New Types of Fuzzy Topological Maps

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***Abstract*--In this paper, we introduce fuzzy $\ddot{g}$ -closed map and fuzzy $\ddot{g}$ -open map from a fuzzy topological space X to a fuzzy topological space Y. We also obtain some properties of fuzzy $\ddot{g}$ -closed and open maps.**

Key words and Phrases: Fuzzy Topological space, fuzzy $\ddot{g}$ -closed map, fuzzy $\ddot{g}$ *-closed map and fuzzy $\ddot{g}$ -open map, fuzzy $\ddot{g}$ *-open map.

1. INTRODUCTION

In the classical paper [19] of 1965, L. A. Zadeh generalized the usual notion of a set by introducing the important and useful notion of fuzzy sets. Subsequently many researchers have worked on various basic concepts from general topology using fuzzy sets and developed the theory of fuzzy topological spaces. The notion of fuzzy sets naturally plays a very significant role in the study of fuzzy topology introduced by C. L. Chang [5].

Fuzzy continuous functions is one of the main topics in fuzzy topology. Various authors introduce various types of fuzzy continuity. The decomposition of fuzzy continuity is one of the many problems in fuzzy topology. Tong [16] obtained a decomposition of fuzzy continuity by introducing two weak notions of fuzzy continuity namely, fuzzy strong semi-continuity and fuzzy precontinuity. Rajamani [9] obtained a decomposition of fuzzy continuity.

In this section, we introduce fuzzy $\ddot{g}$ -closed maps, fuzzy $\ddot{g}$ -open maps, fuzzy $\ddot{g}$ *-closed maps and fuzzy $\ddot{g}$ *-open maps in fuzzy topological spaces and obtain certain characterizations of these maps.

2. PRELIMINARIES

Throughout this paper, (X, τ), (Y, σ) and (Z, η) (or X, Y and Z) represent fuzzy topological spaces on which no separation axioms are assumed unless otherwise mentioned. For any fuzzy subset A of a space (X, τ), the closure of A, the interior of A and the complement of A are denoted by cl(A), int(A) and A' respectively.

We recall the following definitions which are useful in the sequel.

Definition 2.1 [14, 19]

If X is a set, then any function A: X→[0,1] (from X to the closed unit interval [0,1]) is called a fuzzy set in X.

Definition 2.2 [9]

If X is a set, then A, B : X →[0,1] are fuzzy sets in X.

(i) The complement of a fuzzy set A, denoted by A′, is defined by A′(x) =1–A(x), for all x ϵ X.

(ii) Union of two fuzzy sets A and B, denoted by A ∨ B, is defined by

$(A \vee B)(x) = \max \{A(x), B(x)\}$, for all x ϵ X.

(iii) Intersection of two fuzzy sets A and B, denoted by A ∧ B, is defined by

$(A \wedge B)(x) = \min \{A(x), B(x)\}$, for all x ϵ X.

Definition 2.3 [14]

Let A be a fuzzy set in a fts (X, τ). Then,

(i) the closure of A, denoted by cl(A), is defined by

$cl(A) = \bigwedge \{ F: A \leq F$ and F is a fuzzy closed $\}$;

(ii) the interior of A, denoted by int(A), is defined by

$int(A) = \bigvee \{G: G \leq A$ and G is a fuzzy open$\}$

Definition 2.4

A fuzzy subset A of a space (X, τ) is called:

fuzzy semi-open set [1] if A ≤ cl(int(A))

The complement of fuzzy semi-open set is fuzzy semi-closed.

The fuzzy semi-closure [18] of a fuzzy subset A of X, denoted by scl(A), is defined to be the intersection of all fuzzy semi-closed sets of (X, τ) containing A. It is known that scl(A) is a fuzzy semi-closed set.

