

New PI Controller Design and Applications

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Abstract— This paper proposes a new PI controller design using the “product form”. The method is called General Method. It offers more possibilities to design the controller. One detailed application is on the DC Motor with Permanent Magnet. On this subject, some new considerations are taken into account. Simulation results show that all propositions are realizable and leads on good performances.

Keywords—PI controller, design, optimization, DC motor.

I. INTRODUCTION

It is well-known that the method of Ziegler and Nichols [1] constitutes the bread and butter of PID tuning. It can be said that PID controller is widely used in all areas where control is needed and applied. Currently, several design methods are given for this controller to improve or to bring optimization of the system performance, especially for the PI controller [2], [3], [4], [5]. A Variable Gain PI (VGPI) using “sum form” is introduced in [6] and [7]. It uses an entire degree n for the gains K_p and K_i . In [8], a non-entire degree is proposed and applied on AC-DC converter with power factor correction. In [9], the combination with Fuzzy Logic and VGPI is given.

In this paper, new PI controller design is presented and especially for linear system. An application for first order system with time delay is also given. First, theory about the General Method (GM) is showed. Some examples are chosen to support the reasoning. Finally, a speed control of a DC motor with Permanent Magnet is taken. Some proposals are advanced to reduce the inrush current.

II. NEW PI CONTROLLER DESIGN

Three forms of the transfer functions of PI controller are often used, one “product form” and two “sum forms” as shown in (1)

$$\begin{cases} G_R(p) = g \left(1 + \frac{1}{pT_i} \right) \\ G_R(p) = K_p + \frac{K_i}{p} \\ G_R(p) = \frac{1 + pT_n}{pT_i} \end{cases} \quad (1)$$

Where g is the gain of the controller, T_i the integral constant time, K_p and K_i represent respectively the proportional and integral constants and T_n is the proportioning integral time.

The last expression in (1) is the “product form”. Having one of these expressions allows to pass to the others. It may be noted that the integral constant times T_i in the first and the third expression have not the same value.

A. Case of a First Order System

The flat and symmetrical criteria (FC, SC) [10] are commonly used in electric machine drives. Their design rest especially with the little constant time of the system. But these criteria are at fault with a first order system. The transfer function (TF) is:

$$G(p) = \frac{K}{1 + pT} \quad (2)$$

Where K is the static gain, T the constant time.

- Using the “sum form”

The second expression of (1) gives the TF:

$$G_R(p) = K_p + \frac{K_i}{p} \quad (3)$$

Then, the transfer function in open loop (FTOL) is:

$$G_o(p) = \left(K_p + \frac{K_i}{p} \right) \cdot \frac{K}{1 + pT_i} \quad (4)$$

The goal is to find the gains K_p and K_i . There are two possibilities:

- Compensating the constant time T and imposing a constant time T_f in closed loop. It gives,

$$\begin{cases} K_p = \frac{T}{K T_f} \\ K_i = \frac{K_p}{T} \end{cases} \quad (5)$$

- b. The constant time T is not compensated. The damping factor ζ and the non-deadened natural pulsation ω_n are imposed. Then,

$$\begin{cases} K_i = \omega_n \sqrt{\frac{T}{K}} \\ K_p = \frac{2\zeta\omega_n T - 1}{K} \end{cases} \quad (6)$$

If positive constants are wanted, (7) must be respected.

$$T > \frac{1}{2\zeta\omega_n} \quad (7)$$

It will see as follow that the GM generalizes these two possibilities.

The adopted form of the PI controller is the third expression of (1):

$$G_R(p) = \frac{1 + pT_n}{pT_i} \quad (8)$$

By the GM, T_n and T_i are given as follow:

$$\begin{cases} T_n = aT \\ T_i = b.K.T \end{cases} \quad (9)$$

With $a \geq 0$ and $b > 0$

The TFOL is then,

$$G_o(p) = \frac{1 + paT}{pbKT} \cdot \frac{K}{1 + pT} = \frac{1 + paT}{pbT} \cdot \frac{1}{1 + pT} \quad (10)$$

According the value of a , three possibilities are presented.

- $a = 1$

It means that the constant time T is compensated. The TFOL $G_o(p)$ and the transfer function in closed loop (TFCL) $H(p)$ are both first order.

$$G_o(p) = \frac{1}{pbT} \quad (11)$$

And,

$$H(p) = \frac{1}{1 + pbT} \quad (12)$$

It is clear here that the constant b determines the speed of the step response as shown in Fig. 1. In this example,

$K = 2$ and $T = 1$ [s].

