New Function With "Gem-Set" in Topological Space

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***Abstract*—In this paper we introduce a new class of maps called A-map , AO – map and Am – map under the idea of '' Gem-set'' in topological spaces and study some of it's basic properties and relations among them .**

I. INTRODUCTION

The impression of ideals in topological spaces in treated in the standard text by Kuratowski [2] and Vaidyanathaswamy [5] ,O.Njastad was introduced the idea of compatible ideals in 1966 . This ideal was also called as super compact by R.Vaidyanathaswamy . Furher D.Jankovic and T.R.Hamlett are also worked in this area , Jankovic and Hamlett [1] investigated supplementary properties of ideal spaces . An ideal I on a topological space (X,τ) is a non-empty collection of subsets of X which satisfies the following properties : (1) $A \in I$ and $B \in A$ implies $B \in I$; (2) $A \in I$ and $B \in I$ implies $A \cup B \in I$. An ideal topological space is a topological space (X,τ) with an ideal I on X and is denoted by (X,τ,I) , For a subsets $A \subseteq X$, $A^*(I,\tau) = \{x \in X ; A \cap U \notin I \; for\; any\; U \in \tau(X,x)\}$ is called the locall function of A with respect to o I and τ [2] .In 2012 [4] , Al-Swidi and Al-Sada introduced a new type of ideals for a one point and denoted by I_x , they defined the I_x is an ideal on a topological space (X,τ) at point x is defined by $I_x = \{U \subseteq X : x \in U^c\}$, where U is non-empty set of X . In a topical paper , Al-Swidi and Al- Naff [2013] [3] hare considered a new set in topological space namely "Gem-set" and depending on the I_x , and, and some its properties are studied , define a new separation axioms by using the idea of "Gem-set" namely "$I^* - T_i$- space" and "$I^{**} - T_i$- space", i=0,1,2 . and defines two mapping is called "$I^* - map$" and "$I^{**} - map$" to carry properties of "Gem-Set" from a space to other space and give more properties for new separation axioms . The aim of this paper is to introduce and study the concepts of new class of maps namely $A - map$, $AO - map$ and $Am - map$ and Study of the most important properties and the relationship between them, as well as connect the properties of the separation axioms type of $I^* - T_i$- space" and "$I^{**} - T_i$- space", i=0,1,2 with the functions and their effect upon . For a sub set A of X , A^{*x} is the Gem-set of A at the point $x \in X$.

II. PRELIMINARIES :

We recall the folloing definitions and results :-

Definition (2.1) [3] : Let (X,τ) is a topological space , $A \subseteq X , x \in X$, we defined A^{*x} with respect to space (X,τ) as follows : $A^{*x} = \{ y \in X : G \cap A \notin I_x , for\; every\; G \in T(y)\}\; where\; T(y) = \{ G \in T : y \in G \}$, A set A^{*x} is called '' Gem-Set " .

Definition (2.2) [3]: Let (X,τ) is a topological space , $A \subseteq X$, We defineb $^{*x}pr(A) = A^{*x} \cup A$, for each $x \in X$.

Definition (2.3) [3]: A subset A of a topological space (X,τ) is called perfected set if $A^{*x} \subseteq A$, for each $x \in X$, and called coperfect if A^c is a perfected set .

Definition (2.4) [3]: A topological space (X,τ) is called :

- $I^* - T_0$- space if and only if for each pair if distinct points x,y of X, there non-empty subsets A,B of X such that $y \notin A^{*x}\; or\; x \notin B^{*y}$.
- $I^* - T_1$- space if and only if for each pair if distinct points x,y of X, there non-empty subsets A,B of X such that $y \notin A^{*x}\; and\; x \notin B^{*y}$.

$I^* - T_2$- space if and only if for each pair if distinct points x,y of X, there non-empty subsets A,B of X such that $A^{*x} \cap B^{*y} = \emptyset, with\; y \notin A^{*x}\; and\; x \notin B^{*y}$

- $I^{**} - T_0$- space if and only if for each pair if distinct points x,y of X, there non-empty subset A of X such that $y \notin A^{*x}\; or\; x \notin A^{*y}$.
- $I^{**} - T_1$- space if and only if for each pair if distinct points x,y of X, there non-empty subset A of X such that $y \notin A^{*x}\; and\; x \notin A^{*y}$.
- $I^{**} - T_2$- space if and only if for each pair if distinct points x,y of X, there non-empty subset A of X such that $A^{*x} \cap A^{*y} = \emptyset, with\; y \notin A^{*x}\; and\; x \notin A^{*y}$.