Definition 2.5

A fuzzy subset A of a space (X, τ) is called:

(i) a fuzzy generalized closed (briefly fg-closed) set [2] if cl(A) ≤U whenever A ≤ U and U is fuzzy open in (X, τ). The complement of fg-closed set is called fg-open set;

(ii) a fuzzy ω-closed set (= fω-closed set)[13] if cl(A) ≤ U whenever A ≤ U and U is fuzzy semi-open in (X, τ). The complement of fuzzy ω-closed set is called fuzzy ω-open set;

(iii) a fuzzy semi-generalized closed (briefly fsg-closed) set [3] if scl(A) ≤ U whenever A ≤ U and U is fuzzy semi-open in (X, τ). The complement of fsg-closed set is called fsg-open set;

(iv) a generalized fuzzy semi-closed (briefly fgs-closed) set [11] if scl(A) ≤ U whenever A ≤ U and U is open in (X, τ). The complement of fgs-closed set is called fgs-open set;

(v) a fuzzy $\ddot{g}$-closed set [7] if cl(A) ≤ U whenever A ≤ U and U is fsg-open in (X, τ). The complement of fuzzy $\ddot{g}$-closed set is called fuzzy $\ddot{g}$-open set;

(vi) a fuzzy sg*-closed set [7] if scl(A) ≤ U whenever A ≤ U and U is fsg-open in (X, τ). The complement of fsg*-closed set is called fsg*-open set.

The collection of all fuzzy $\ddot{g}$-closed sets is denoted by F$\ddot{G}$C(X).

Remark 2.6[7]

Every fuzzy closed set is fuzzy $\ddot{g}$-closed set but not conversely.

Lemma 2.7[5]

Let f : (X, τ) → (Y, σ) be a fuzzy function. For fuzzy sets A and B of X and Y respectively, the following statements hold:

(i) $ff^{-1}(B) \leq B$;
(ii) $f^{-1}f(A) \geq A$;
(iii) $f(A') \geq (f(A))'$;
(iv) $f^{-1}(B') = (f^{-1}(B))'$;
(v) if f is injective, then $f^{-1}(f(A)) = A$;
(vi) if f is surjective, then $f f^{-1}(B) = B$;
(vii) if f is bijective, then $f(A') = (f(A))'$.

Definition 2.8

A fuzzy function f : (X, τ) → (Y, σ) is called

(i) fuzzy closed [5] if the image of every fuzzy closed set of X is a fuzzy closed in Y.

(ii) fuzzy open [5] if the image of every fuzzy open set of X is a fuzzy open in Y.

(iii) fuzzy continuous [5] if the inverse image of every fuzzy open set in (Y, σ) is fuzzy open set in (X, τ).

(iv) fuzzy ω-continuous [13] if the inverse image of every fuzzy close Set in (Y, σ) is fω-closed set in (X, τ).

3. FUZZY $\ddot{g}$-INTERIOR AND FUZZY $\ddot{g}$-CLOSURE

Definition 3.1

(i) For any fuzzy set A of X, fuzzy $\ddot{g}$-int(A) is defined as the union of all fuzzy $\ddot{g}$-open sets contained in A.

i.e., f$\ddot{g}$-int(A) =$\bigvee$\{G : G ≤ A and G is fuzzy $\ddot{g}$-open\}.

(ii) For every fuzzy set A of X, we define the fuzzy $\ddot{g}$-closure of A to be the intersection of all fuzzy $\ddot{g}$-closed sets containing A.

In symbols, f$\ddot{g}$-cl(A) = $\bigwedge$ \{F : A ≤ F ∊ F$\ddot{G}$C(X)\}.

Definition 3.2

Let (X, τ) be a fuzzy topological space. Let G be a fuzzy subset of X. Then G is called an fuzzy $\ddot{g}$-neighborhood of A (briefly, f$\ddot{g}$-nbhd of A) iff there exists an f$\ddot{g}$-open set U of X such that A< U < G.

Definition 3.3

A fuzzy topological space (X, τ) is called a

(i) T fω-space if every fω-closed set in it is fuzzy closed.

(ii) T f$\ddot{g}$-space if every f$\ddot{g}$-closed set in it is fuzzy closed.

Example 3.4

Let X = \{a, b\} with τ = \{0_x, λ, 1_x\} where λ is fuzzy set in X defined by λ(a)=0.4, λ(b)=0.5 . Then (X, τ) is a fuzzy topological space. Clearly (X, τ) is a T fω-space.

Example 3.5

Let X = \{a, b\} with τ = \{0_x, λ, 1_x\} where λ is fuzzy set in X defined by λ(a)=0.5, λ(b)=0.5 .Then (X, τ) is a fuzzy topological space. Clearly (X, τ) is a T f$\ddot{g}$-space.