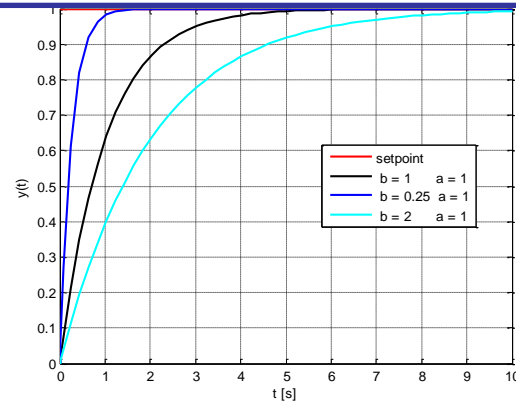


Fig. 1. Step response with different values of the constant b .

The more the constant b decreases, the more the response is fast.

- $a = 0$

The controller is not truly a PI one but has an effect I.

$$G_R(p) = \frac{1}{pbT} \quad (13)$$

The TFOL is then,

$$G_o(p) = \frac{1}{pbT} \cdot \frac{1}{1 + pT} \quad (14)$$

The characteristic equation in closed loop is a second order polynomial:

$$d_c(p) = p^2 + p \frac{1}{T} + \frac{1}{bT^2} = 0 \quad (15)$$

The canonic form, $dc^*(p)$ of the characteristic equation is,

$$d_c^*(p) = p^2 + 2\zeta\omega_n p + \omega_n^2 = 0 \quad (16)$$

With ζ the damping factor and ω_n the non-deadened natural pulsation, (17).

By comparing (15) and (17),

$$\begin{cases} 2\zeta\omega_n = \frac{1}{T} \\ \omega_n^2 = \frac{1}{bT^2} \end{cases} \quad (17)$$

Relation (17) allows to obtain ζ and ω_n . For instance,

$$\zeta = \frac{\sqrt{2}}{2} \Rightarrow b = 2. \quad (18)$$

- $a > 0$

In this case, it is assumed that $a \neq 1$. The TFOL is,

$$G_o(p) = \frac{1 + paT}{pbT(1 + pT)} \quad (19)$$

This TF has one zero z_0 and two poles p_1 and p_2 .

$$\begin{cases} z_o = -\frac{1}{aT} \\ p_1 = 0; p_2 = -\frac{1}{T} \end{cases} \quad (20)$$

Fig.2 and Fig.3 show the root locus map for different values of a with $K = 2$ and $T = 1$ [s].

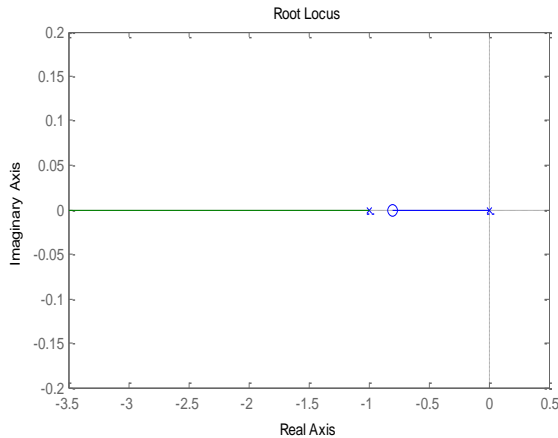


Fig. 2. Root locus with $a > 1$.

It is highlighted that the case in Fig.3 will generate an oscillatory deadened response and the case in Fig.2, an aperiodic response one.

By (19), the TFCL is,

$$H(p) = \frac{1 + paT}{p^2bT^2 + pT(a+b) + 1} \quad (21)$$

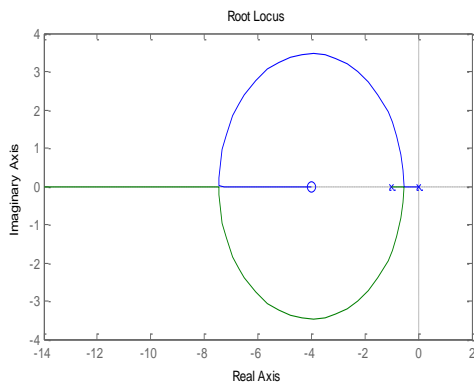


Fig. 3. Root locus with $0 < a < 1$

The characteristic equation is given by (22).

$$d_c(p) = p^2 + p \cdot \frac{(a+b)}{bT} + \frac{1}{bT^2} = 0 \quad (22)$$

In comparison with (16),

$$\begin{cases} 2\zeta\omega_n = \frac{a+b}{bT} \\ \omega_n^2 = \frac{1}{bT^2} \end{cases} \quad (23)$$

By the second expression of (23), it can be noticed that the step response speed depends of the value of the constant b . Fig. 4 shows step responses with different values of a .