Theorem (2.5) [3]: For a topological space (X,τ), then the following properties hold :

1- Every T_0- space is a $I^* - T_0$- space.
2- Every T_1- space is a $I^* - T_1$- space.
3- Every T_2- space is a $I^* - T_2$- space.
4- Every T_0- space is a $I^{**} - T_0$- space.
5- Every T_1- space is a $I^{**} - T_1$- space.
6- Every T_2- space is a $I^{**} - T_2$- space.

Definition (2.6) [3] : A mapping $f : (X,\tau) \longrightarrow (Y,\rho)$ is called I^*- map if and only if , for every subset A of X , $x \in X$, $f(A^{*x}) = (f(A))^{*f(x)}$.

Definition (2.7) [3]: A mapping $f : (X,\tau) \longrightarrow (Y,\rho)$ is called I^{**}- map if and only if , for every subset A of Y , $y \in Y$,$f^{-1}(A^{*y}) = (f^{-1}(A))^{*f^{-1}(y)}$.

III. THE NEW FUNCTION AND IT'S PROPERTIES:

In this section we introduce a new class of functions namely $A-map$, $AO-map$ and $Am-map$, and study their properties and relationships .

Definition (3.1) :
A mapping $f:(X,\tau)\longrightarrow(Y,\sigma)$ is said to be :

- f is $A-map$ at $x\in X$ iff $\forall B\subseteq Y$,$\exists A\subseteq X$ $s.t$ $f(A^{*x})\subseteq B^{*f(x)}$.
- Also f is said to be A − map on X iff it is A − map at each point on X .
- f^{-1} is A − map iff $\forall A\subseteq X\ \exists B\subseteq Y\ s.t\ f^{-1}(B^{*y})\subseteq A^{*f^{-1}(y)}$.
- f is $AO-map$ iff $\forall A\subseteq X\ \exists B\subseteq Y\ s.t\ B^{*y}\subseteq f[A^{*f^{-1}(y)}]$.
- $f$is $Am-map\ iff$ f $is\ A-map$, f^{-1} $is\ A-map$ and f is bijective .

Theorem (3.2) :_ A mapping f from a topological space (X,τ) into a topological space (Y,ρ) is $A-map$ at x ∈ X . iff for each $B\subseteq Y$there exist $A\subseteq X$ such that $H_y\cap B\notin I_{f(x)}$,$\forall H_y\in\tau_Y(y)$ whenever $y\in f(A^{*x})$.

Proof:- By definition of $A-map$.

Theorem (3.3) :_ A mapping f from a topological space (X,τ) into a topological space (Y,ρ) is $A-map$ at x ∈ X . iff for each $B\subseteq Y$there exist $A\subseteq X$ such that $y\in f(A^{*x})$ there exist $a\in A^{*x}$ and $V_y\cap B\notin I_{f(x)}$ whenever $V_y\in N(y)$ and $y=f(a)$.

Proof:- By definition of $A-map$ & theorem (1.6) .

Theorem (3.4) :_ A mapping f from a topological space (X,τ) into a topological space (Y,ρ) is $A-map$ iff $[f^{-1}(B)]^{*f^{-1}(y)}\subseteq f^{-1}(B^{*y})$.

Proof :- Suppose that f $A-map$ and let $B\subseteq Y$, Put $A=f^{-1}(B)$ and $x=f^{-1}(y)$, Since f is A − map so we get that $f(A^{*x})\subseteq B^{*y}$ but f is one to one hence $f([f^{-1}(B)]^{*x})\subseteq(B^{*y})$, it follows $[f^{-1}(B)]^{*x}\subseteq f^{-1}(B^{*y})$.

Conversely : Let $B\subseteq Y$ and $y\in Y$, Put $A=f^{-1}(B)$ and $x=f^{-1}(y)$, By hypotheses : $[f^{-1}(B)]^{*x}\subseteq f^{-1}(B^{*y})$ it follows that $f(A^{*x})\subseteq f(f^{-1}(B^{*y}))=B^{*y}$. Thus f is A − map .