Definition 3.6

A fuzzy map f : (X, τ) → (Y, σ) is called

(i) fuzzy f$\ddot{g}$-continuous[8] if the inverse image of every fuzzy closed set in (Y, σ) is f$\ddot{g}$-closed in (X, τ).

(ii) fuzzy f$\ddot{g}$-irresolute if the inverse image of every f$\ddot{g}$-closed set in (Y, σ) is f$\ddot{g}$-closed in (X, τ).

(iii) strongly fuzzy f $\ddot{g}$ -continuous if the inverse image of every f $\ddot{g}$ -open set in (Y, σ) is fuzzy open in (X, τ).

(iv) fuzzy sg-irresolute if $f^{-1}(V)$ is fsg-open in (X, τ) for every fsg-open subset V in (Y, σ).

Proposition 3.7

If A is fuzzy $\ddot{g}$ -open, then fuzzy $\ddot{g}$ -int(A) = A.

proof

Let A ≤ A and A is f $\ddot{g}$ -open in (X, τ). Hence by definition of f $\ddot{g}$ -int(A). Thus f $\ddot{g}$ -int(A) = A.

Remark 3.8

Converse of proposition 3.7 is not true as illustrated in the following example.

Example 3.9

Let X = {a, b} and α, β : X ⟶ [0,1] be defined by α (a) = 0.6, α (b) = 0 and β (a) = 0, β (b) = 0.3. Then (X, τ) is a fuzzy topological spaces with τ = {0_x, α, β, $\alpha \vee \beta$, 1_x}. For λ = (0.7, 0.3), f $\ddot{g}$ -int(A) = A. But not f $\ddot{g}$ -open in (X, τ).

Proposition 3.10

If A is fuzzy f $\ddot{g}$ -closed, then fuzzy f $\ddot{g}$ -cl(A) = A. But the converse is not true.

Proposition 3.11

For any two fuzzy subsets A and B of (X, τ), the following hold:

(i) If A ≤ B, then f $\ddot{g}$ -int(A) ≤ f $\ddot{g}$ -int(B).

(ii) f $\ddot{g}$ -int(A) ∨ f $\ddot{g}$ -int(B) ≤ f $\ddot{g}$ -int(A∨B).

proof

(i) Since A ≤ B, a f $\ddot{g}$ -open subset of a A is also a f $\ddot{g}$ -open subaet of B, we have f $\ddot{g}$ -int(A) ≤ f $\ddot{g}$ -int(B).

(ii) A ≤ A∨ B and B≤ A∨ B imply f $\ddot{g}$ -int(A) ≤ f $\ddot{g}$ -int(A∨B) and f $\ddot{g}$ -int(B) ≤ f $\ddot{g}$ -int(A∨B) by (i). Hence f $\ddot{g}$ -int(A) ∨ f $\ddot{g}$ -int(B) ≤ f $\ddot{g}$ -int(A∨B).

Proposition 3.12

For any two fuzzy subsets A and B of (X, τ), the following hold:

(i) If A ≤ B, then f $\ddot{g}$ -cl(A) ≤ f $\ddot{g}$ -cl(B).

(ii) f $\ddot{g}$ -cl(A ∧ B) ≤ f $\ddot{g}$ -cl(A) ∧ f $\ddot{g}$ -cl(B).

Definition 3.1

A fuzzy map f : (X, τ) → (Y, σ) is called:

(i) fg-closed if f(V) is fg-closed in (Y, σ) for every fuzzy closed set V of (X, τ).
(ii) fsg-closed if f(V) is fsg-closed in (Y, σ) for every fuzzy closed set V of (X, τ).
(iii) fgs-closed if f(V) is fgs-closed in (Y, σ) for every fuzzy closed set V of (X, τ).
(iv) fsg*-closed if f(V) is fsg*-closed in (Y, σ) for every fuzzy closed set V of (X, τ).

4. FUZZY $\ddot{g}$ -CLOSED MAPS

We introduce the following definition:

Definition 4.1

A fuzzy map f : (X, τ) → (Y, σ) is said to be fuzzy $\ddot{g}$ -closed if the image of every fuzzy closed set in (X, τ) is f $\ddot{g}$ -closed in (Y, σ).