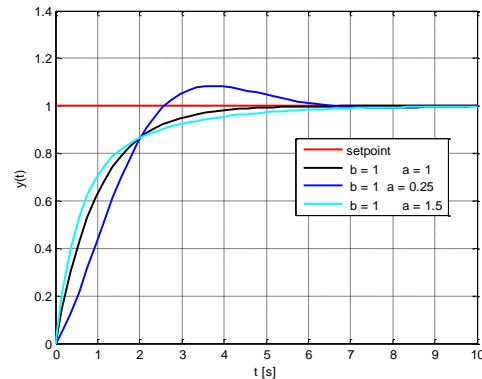


Fig. 4. Step responses according to a .

The more a decreases, the more the overshoot increases. It may be noted, that changing the value of the constant b gives more possibilities.

Fig.5 presents the effect of varying b with a fixed. When b decreases, the response is fast and the overshoot increases. When $b > 1$, the response becomes aperiodic. It can be said that there are several possibilities for the combinations.

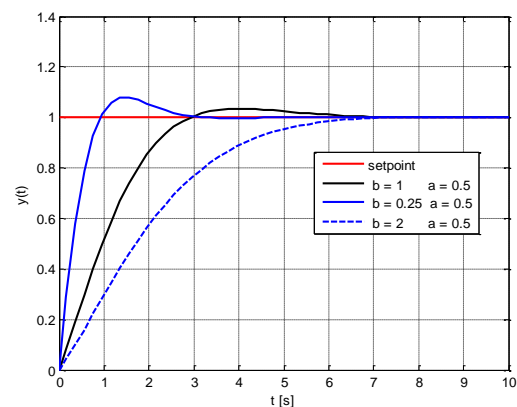


Fig.5. Step responses according to b with a fixed.

B. First Order System with Time Delay

The TF is here,

$$G(p) = K \frac{e^{-pT_o}}{1 + pT} \quad (24)$$

Where, T_o is the time delay, K the gain and T the constant time.

If $T_o \ll T$, the TF can be approximated as,

$$G(p) \approx \frac{K}{(1 + pT_o)(1 + pT)} \quad (25)$$

Fig. 6 and Fig. 7 show the simulation results obtained by (19) and (20) using the flat criteria (FC) [10] and GM.

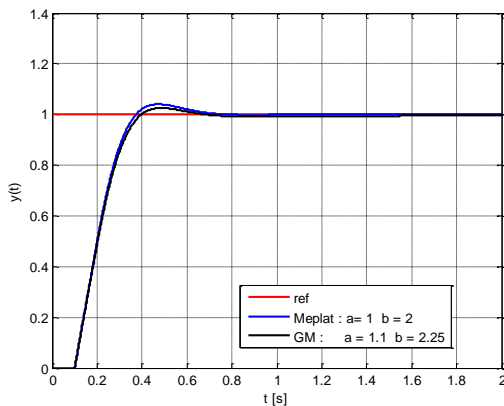


Fig. 6. Simulation results with PI controllers using (24)

By Fig.6 and Fig.7, it is highlighted that GM gives more possibilities and can improve performances obtained by the FC

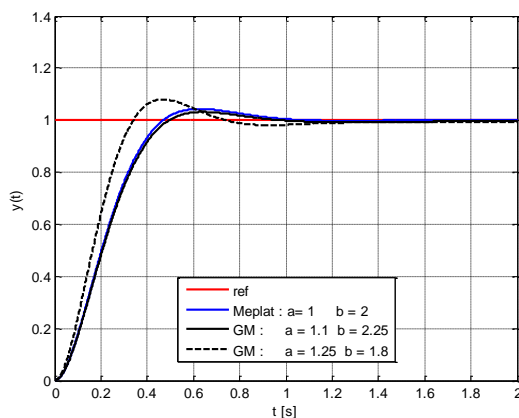


Fig. 7. Step responses with PI controllers using (25)

Putting aside the use of Smith predictor, [11], [12] propose a PI controller design using the pole dominant method (POLDOM) in the domain frequency and taking into account T_o .