Theorem (3.5) :_ An injective map f from a topological space (X,τ) onto a topological space (Y,ρ) is $A-map$ iff $\forall B\subseteq Y\ s.t\ pr^{*x}(f^{-1}(B))\subseteq f^{-1}(pr^{*f(x)}(B))$ whenever $B\subseteq Y$.

Proof:- By definition of $A-map$ and identifiable of $pr^{*x}(A)$.

Theorem (3,6) :_ A function $f:(X,\tau)\longrightarrow(Y,\sigma)$, $is\ A-map$ if and only if $f(pr^{*x}(A))\subseteq pr^{*f(x)}(f(A))$ for $x\in X$ and $A\subseteq X$.

Proof :-_ By definition of $A-map$ and identifiable of $pr^{*x}(A)$.

Note (3.7) :-

(1) Let $f:(X,\tau)\longrightarrow(Y,\sigma)$, if f is injective map , then $f(I_x)=I_{f(x)}$.

Proof:- Since the map f is injective , then for $f(I_x)=\{f(G): G\subseteq I_x\}$ $=\{f(G): x\in G^c\}$ $=\{f(G): f(x)\in(f(G))^c\}=I_{f(x)}$.

(2) In the function $f:(X,\tau)\longrightarrow(Y,\sigma)$ is bijective , then $f^{-1}(I_y)=I_{f^{-1}(y)}$.

Proof :- By bijective of the map f , we accomplished $f^{-1}(I_y)=\{f^{-1}(A): A\in I_y\}$ $=\{f^{-1}(A): y\in A^c\}$ $=\{f^{-1}(A): f^{-1}(y)\in(f^{-1}(A))^c\}=I_{f^{-1}(y)}$.

Theorem (3.8) :_ If a bijective mapping $f:(X,\tau)\longrightarrow(Y,\sigma)$ $I^{**}-map\ then\ f\ is\ A-map$.

Proof :- By noti function of $I^{**}-map$.

Theorem (3.9) :_ An injective map f from a topological space (X,τ) onto a topological space (Y,ρ) is $AO-map$ iff f^{-1} $is\ A-map$.

Proof :-_ Assume that f be $AO-map$ map ,such that for $y\in Y$ and $A\subseteq X$, we gotting that , there exists $B\subseteq Y$ such that $B^{*y}\subseteq f[A^{*f^{-1}(y)}]$. But f $is\ injective$, so $f^{-1}(B^{*y})\subseteq f^{-1}(f[A^{*f^{-1}(y)}])=A^{*f^{-1}(y)}$ and $f^{-1}(B^{*y})\subseteq A^{*f^{-1}(y)}$. Thus f^{-1} $is\ A-map$.

Conversely:- For $A\subseteq X$ & $y\in Y$ and by identifiable of $A-map$ of f^{-1}, we have there exist $B\subseteq Y$ such that $f^{-1}(B^{*y})\subseteq A^{*f^{-1}(y)}$ so we get that $f[f^{-1}(B^{*y})]\subseteq f[A^{*f^{-1}(y)}]$ and by surjective of f , we getting that $(B^{*y})\subseteq f[A^{*f^{-1}(y)}]$. Then f is $AO-map$.

Theorem (3.10) :_ A bijective $A-map$ function is $AO-map$ iff its $Am-map$.

proof :- By definition of $A-map$ and theorem (3.9) .

Theorem (3.11) :_ Every injective $I^{*}-map$ function is $A-map$.

Proof :- Firsthand from definition of $I^{*}-map$ function .

Theorem (3.12) : A bijective is $A-map$ function f $I^{*}-map$ if f is continuous function .

Proof :- By continuity of f and definition of $A-map$ and used the note (3.7) we can prove this theorem .

Theorem (3.13) :_ Every $I^{*}-map$ is $AO-map$ if the function f is injective .

Proof :- Let $A\subseteq X$, $y\in Y$ Put $B=f(A)$ & $x=f^{-1}(y)$ [Since f is injective] , So by $I^{*}-map$, we get that $f(A^{*x})=(f(A))^{*f(x)}=B^{*y}$ then $B^{*y}\subseteq f(A^{*f^{-1}(y)})$. Hence f is $AO-map$.