Example 4.2

Let X = Y = {a, b} with τ = {0_x , α , 1_x } where α(a)= 0.5, α(b)= 0 and σ = {0_x , β, 1_x } where β(a) =1, β(b)= 0. Then (X, τ) and (Y, σ) are fuzzy topological spaces. Let f : (X, τ) → (Y, σ) be the identity map. Clearly f is an fuzzy $\ddot{g}$ -closed map.

Proposition 4.3
A fuzzy map f : (X, τ) → (Y, σ) is fuzzy $\ddot{g}$ -closed if and only if fuzzy $\ddot{g}$ -cl(f(A)) ≤ f(cl(A)) for every fuzzy subset A of (X, τ).

Proof
Suppose that f is fuzzy $\ddot{g}$ -closed and A ≤ X. Then cl(A) is fuzzy closed in X and so f(cl(A)) is fuzzy $\ddot{g}$ -closed in (Y, σ). We have f(A) ≤ f(cl(A)) and by Propositions 3.10 and 3.11, f $\ddot{g}$ -cl(f(A)) ≤ f $\ddot{g}$ -cl(f(cl(A))) = f(cl(A)).

Conversely, let A be any fuzzy closed set in (X, τ). Then A = cl(A) and so f(A) = f(cl(A)) ≥ f $\ddot{g}$ -cl(f(A)), by hypothesis. We have f(A) ≤ f $\ddot{g}$ -cl(f(A)) by Proposition 3.11. Therefore f(A) = f $\ddot{g}$ -cl(f(A)). i.e., f(A) is f $\ddot{g}$ -closed by Proposition 3.10 and hence f is f $\ddot{g}$ -closed.

Proposition 4.4

Let f : (X, τ) → (Y, σ) be a map such that f $\ddot{g}$ -cl(f(A)) ≤ f(cl(A)) for a fuzzy closed subset A of X. Then the image f(A) is f $\ddot{g}$ -closed in (Y, σ).

Proof

Let A be a fuzzy closed set in (X, τ). Then by hypothesis f $\ddot{g}$ -cl(f(A)) ≤ f(cl(A)) = f(A) and so f $\ddot{g}$ -cl(f(A)) = f(A). Therefore f(A) is f $\ddot{g}$ -closed in (Y, σ).

Theorem 4.5

A map f : (X, τ) → (Y, σ) is fuzzy $\ddot{g}$ -closed if and only if for each fuzzy subset S of (Y, σ) and each fuzzy open set U containing $f^{-1}(S)$ there is an fuzzy $\ddot{g}$ -open set V of (Y, σ) such that S ≤ V and $f^{-1}(V)$ ≤ U.

Proof

Suppose f is fuzzy $\ddot{g}$ -closed. A fuzzy subset S of Y and U be an fuzzy open set of (X, τ) such that $f^{-1}(S)$ ≤ U. Then V = (f(U′))′ is an fuzzy $\ddot{g}$ -open set containing S such that $f^{-1}(V)$ ≤ U.

For the converse, let F be a fuzzy closed set of (X, τ). Then $f^{-1}((f(F))')$ ≤ F′ and F′ is fuzzy open. By assumption, there exists an fuzzy $\ddot{g}$ -open set V in (Y, σ) such that (f(F))′ ≤ V and $f^{-1}(V)$ ≤ F′ and so F ≤ ($f^{-1}(V)$) ′. Hence V′ ≤ f(F) ≤ f(($f^{-1}(V)$) ′) ≤ V′ which implies f(F) = V′. Since V′ is fuzzy $\ddot{g}$ -closed, f(F) is f $\ddot{g}$ -closed and therefore f is f $\ddot{g}$ -closed.

Proposition 4.6

If f : (X, τ) → (Y, σ) is fuzzy sg-irresolute fuzzy $\ddot{g}$ -closed and A is an fuzzy $\ddot{g}$ -closed subset of (X, τ), then f(A) is fuzzy $\ddot{g}$ -closed in (Y, σ).

Proof

Let U be a fsg-open set in (Y, σ) such that f(A) ≤ U. Since f is fsg-irresolute, $f^{-1}(U)$ is a fsg-open set containing A. Hence cl(A) ≤ $f^{-1}(U)$ as A is fuzzy $\ddot{g}$ -closed in (X, τ). Since f is fuzzy $\ddot{g}$ -closed, f(cl(A)) is an fuzzy $\ddot{g}$ -closed set contained in the fsg-open set U, which implies that cl(f(cl(A))) ≤ U and hence cl(f(A)) ≤ U. Therefore, f(A) is an fuzzy $\ddot{g}$ -closed set in (Y, σ).