C. System with Integral Behavior

For one application, a system defined by the TF given in (26) is chosen.

$$G(p) = \frac{K}{pT_i(1+pT_p)} \quad (26)$$

For this kind of system [10] recommends that the FC is not applicable. The PI controllers obtained by symmetric criteria (SC) and GM design are applied.

$$\text{SC:} \quad \begin{cases} T_n = 4T_p \\ T_i = 8K \frac{T_p^2}{T_1} \end{cases} \quad (27)$$

$$\text{GM:} \quad \begin{cases} T_n = aT_1 \\ T_i = bKT_p \end{cases} \quad (28)$$

Fig. 8 shows the simulation results obtained by the two methods.

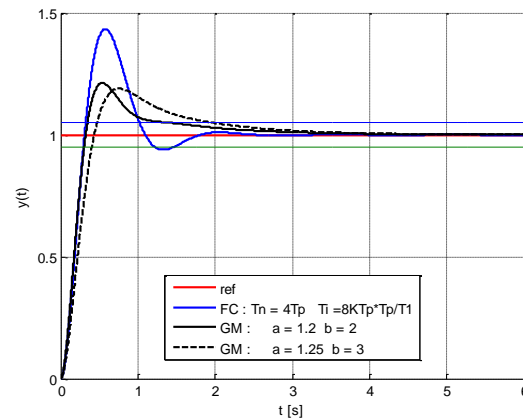


Fig. 8. Step responses by SC and GM

The characteristics of the responses are resumed in Table 1.

TABLE I CHARACTERISTICS

	Step response characteristics		
	PI - SC	PI- GM $a = 1,2 \quad b = 2$	PI- GM $a = 1,25 \quad b = 3$
D_I	43,3 %	21,2 %	11,87 %
tp	0,59 [s]	0,55 [s]	0,74 [s]
Tr ($\pm 5\%$)	1,53 [s]	1,38 [s]	1,98 [s]

D_I : overshoot tp : peak time Tr settling time for $\pm 5\%$

It is here highlighted that the SC leads to a high overshoot. For the GM, the more b increases, the more $D_I\%$ decreases but the response becomes slower. In anytime, the GM presents better performances and permits more possibilities.

III. APPLICATION ON DC MOTOR SPEED CONTROL

The DC Motor is with Permanent Magnet one. The speed control of the motor needs two loops: an inner loop constituted by the current loop and the principal loop which is the speed loop. The PI control design needs the modeling of the system. The motor provided with an inertial load and a viscous friction is taken into account.

Such system is rather current and presents also a teaching interest and industrial applications. Fig. 9 shows the system.

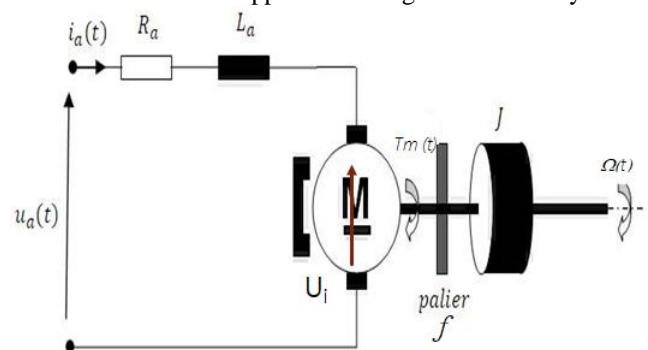


Fig. 9. The DC-Motor with its load

With R_a , L_a the resistance and inductance armatures, T_m the motor torque, f the constant friction, J the inertia and U_i the back electromotive force.

The equations governing the system are as follows:

$$\begin{cases} u_a(t) = R_a i_a(t) + L_a \frac{di_a(t)}{dt} + u_i(t) \\ T_{em}(t) = K_T i_a(t) \\ u_i(t) = K_V \Omega(t) \end{cases} \quad (29)$$

With K_V , the speed constant and K_T the constant torque

It is here assumed that K_V and K_T are the same,

$$K_V = K_T \quad (30)$$

The motion equation is:

$$T_{em}(t) - T_r(t) = J \frac{d\Omega(t)}{dt} + f\Omega(t) \quad (31)$$

Where, T_r is a resistive torque.

