLE V. DIMENSIONS FOR CONCRETE**

<table>
<thead>
<tr>
<th>ANSYS</th>
<th>Concrete(mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>X1,X2,X-coordinates</td>
<td>0, 700</td>
</tr>
<tr>
<td>Y1,Y2,Y-coordinates</td>
<td>0, 350</td>
</tr>
<tr>
<td>Z1,Z2,Z-coordinates</td>
<td>0, 150</td>
</tr>
</tbody>
</table>
I. Deformed Shape

![Deformed Shape Image]

V. RESULTS & DISCUSSION

Design of deep beams by using IS 456-2000 for different Span to depth ratio the flexural steel required has nearly Same for beams. The deflection cracking and stress Distribution obtained after analysis is as below.

<table>
<thead>
<tr>
<th>Depth</th>
<th>400mm</th>
<th>375mm</th>
<th>350mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Span to depth ratio</td>
<td>1.5</td>
<td>1.6</td>
<td>1.71</td>
</tr>
<tr>
<td>Flexural steel required in</td>
<td>85.06</td>
<td>95.862</td>
<td>102.689</td>
</tr>
<tr>
<td>Flexural steel provided in</td>
<td>157</td>
<td>157</td>
<td>157</td>
</tr>
<tr>
<td>Load at first crack</td>
<td>172 2-10</td>
<td>164 2-10</td>
<td>153 2-10</td>
</tr>
<tr>
<td>Load at failure</td>
<td>303</td>
<td>289</td>
<td>267</td>
</tr>
<tr>
<td>Deflection at first crack,mm</td>
<td>0.557</td>
<td>0.538</td>
<td>0.434</td>
</tr>
<tr>
<td>Deflection at failure,mm</td>
<td>1.269</td>
<td>1.186</td>
<td>1.230</td>
</tr>
</tbody>
</table>

![Deflection of deep beam Image]

Fig. 7. Deformed Shape

![Stress distribution of deep beam Image]

Fig. 8. Stress distribution of deep beam

VI. CONCLUSION

It is well recognized that the exact analysis of concrete deep beams is a complex problem. This ease can be attributed to the use of computer programmes. In principle ANSYS has the capacity of idealizing any continuum into finer mesh which in turn enhances the results obtained, and with a high speed of operation. Starting from the literature survey regarding analysis of deep beams the different parameters that affect properties of deep beam were studied. After doing non linear finite element analysis of deep beam using ANSYS 13.0 following points to be concluded.

1. Deflection of beams increases as span to depth ratio decreases.
2. As span to depth ratio goes on decreasing the load at failure goes decreasing.

REFERENCES