Theorem (3.14) : Every $I^{**}-map$ is $AO-map$ if the function f is bijective .

Proof :- Let $B\subseteq Y$, $x\in X$ Put $A=f^{-1}(B)$ & $y=f(x)$ [Since f is bijective] , So by $I^{**}-map$, we get that $f^{-1}(B^{*y})=[f^{-1}(B)]^{*f^{-1}(y)}=A^{*x}$ then $f(f^{-1}(B^{*y}))=f(A^{*x})$ so we get that $B^{*y}\subseteq f(A^{*x})$. Hence f is $AO-map$.

Theorem (3.15) : If $f:(X,\tau)\longrightarrow(Y,\sigma)$ is $AO-map$, open. and bijective map then f is $I^{**}-map$.

Proof :- Let $B\subseteq Y$, $y\in Y$, Put $f^{-1}(B)=A$, $f^{-1}(y)=x$. Since f is $AO-map$ so $B^{*y}\subseteq f(A^{*x})$ .Now we must prove that $f(A^{*x})\subseteq B^{*y}$ .Let $b\in f(A^{*x})$ so there exists $a\in X$ $such\ that\ b=f(a)$ and $a\in A^{*x}$ .Therefore $G_a\cap A\notin I_x$,$\forall G_a\in\tau$ .The $f(G_a\cap A\notin I_x)$ it follows that $f(G_a)\cap f(A)\notin f(I_x)$ .By note (3.7)(1) we have $f(I_x)=I_{f(x)}$, $f(A)=B$ since f is bijective .So we get that $f(G_a)_b\cap B\notin I_{f(x)}$ for $f(G_a)_b\in\sigma$, Hence $b\in B^{*y}$. Therefore $f(A^{*x})\subseteq B^{*y}$. Thus f is $I^{**}-map$.

Theorem (3.16) :_ Let $f: (X, \tau) \longrightarrow (Y, \sigma)$ be bijective function . Then f is $Am - map$ iff f is $I^* - map$.

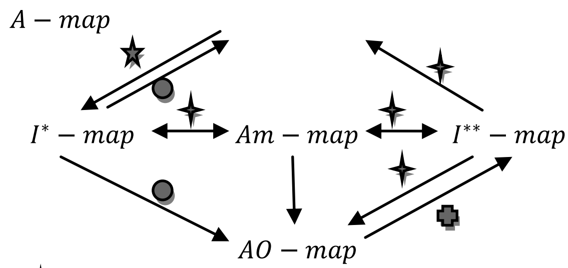
Proof :- By definition of $Am - map$ and $I^* - map$.

Theorem (3.17) :_ Let $f: (X, \tau) \longrightarrow (Y, \sigma)$ be bijective function . Then f is $Am - map$ iff f is $I^{**} - map$.

Proof :- Straight from definition of $Am - map$ and $I^{**} - map$.

Remark (3.18) : *For a function* $f: (X, \tau) \longrightarrow (Y, \sigma)$ *, the following diagram holds :*

Diagram (3.1) : The relationship between the maps .



Bijective .
Injective .
Bijective and open .
Bijective and continuous .

Diagram (3.1) : The relationship between the maps .

Remark (3.17): The converse of theorems need not true as seen from the following examples .

Example(3.18) (1) : Let $f: (X, \tau) \longrightarrow (Y, \sigma)$ if f is $A - map$ then f need not $I^* - map$.

Let $X = \{a, b, c, d\}$, $and\ f: (X, \tau) \longrightarrow (X, \sigma)\ such\ that\ \tau = \{X, \emptyset, \{a\}, \{b, c\}, \{a, b, c\}, \{b, c, d\}\}$, $\sigma = \{X, \emptyset\}$, $f(a) = c$, $f(b) = a$, $f(c) = d$, $f(d) = b$.

$f\ is\ A - map$ but f is not $I^* - map$ since let $A = \{a, b\}$, $x = a \rightarrow f(A) = \{c, d\}, f(a) = c$

$f(A^{*a}) = f(\{a\}) = \{c\}$, $(f(A))^{*c} = \{b, c, d\}$, $\therefore f(A^{*a}) \neq (f(A))^{*c}$.

(2) : Let $f: (X, \tau) \longrightarrow (Y, \sigma)$ if f is $A - map$ then f need not $I^{**} - map$.