Using the Laplace's transformation, these equations are given by (32) and (33):

$$\begin{cases} U_a(p) - U_i(p) = [R_a + pL_a] I_a(p) \\ T_{em}(p) = K_T I_a(p) \\ U_i(p) = K_V \Omega(p) \end{cases} \quad (32)$$

And,

$$T_{em}(p) - T_r(p) = [Jp + f] \Omega(p) \quad (33)$$

For the speed control, a cascade scheme is adopted which contains two loops: the current loop and the speed one. First, the inner loop, which is the current loop is studied then the speed loop. The technical diagram is showed in Fig. 10.

Where 1: DC-Motor, 2: buck DC converter, 3: control unit, 4: Current PI controller, 5: current sensor, 6: current controller, 7: speed sensor, 8: setpoint circuit, 9: Permanent

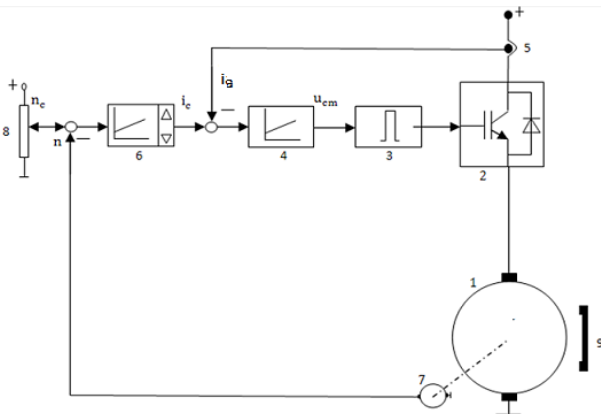


Fig. 10. Technical diagram of the complete system

The Motor Organ Command (MOC) is constituted by the control unit, the buck DC converter. It is defined by the TF,

$$G_{cm}(p) = \frac{K_{cm}}{1 + pT_{cm}} \quad (34)$$

Several cases will be taken into account by the consideration of this MOC. Fig. 11 shows the functional diagram.

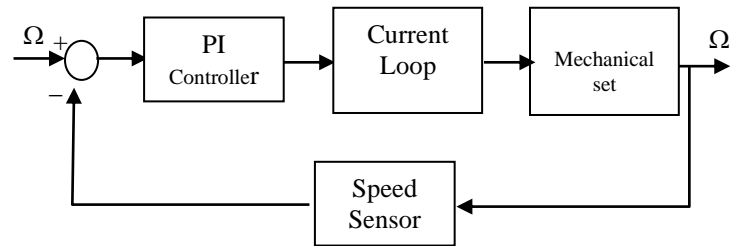


Fig. 11. Functional diagram

As already said, the current loop will be studied first then the speed loop will be analyzed.

A. Current Loop Analysis

The current loop diagram is given by Fig. 12.

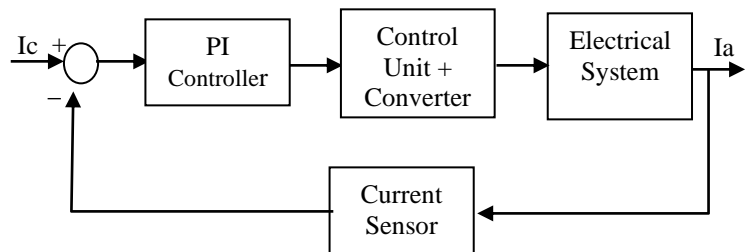


Fig. 11. Current loop diagram

Equation (34) gives the TF of the MOC. For the electrical set,

$$G_i(p) = \frac{1}{R_a + pL_a} \quad (35)$$

The current sensor is assumed to be ideal. Its TF is,

$$G_{cs}(p) = 1 \quad (36)$$

Relation (35) can be written as,

$$G_i(p) = \frac{K_a}{1 + pT_a} \quad (37)$$

With,

$$\begin{cases} K_a = \frac{1}{R_a} \\ T_a = \frac{L_a}{R_a} \end{cases} \quad (38)$$

Relation (8) is for the PI controller. The TFOL of the electrical set with the MOC is,

$$G_{ci}(p) = \frac{K_1}{(1 + pT_{cm})(1 + pT_a)} \quad (39)$$

With, $K_1 = K_{cm} \cdot K_a$.

Thus, the TFOL with the PI controller is given by (40).

$$G_{oi}(p) = \frac{1 + pT_n}{pT_i} \cdot \frac{K_1}{(1 + pT_{cm})(1 + pT_a)} \quad (40)$$

Several cases will be considered according the MOC TF.