The following example shows that the composition of two fuzzy $\ddot{g}$ -closed maps need not be fuzzy $\ddot{g}$ -closed.

Example 4.7

Let X = Y = Z = {a, b} with τ = {0_x ,α , 1_x } where α(a)= 0.4, α(b)= 0 , σ = {0_x , β, 1_x } where β(a) =1, β(b)= 0 and η = {0_x , γ, β, 1_x } where γ(a) = 0.5, γ(b) = 0. Then (X, τ), (Y, σ) and (Z, η) are fuzzy topological spaces. Let f : (X, τ) → (Y, σ) be the identity map and g : (Y, σ) → (Z, η) be the identity fuzzy map. Clearly both f and g are fuzzy $\ddot{g}$ -closed maps but their composition g o f : (X, τ) → (Z, η) is not an fuzzy $\ddot{g}$ -closed map.

Corollary 4.8

Let f : (X, τ) → (Y, σ) be fuzzy $\ddot{g}$ -closed and g : (Y, σ) → (Z, η) be fuzzy $\ddot{g}$ -closed and fuzzy sg-irresolute, then their composition g o f : (X, τ) → (Z, η) is fuzzy $\ddot{g}$ -closed.

Proof

Let A be a fuzzy closed set of (X, τ). Then by hypothesis f(A) is an fuzzy $\ddot{g}$ -closed set in (Y, σ). Since g is both fuzzy $\ddot{g}$ -closed and fsg-fuzzy irresolute by Proposition 4.6, g(f(A)) = (g o f) (A) is fuzzy $\ddot{g}$ -closed in (Z, η) and therefore g o f is fuzzy $\ddot{g}$ -closed.

Proposition 4.9

Let f : (X, τ) → (Y, σ), g : (Y, σ) → (Z, η) be fuzzy $\ddot{g}$ -closed maps and (Y, σ) be a f $\ddot{g}$ -space. Then their composition g o f : (X, τ) → (Z, η) is fuzzy $\ddot{g}$ -closed.

Proof

Let A be a fuzzy closed set of (X, τ). Then by assumption f(A) is fuzzy $\ddot{g}$ -closed in (Y, σ). Since (Y, σ) is a fuzzy T $\ddot{g}$ -space, f(A) is fuzzy closed in (Y, σ) and again by assumption g(f(A)) is fuzzy $\ddot{g}$ -closed in (Z, η). i.e., (g o f) (A) is fuzzy $\ddot{g}$ -closed in (Z, η) and so g o f is fuzzy $\ddot{g}$ -closed.

Proposition 4.10

If f : (X, τ) → (Y, σ) is fuzzy $\ddot{g}$ -closed, g : (Y, σ) → (Z, η) is fuzzy $\ddot{g}$ -closed (resp. fg-closed, fsg*-closed, fsg-closed and fgs-closed) and (Y, σ) is a T f $\ddot{g}$ -space, then their composition g o f : (X, τ) → (Z, η) is fuzzy $\ddot{g}$ -closed (resp. fg-closed, fsg*-closed, fsg-closed and fgs-closed).

Proof

Similar to Proposition 4.9.

Proposition 4.11

Let f : (X, τ) → (Y, σ) be a fuzzy closed map and g : (Y, σ) → (Z, η) be an fuzzy $\ddot{g}$ -closed map, then their composition g o f : (X, τ) → (Z, η) is fuzzy $\ddot{g}$ -closed.

Proof

Similar to Proposition 4.9.

Remark 4.12

If f : (X, τ) → (Y, σ) is a fuzzy $\ddot{g}$ -closed and g : (Y, σ) → (Z, η) is fuzzy closed, then their composition need not be an fuzzy $\ddot{g}$ -closed map as seen from the following example.