Let $X = \{x, y, z\}\ and\ f: (X, p(X)) \longrightarrow (X, \tau)\ such\ that\ \tau = \{X, \emptyset, \{z\}, \{z, x\}\}$, $f(x) = y$, $f(y) = x$, $f(z) = z$.

$f\ is\ A - map$ but not $I^{**} - map$ since , let $B = \{x\} \rightarrow B^{*y} = \{x, y\} \rightarrow f^{-1}(B^{*y}) = \{x, y\}$.

$f^{-1}(B) = \{y\} \rightarrow [f^{-1}(B)]^{*x} = \{y\}b$, So we get that $f^{-1}(B^{*y}) \neq [f^{-1}(B)]^{*x}$.

(3) : Let $f: (X, \tau) \overset{1-1}{\longrightarrow} (Y, \sigma)$ if f is $AO - map$ then f is $I^* - map$.

Let $X = \{x, y, z\}\ and\ f: (X, \tau_1) \longrightarrow (X, \tau_2)\ such\ that\ \tau_1 = \{X, \emptyset, \{x\}, \{x, z\}\}$,

$\tau_2 = \{X, \emptyset, \{x\}, \{x, z\}, \{x, y\}, \{y\}\}$, $f(x) = x$, $f(y) = y$, $f(z) = z$. so f^{-1} is $A - map$, by theorem (3.9) we get that $f\ is\ AO - map$. $Let\ A = \{x\} \rightarrow A^{*x} = \{x, z\} \rightarrow f(A^{*x}) = \{y, z\}$, $f(A) = \{y\} \rightarrow [f(A)]^{*y} = \{x, y, z\}$, So we get that $f(A^{*x}) \neq [f(A)]^{*y}$. Therefore $f\ is\ not\ I^* - map$.

(4) : Let $f: (X, \tau) \longrightarrow (Y, \sigma)$ if f is $AO - map$ then f is $I^{**} - map$.

Let $X = \{x, y, z\}$, $Y = \{a, b, c\}\ and\ f: (X, \tau) \longrightarrow (Y, \sigma)\ such\ that\ \tau = \{X, \emptyset, \{y\}, \{x, y\}\}$, $\sigma = \{Y, , \{c\}, \{a, c\}, \{b, c\}, \{b\}\}$ $f(x) = c$, $f(y) = b$, $f(z) = a$. $f\ is\ AO - map$ but f is not $I^{**} - map$ since $B = \{z\}, B^{*c} = \{a, c\}$, $f^{-1}(\{a, c\}) = \{x, z\}$, $[f^{-1}(\{z\})]^{*f^{-1}(c)} = \{x\}^{*x} = \{x\}$, $\{x\} \neq \{x, z\}$.

(5) : Is the same example (3.18) (1) $f\ is\ A - map$ but f is not $AO - map$ since if $B = \{x\}$, $\exists\ A = \{y\}$

$f^{-1}(y) = x \rightarrow A^{*x} = y, B^{*y} = \{x, y\}$, $f^{-1}(B^{*y}) = \{x, y\}$, so $f^{-1}(B^{*y}) \nsubseteq A^{*x}$ and $\nexists\ A\ in\ X$ such that $f^{-1}(B^{*y}) \subseteq A^{*x}$.

(6) : Is the same example (3.18) (3) f^{-1} is $A - map$ but f is not $A - map$ since if $B = \{x\}$ so $\nexists\ A\ in\ X$ such that $f(A^{*x}) \subseteq B^{*f(x)}$.

Theorem (3.18) : Let $f: (X, \tau) \longrightarrow (Y, \sigma)$ be A − map $at\ x \in X\ and\ g: (Y, \sigma) \longrightarrow (Z, \rho) is$ A − map $at\ f(x) \in Y, then:$ $g \circ f: (X, \tau) \longrightarrow (Z, \rho)\ be$ A − map at $x \in X$.

Proof:- Let $C \subseteq Z\ then\ \exists\ B \subseteq Y\ s.t\ g(B^{f(x)}) \subseteq C^{g(f(x))}$ by $A - map$ of $g\ at\ f(x) \in Y$. But $B \subseteq Y$ then there exist $A \subseteq X$ by $A - map$ of $f\ at\ x \in X$. so we get that $f(A^{*x}) \subseteq B^{*f(x)}$ it follows that $g(f(A^{*x})) \subseteq g(B^{*f(x)}) \subseteq C^{*g(f(x))}$so $g(f(A^{*x})) \subseteq C^{*g(f(x))}$. Thus $g \circ f: (X, \tau) \longrightarrow (Z, \rho)\ be$ A − map $x \in X$.