- The little constant time T_{cm} is taken into account
FC and GM are applied.

$$\text{FC:} \quad \begin{cases} T_n = T_a \\ T_i = 2 \cdot K_1 T_{cm} \end{cases} \quad (41)$$

$$\text{GM:} \quad \begin{cases} T_n = a \cdot T_a \\ T_i = b \cdot K_1 T_{cm} \end{cases} \quad (42)$$

Here, $T_{cm} \ll T_a$.

Fig. 12 shows the simulation results of these cases.

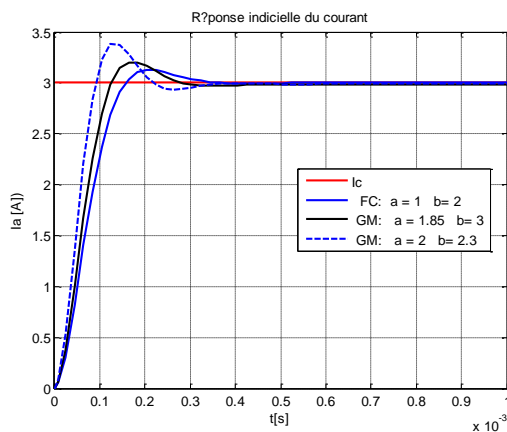


Fig. 12. Current curves with FC and GM

By combination of (a, b) , GM offers more possibilities. Generally, the overshoot increases with a .

- The MOC is assumed to be ideal

In this cas, T_{cm} is not taken into account. Then, the MOC TF is as,

$$G_{cm}(p) = K_{cm} \quad (43)$$

The TFOL becomes,

$$G_{ci}(p) = \frac{K_1}{1 + pT_a} \quad (44)$$

K_1 is always given by (39).

SC or FC are at fault. The PI controller can be only given by GM. The reasoning is exactly in section II-A.

For the PI controller, T_n and T_i are obtained by (9).

The speed responses is dictated by the constant time T_a . Fig. 13 shows the simulations results with different combinations of (a, b) .

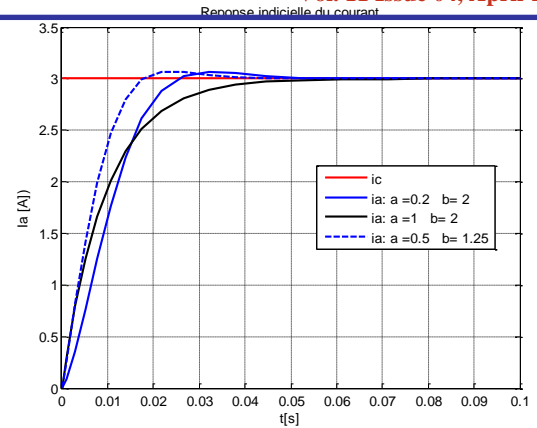


Fig. 13. Current curves according (a, b) .

The responses are largely slower than the system with the little constant time T_{cm} . For the speed loop, the couple $(a=0,5; b=1,25)$ will be chosen as an application.

- Compensating the little time -constant T_{cm} .

In [10], it is formally recommended not to compensate the little constant-time by the reason that noises in high frequencies are badly known. However, an equivalent constant-time is proposed.

$$T_p = \sum_{j=1}^{n_p} T_{pj} \quad (45)$$

Where T_{pj} is a small constant-time.

Then, it can be posed,

$$\begin{cases} T_n = T_p \\ T_i = b K_1 T_a \end{cases} \quad (46)$$

In this case, $T_p = T_{cm}$.

The FTOL with the PI controller is,

$$G_{oi}(p) = \frac{1}{pbT_a(1 + pT_a)} \quad (47)$$

Simulation results are given in Fig. 14.

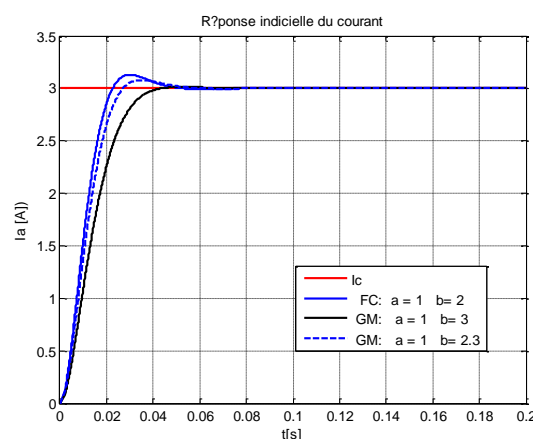


Fig. 14. Current curves with different values of b ; $(a=1)$

With the same value of a , the speed response is slower when b increases.