Example 4.13

Let X = Y = Z = {a, b} with $\tau = \{0_x, \alpha, 1_x\}$ where α(a)= 0.4, α(b)= 0 , $\sigma = \{0_x, \beta, 1_x\}$ where β(a) =1, β(b)= 0 and $\eta = \{0_x, \gamma, , 1_x\}$ where γ(a) = 0.5, γ(b) = 0. Then (X, τ), (Y, σ) and (Z, η) are fuzzy topological spaces. Let f : (X, τ) → (Y, σ) be the identity map and g : (Y, σ) → (Z, η) be the identity fuzzy map. Clearly both f is a fuzzy $\ddot{g}$ -closed map and g is fuzzy closed map but their composition
g o f : (X, τ) → (Z, η) is not an fuzzy $\ddot{g}$ -closed map.

Definition 4.14

A map f : (X, τ) → (Y, σ) is said to be an fuzzy $\ddot{g}$ -open map if the image f(A) is fuzzy $\ddot{g}$ -open in (Y, σ) for each fuzzy open set A in (X, τ).

Proposition 4.15

For any bijection f : (X, τ) → (Y, σ), the following statements are equivalent:

(i) f^{-1} : (Y, σ) → (X, τ) is fuzzy $\ddot{g}$ -continuous.

(ii) f is fuzzy $\ddot{g}$ -open map.

(iii) f is fuzzy $\ddot{g}$ -closed map.

Proof

(i) ⇒ (ii). Let U be an fuzzy open set of (X, τ). By assumption, $(f^{-1})^{-1}$(U) = f(U) is fuzzy $\ddot{g}$ -open in (Y, σ) and so f is fuzzy $\ddot{g}$ -open.

(ii) ⇒ (iii). Let F be a fuzzy closed set of (X, τ). Then F^c is fuzzy open set in (X, τ). By assumption, f(F′) is fuzzy $\ddot{g}$ -open in (Y, σ). That is f(F′) = (f(F)) ′ is fuzzy $\ddot{g}$ -open in (Y, σ) and therefore f(F) is fuzzy $\ddot{g}$ -closed in (Y, σ). Hence f is fuzzy $\ddot{g}$ -closed.

(iii) ⇒ (i). Let F be a fuzzy closed set of (X, τ). By assumption, f(F) is fuzzy $\ddot{g}$ -closed in (Y, σ). But f(F) = $(f^{-1})^{-1}$(F) and therefore f^{-1} is fuzzy $\ddot{g}$ -continuous.

In the next two theorems, we obtain various characterizations of fuzzy $\ddot{g}$ -open maps.

Theorem 4.16

Let f : (X, τ) → (Y, σ) be a map. Then the following statements are equivalent:

(i) f is an fuzzy $\ddot{g}$ -open map.

(ii) For a fuzzy subset A of (X, τ), f(int(A)) ≤f $\ddot{g}$ -int(f(A)).

(iii) For each fuzzy set A and for each neighborhood U of A in (X, τ), there exists a fuzzy $\ddot{g}$ -neighborhood W of f(A) in (Y, σ) such that W < f(U).

Proof

(i) ⇒ (ii). Suppose f is fuzzy $\ddot{g}$ -open. Let A ≤ X. Then f int(A) is fuzzy open in (X, τ) and so f(int(A)) is f $\ddot{g}$ -open in (Y, σ). We have f(int(A)) ≤ f(A). Therefore by Proposition 3.7, f(int(A)) ≤ f $\ddot{g}$ -int(f(A)).

(ii) ⇒ (iii). Suppose (ii) holds. Let A be a fuzzy set and U be an arbitrary neighborhood of A in (X, τ). Then there exists a fuzzy open set G such that A ≤ G ≤ U. By assumption, f(G) = f(int(G)) ≤ f $\ddot{g}$ -int(f(G)). This implies f(G) = f $\ddot{g}$ -int(f(G)). By Proposition 3.7, we have f(G) is fuzzy $\ddot{g}$ -open in (Y, σ). Further, f(A) ≤ f(G) ≤ f(U) and so (iii) holds, by taking W = f(G).

(iii) ⇒ (i). Suppose (iii) holds. Let U be any fuzzy open set in (X, τ), A ≤ U and f(A) = B. Then B ≤ f(U) and for each B ≤ f(U), by assumption there exists an f $\ddot{g}$ -neighborhood W of B in (Y, σ) such that W ≤ f(U). Since W is a f $\ddot{g}$ -neighborhood of B, there exists an f $\ddot{g}$ -open set V in (Y, σ) such that B ≤ V ≤ W. Therefore, f(U) = ∨ {V : B ≤ f(U)} is a f $\ddot{g}$ -open set in (Y, σ) by proposition 3.11. Thus f is an f $\ddot{g}$ -open map.