Theorem (3.19) : Let $f: (X, \tau) \longrightarrow (Y, \sigma)$ be $A - map$ Then for $A \subseteq X$, the following statements are true :-

1)) The inclusion function :
$i: (A, \tau_A) \longrightarrow (X, \tau)$ is $A - map$.

2)) $f/_A: (A, \tau_A) \longrightarrow (Y, \sigma)$ is $A - map$.

3)) $i_d: (X, \tau) \longrightarrow (X, \tau)$ is always $A - map$.

IV. THE FUNCTION AND SEPARATION AXIOMS :

In this section we study the relationships between separations axioms T_i_space , $i = 0,1,2$; $I^*\_T_i space$, $i = 0,1,2$ and $I^{**}\_T_i space$, $i = 0,1,2$; by using the maps .

Theorem (4.1) : Let $f: (X, \tau) \longrightarrow (Y, \sigma)$ be bijective open mapping . Then :

(1) if (X, τ) is strongly and $I^{**}\_T_o space$ then $(Y, \rho)\ is\ T_o\_space$.

(2) if (X, τ) is strongly and $I^{**}\_T_1 space$ then $(Y, \rho)\ is\ T_1\_space$.

(3) if (X, τ) is strongly and $I^{**}\_T_2 space$ then $(Y, \rho)\ is\ T_2\_space$.

Proof:- Closely from use the definition of strongly space , open map and $I^{**}\_T_i space, i = 0, 1, 2$, we evidential this theorem .

Remark (4.2): The converse of theorem need not true as seen from the following examples .

Example (4.3) :

(1) Let $X = \{x, y, z\}$ and $f: (X, \tau_1) \longrightarrow (X, \tau_2)$s.t $\tau_1 = \{X, \emptyset, \{x\}, \{x, y\}\}$ & $\tau_2 = \{X, \emptyset, \{x\}, \{y\}, \{x, y\}, \{y, z\}\}$and $f(x) = x$, $f(y) = y$, $f(z) = z$ .Then $(X, \tau_2)\ is\ T_0 - space$ .Let $A = \{x\} \rightarrow A^{*x} = \{x, y, z\}$ is not open set in (X, τ_1), So we get that (X, τ_1) is not strongly space .

(2) Let $X = \{x, y, z\}$ and $f: (X, \tau_1) \longrightarrow (X, p(x))$ s.t $\tau_1 = \{X, \emptyset, \{x\}, \{x, y\}\}$ and $f(x) = x, f(y) = y, f(z) = z$ .Then $(X, p(x))$ $is\ T_1 - space$ .Let $A = \{x\} \rightarrow A^{*x} = \{x, y, z\}$ is not open set in (X, τ_1) So we get that (X, τ_1) is not strongly space .

(3) Let $X = \{x, y, z\}$ and $f: (X, \tau_1) \longrightarrow (X, p(x))$ s.t $\tau_1 = \{X, \emptyset, \{x\}, \{x, y\}\}$ and $f(x) = x, f(y) = y, f(z) = z$ .Then $(X, p(x))$ $is\ T_2 - space$ .Let $A = \{x\} \rightarrow A^{*x} = \{x, y, z\}$ is not open set in (X, τ_1) So we get that (X, τ_1) is not strongly space .

Corollary (4.4) : Let $f: (X, \tau) \longrightarrow (Y, \sigma)$ is bijective and open maping , Then :

1- if (X, τ) is strongly and $T_o space$ then (Y, ρ) $is\ T_o\_space$.

2- if (X, τ) is strongly and $T_1 space$ then (Y, ρ) $is\ T_1\_space$.

3- if (X, τ) is strongly and $T_2 space$ then (Y, ρ) $is\ T_2\_space$.

Proof:- By theorem (2.5) and (4.1) .

Theorem (4.5) : Let $f: (X, \tau) \longrightarrow (Y, \sigma)$ is bijective and continuous mapping , Then :

(1) if (Y, ρ) is strongly & $I^{**}\_T_0 space$ then (X, τ) $is\ T_0\_space$.