B. Speed loop analysis

The mechanical set is defined by the TF as below,

$$G_m(p) = \frac{1}{Jp + f} = \frac{K_m}{1 + pT_m} \quad (48)$$

With,

$$\begin{cases} K_m = \frac{1}{f} \\ T_m = \frac{J}{f} \end{cases} \quad (49)$$

Fig. 11 shows the general functional diagram. The speed loop is analyzed according to the three cases:

- Case 1: T_{cm} is taken into account

Using (38), and (41), the TFLO with $a = 1$ is,

$$G_{oi}(p) = \frac{1}{pbT_{cm}(1 + pT_{cm})} \quad (50)$$

Here (41) is a particular case with $b = 2$. Then, the TFCL is,

$$H_i(p) = \frac{1}{pbT_{cm}^2 + pbT_{cm} + 1} \quad (51)$$

Because T_{cm} has a very small value ($T_{cm} = 33,3 [\mu s]$), it be assumed that,

$$H_i(p) \approx \frac{1}{pbT_{cm} + 1} \quad (52)$$

Fig. 15 gives the speed curves obtained with FC and GM.

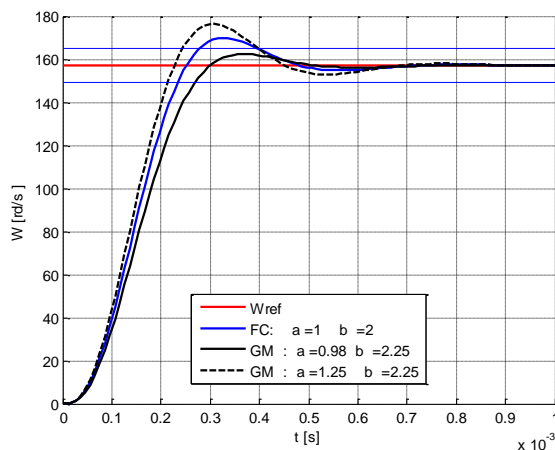


Fig. 15. Speed responses with FC and GM

The speed responses are very fast. The characteristics are resumed in Table II.

TABLE II. CHARACTERISC RESPONSES

	PI FC	PI GM $a = 1,25 \quad b = 2,25$	PI GM $a = 0,98 \quad b = 2,25$
D1 overshoot	8,15 %	12,28 %	3,37 %
tp Peak-time	0,34 [ms]	0,3 [ms]	0,36 [ms]
tr ($\pm 5\%$)	0,4 [ms]	0,4 [ms]	0,26 [ms]

With a fixed value of b , the overshoot decreases when the constant a decreases by GM. Current curves in closed loop are showed in Fig. 16

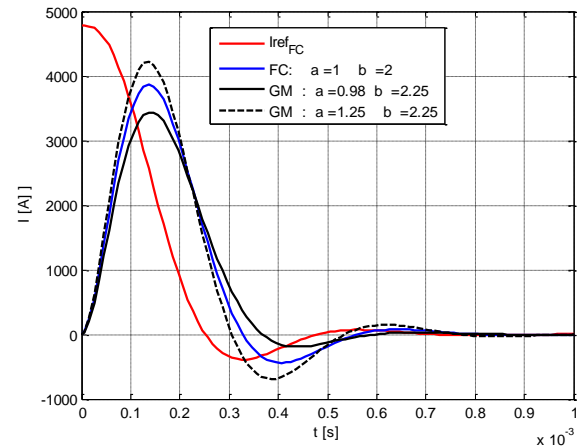


Fig. 16. Currents in speed closed loop

It is here highlighted that the inrush currents are too high.

Even the duration is very short, the values are not realistic. It is due by taking into account the little constant-time T_{cm} which needs a fast control. Inserting a filter at the setpoint constitutes a solution but in this case inrush current is always high. Two methods to reduce this inrush current are now proposed.

- Case 2: The MOC is assumed as ideal

Relation (43) gives the FT of the MOC. It is a simple gain and this method is already a usual one. For the current loop, FC and SC are in fault but the GM can be applied. Fig. 17 and Fig. 18 give respectively speed curves and currents in closed loop.