Theorem 4.17

A map f : (X, τ) → (Y, σ) is f $\ddot{g}$ -open if and only if for any fuzzy subset S of (Y, σ) and for any fuzzy closed set F containing $f^{-1}(S)$, there exists an f$\ddot{g}$-closed set K of (Y, σ) containing S such that $f^{-1}(K) \leq F$.

Proof

Similar to Theorem 4.5.

Corollary 4.18

A map f : (X, τ) → (Y, σ) is fuzzy $\ddot{g}$-open if and only if $f^{-1}(\ddot{g}\text{-cl}(B)) \leq \text{cl}(f^{-1}(B))$ for each fuzzy subset B of (Y, σ).

Proof

Suppose that f is fuzzy $\ddot{g}$-open. Then for any fuzzy subset B of (Y, σ), $f^{-1}(B) \leq \text{cl}(f^{-1}(B))$. By Theorem 4.18, there exists a fuzzy $\ddot{g}$-closed set K of (Y, σ) such that $B \leq K$ and $f^{-1}(K) \leq \text{cl}(f^{-1}(B))$. Therefore, $f^{-1}(\ddot{g}\text{-cl}(B)) \leq (f^{-1}(K)) \leq \text{cl}(f^{-1}(B))$, since K is an fuzzy $\ddot{g}$-closed set in (Y, σ).

Conversely, let S be any fuzzy subset of (Y, σ) and F be any fuzzy closed set containing $f^{-1}(S)$. Put K = $\ddot{g}$-cl(S). Then K is an fuzzy $\ddot{g}$-closed set and $S \leq K$. By assumption, $f^{-1}(K) = f^{-1}(\ddot{g}\text{-cl}(S)) \leq \text{cl}(f^{-1}(S)) \leq F$ and therefore by Theorem 4.18, f is fuzzy $\ddot{g}$-open.

Finally in this section, we define another new class of maps called f$\ddot{g}$*-closed maps which are stronger than f$\ddot{g}$ closed maps.

Definition 4.19

A map f : (X, τ) → (Y, σ) is said to be f$\ddot{g}$*-closed if the image f(A) is fuzzy $\ddot{g}$-closed in (Y, σ) for every fuzzy $\ddot{g}$-closed set A in (X, τ).

For example the map f in Example 4.2 is an f$\ddot{g}$*-closed map.

Remark 4.20

Since every fuzzy closed set is an fuzzy $\ddot{g}$-closed set we have f$\ddot{g}$*-closed map is an f$\ddot{g}$-closed map. The converse is not true in general as seen from the following example.

Example 4.21

Let X = Y = {a, b} with τ = {0_x, α, 1_x } where α(a) =1, α(b)= 0 and σ = {0_x, β, α, 1_x} where β(a) =0.5, β(b)= 0 . Then (X, τ) and (Y, σ) are fuzzy topological spaces. Let f : (X, τ) → (Y, σ) be the identity map. Then f is a f$\ddot{g}$-closed but not f$\ddot{g}$*-closed map.

Proposition 4.22

A map f : (X, τ) → (Y, σ) is f$\ddot{g}$*-open if and only if $\ddot{g}$-cl(f(A)) ≤ f($\ddot{g}$-cl(A)) for every fuzzy subset A of (X, τ).

Proof
Similar to Proposition 4.3.

Analogous to f$\ddot{g}$*-closed map we can also define f$\ddot{g}$*-open map.

Proposition 4.23

For any bijection f : (X, τ) → (Y, σ), the following statements are equivalent:

(i) f^{-1} : (Y, σ) → (X, τ) is fuzzy $\ddot{g}$-irresolute.

(ii) f is f$\ddot{g}$*-open map.

(iii) f is f$\ddot{g}$*-closed map.

Proof

Similar to Proposition 4.16.

Proposition 4.25

If f : (X, τ) → (Y, σ) is fgs-irresolute and f$\ddot{g}$-closed, then it is an f$\ddot{g}$*-closed map.

Proof

The Proof follows from Proposition 4.6.

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