(2) if (Y, ρ) is strongly & $I^{**}\_T_1 space$ then (X, τ) $is\ T_1\_space$.

(3) if (Y, ρ) is strongly & $I^{**}\_T_2 space$ then (X, τ) $is\ T_2\_space$.

Proof:- By definition of strongly space , continuous map & $I^{**}\_T_i space, i = 0, 1, 2$.

Remark (4.6): The converse of theorem need not true as seen from the following example .

Example(4.7) :

(1) Let $X = \{x, y, z\}$ and $f: (X, \tau_1) \longrightarrow (X, (X, \tau_2))$ s.t $\tau_1 = \{X, \emptyset, \{x\}, \{y\}, \{x, y\}\}$ and $\tau_2 = \{X, \emptyset, \{x\}, \{z\}, \{x, z\}\}$, $f(x) = x, f(y) = z, f(z) = y$. f $is\ cont.$ & $1 - 1$, Then (X, τ_1) $is\ T_0 - space$. Let $A = \{x\} \rightarrow A^{*x} = \{x, y\}$ is not open set in (X, τ_2) , So we get that (X, τ_2) is not strongly space .

(2) Let $X = \{x, y, z\}$ and $f: (X, p(x)) \longrightarrow (X, (X, \tau_1))$ s.t $\tau_1 = \{X, \emptyset\}$ and $f(x) = x, f(y) = y, f(z) = z$. f $is\ cont.$ & $1 - 1$, Then $(X, p(x))$ $is\ T_1 - space$. Let $A = \{x\} \rightarrow A^{*x} = \{x, y, z\}$ is not open set in (X, τ_1) ,So we get that (X, τ_1) is not strongly space .

(3) Let $X = \{x, y, z\}$ and $f: (X, p(x)) \longrightarrow (X, (X, \tau_1))$ s.t $\tau_1 = \{X, \emptyset\}$ and $f(x) = x, f(y) = y, f(z) = z$. f $is\ cont.$ & $1 - 1$, Then $(X, p(x))$ $is\ T_2 - space$. Let $A = \{x\} \rightarrow A^{*x} = \{x, y, z\}$ is not open set in (X, τ_1) , So we get that (X, τ_1) is not strongly space .

Corollary (4.8) :_ Let $f: (X, \tau) \longrightarrow (Y, \sigma)$ is surjective continuous maping , Then :

1- if (Y, ρ) is strongly and $T_o space$ then (X, τ) $is\ T_o\_space$.

2- if (Y, ρ) is strongly and $T_1 space$ then (X, τ) $is\ T_1\_space$.

3- if (Y, ρ) is strongly and $T_2 space$ then (X, τ) $is\ T_2\_space$.

Proof:- By theorem (2.5) and (4.5) .

Theorem (4.9): Let $f: (X, \tau) \longrightarrow (Y, \sigma)$ be bijective open mapping , Then :

(1) if (X, τ) is strongly and $I^{*}\_T_o space$ then (Y, ρ) $is\ T_o\_space$.

(2) if (X, τ) is strongly and $I^{*}\_T_1 space$ then (Y, ρ) $is\ T_1\_space$.

(3) if (X, τ) is strongly and $I^{*}\_T_2 space$ then (Y, ρ) $is\ T_2\_space$.

Proof:- By definition of strongly space , open map and $I^{*}\_T_i space, i = 0, 1, 2$.

Remark (4.10): The converse of theorem need not true as seen from the same example (4.3) .

Theorem (4.11) : Let $f: (X, \tau) \longrightarrow (Y, \sigma)$ is bijective continuous mapping , Then :

(1) if (Y, ρ) is strongly and $I^{*}\_T_0 space$ then (X, τ) $is\ T_0\_space$.

(2) if (Y, ρ) is strongly and $I^{*}\_T_1 space$ then (X, τ) $is\ T_1\_space$.

(3) if (Y, ρ) is strongly and $I^{*}\_T_2 space$ then (X, τ) $is\ T_2\_space$.

Proof:- By definition of strongly space , continuous map and $I^{*}\_T_i space, i = 0, 1, 2$.

Remark (4.12): The converse of theorem need not true as seen from the same example (4.7) .

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