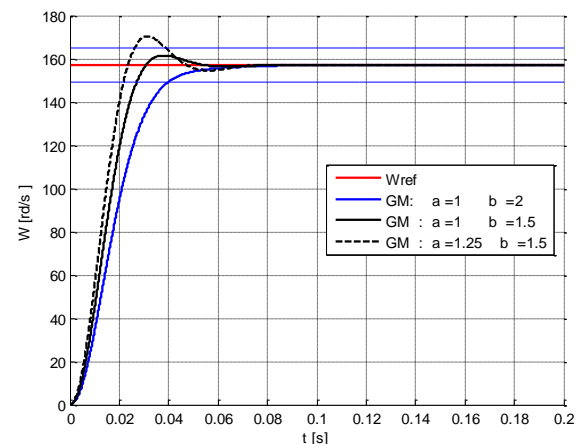


Fig. 17 Speed curves with GM

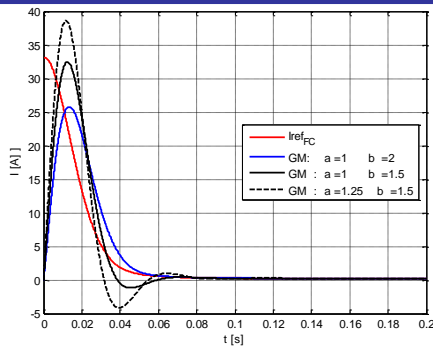


Fig. 18. Currents in closed loop with GM

The inrush currents decrease considerably but the control becomes slower. The constant times in question are only the electrical and mechanical constant times T_a and T_m .

• Case 3: Compensation of T_{cm}

In this case, PI controller parameters are designed as follows in current loop,

$$\begin{cases} T_{ni} = T_{cm} \\ T_{ii} = bKT_a \end{cases} \quad (53)$$

For the speed loop, since $T_a \ll T_m$, it may be assumed,

$$\begin{cases} T_{ni} = T_m \\ T_{ii} = b_1 K_1 T_p \end{cases} \quad (54)$$

With, $T_p = b.T_a$.

Fig. 19 and Fig.20 show speed responses and current curves in closed loop.

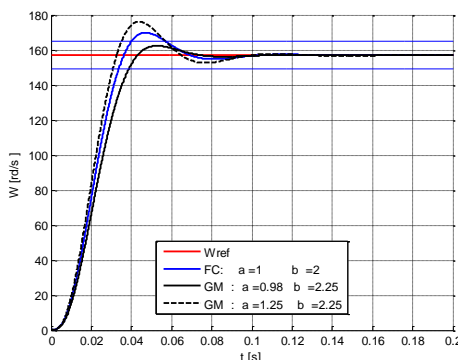


Fig. 19. Speed responses with FC and GM

It can be noticed that the inrush currents leads to better performances by the combination of a and b .

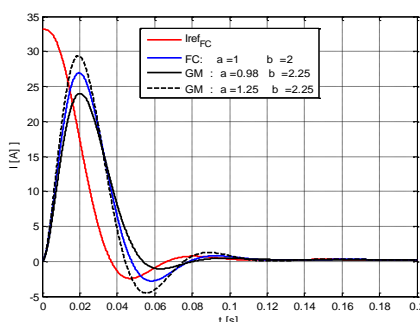


Fig. 20. Currents with FC and GM in closed loop

IV. CONCLUSION

In this paper, new PI controller design is proposed. It is called General Method (GM) which uses the “product form”. It is seen that GM offers more possibilities and can be applied on first order system. In another way, it can improve FC and SC. The application on DC Motor with Permanent Magnet shows that taking into account the little constant time of the MOC leads to a very high inrush current. Simulation shows that compensating this little constant time is one possibility to reduce this inrush current. But, it may be said that considering the MOC as ideal and defined as a simple gain is the better solution.

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DC-Motor parameters: $P_n = 440$ [W]; $N_n = 1500$ [tr/mn]; $I_n = 3$ [A]; $U_n = 175$ [V]; $T_{em} = 3$ [N.m]; $R_a = 5$ [Ω]; $L_a = 0,024$ [H]; $J = 0,004$ [kg.m²]; $f = 0,0016$ uSI; $T_a = 4,86$ [ms]; $T_m = 2,5$ [s]; $K_{cm} = 055$; $T_{cm} = 33,3$ [μ s]; $K_t = 0,986$ [Nm/A];