International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 15 Issue 04 , April - 2026

Neutrosophic Fuzzy Bi magic Labeling of Fan Graphs

Dr. N. Sarala[1] and G. Vidhya[2]
[1]Associate professor and [2]Research Scholar
[1,2]Department of Mathematics, A.D.M. College for Women (Autonomous), Affiliated to Bharathidasan University, Nagapattinam, Tamil Nadu, India.

Abstract - In order to depict the ambiguity and uncertainty in graph structures, this work presents the idea of Neutrosophic Fuzzy Bi Magic Labeling Graphs, which blends fuzzy set theory and neutrosophic logic.We define NFML graphs and investigate their characteristics, such as neotroscopic fuzzy indices and bi magic labeling criteria. Neutrosophic fuzzy bimagic labeling of a fan graph is investigated in this article.

Key words: Fuzzy Labeling, Neutrosophic fuzzy Labeling, Bi magic Labeling, Fan graph.

1.INTRODUCTION

Graph Theory is a fundamental tool for modeling complex systems and relations in various fields, including computer science, engineering, and social networks. However, real world systems often involve uncertainty, ambiguity, and imprecision, which can be challenging to represent using traditional graph theory. To address this limitation, fuzzy graph theory and magic labeling have been introduced. Fuzzy labeling is a technique used to represent uncertainty and imprecision in graph structures. In fuzzy labeling, edges and vertices are assigned fuzzy lables, which are typically represented as fuzzy sets. This allows for a more nuanced representation of relations between nodes, enabling the modeling og complex systems with uncertain or imprecise connections. Magic labeling is a specific type of graph labeling where the sum of labels of edges and vertices satisfies certain conditions. The combination of fuzzy labeling and magic labeling enables the representation of complex systems with uncertain relationships and specific structural properties. Neutrosophic fuzzy labeling is an extension of traditional fuzzy labeling, incorporating neutrosophic logic to represent uncertainty, ambiguity and imprecision in graph structures. Neutrosophic logic, introduced by Florentin Samarandache. In Neutrosophic fuzzy labeling, each label consist of three components Truth, Indeterminacy, Falsity, allowing for a more nuanced representation of complex relationships and uncertain information. This approach enables the modeling of realworld systems with inherent ambiguity, uncertainty and imprecision.

2.PRELIMINARIES

Definition.2. 1

Let $C^* = (V, E)$ be a simple graph. Then $C^* = (\varphi, \psi)$ is called a fuzzy graph on C^*, if $\varphi: V \rightarrow [0, 1]$ and $\psi: E \rightarrow [0, 1]$ and for all $x, y \in E$,$\psi (x, y) \leq \min [\varphi(x), \varphi (y)]$ .A fuzzy graph $C= (\varphi, \psi))$ on C^* is called a fuzzy labeling graph if φ and ψ one to one maps for all $x, y \in E$.

Definition. 2.2

A fuzzy labeling graph $C = (\varphi, \psi)$ on C is called fuzzy magic labeling graph if there exists m, which is called magic value such that for all x, y ∈E.

$\varphi (x) + \varphi (y) + \psi (x, y) = m$

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 15 Issue 04 , April - 2026

Definition. 2.3

If the total membership values of the vertices and edges incident at the vertices are k1 and k2, where k1 and k2 are constants, then a fuzzy labeling graph admits Bi-magic labelling

Definition. 2.4

A Neutrosophic Fuzzy Graph C = (φ, ψ) where $\varphi = \{v_1, v_2, v_3, \dots V_n\}$ such that $T_1: V \to [0, 1]$, $I_1: V \to [0, 1]$ and $F_1: V \to [0, 1]$ denote the degree of truth membership, indeterminacy membership and falsity membership of the element $v_i \in V$ ($i=1,2\dots n$) , $E \subseteq V\times V$ where

$T_2: V\times V \to [0, 1]$, $I_2: V\times V \to [0, 1]$, and $F_2: V\times V \to [0, 1]$ are such that

$T_2 (v_i, v_j) \leq \min [T_1(v_i), T_1(v_j))]$,

$I_2 (v_i, v_j) \leq \min [I_1(v_i), I_1(v_j))]$,

$F_2(v_i, v_j) \leq \max [F_1(v_i), F_1(v_j))]$

$0 \leq T_2 (v_i, v_j) + I_2 (v_i, v_j) + F_2 (v_i , v_j) \leq 3$ for every $(v_i, v_j) \in E$ ($j=1,2\dots n$)

A neutrosophic fuzzy labeling graph is a Neutrosophic fuzzy magic labeling graph if their exist an M such that M equals

$\{T_1(v_i)+ T_1(v_j) + T_2 (v_i, v_j)$,

$[I_1(v_i) +I_1(v_j) + I_2 (v_i, v_j)$,

$F_1(v_i) + F_1(v_j) + F_2 (v_i, v_j)\}$

Definition. 2.5

A fan graph is defined as the graph join of an empty graph on m nodes and a path graph on n nodes. This corresponds to usual fan graphs when m=1 and double fan graphs when m=2.

Definition. 2.6

A Fan graph with fuzzy labeling is called fuzzy Fan graph. A Fan is a fuzzy graph consists of two node sets F and $|F|$ =1 and $|F| > 1$, such that $\mu(F,F_i) >0$, where i=1 to n and $\mu(F_i,F_{i+1}) >0$ where i=1 to n-1.

3.NEUTROSOPHIC FUZZY BI-MAGIC LABELLING OF FAN GRAPH

Preposition:3.1

Every fuzzy fan graph $F_{1, n}$ is a Neutrosophic fuzzy bimagic labeling graph for n=3.

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 15 Issue 04 , April - 2026

Proof:

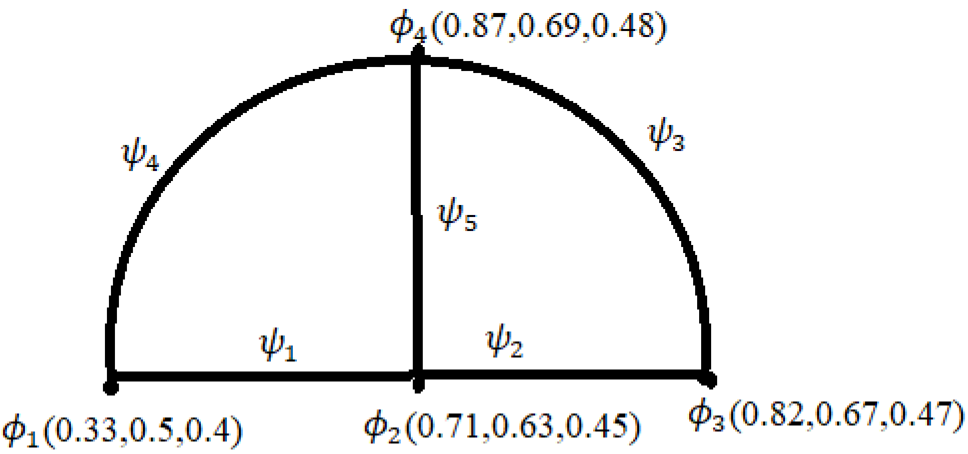
Let $\phi_1, \phi_2, \ldots\ldots \phi_n$ be the vertices and $\psi_1, \psi_{2\ldots}$ ψ_n be the edges of the fan graph.

The vertices are defined for the truth, indeterminacy and false membership functions σ: Ø⟶ [0, 1], ρ: Ø⟶ [0, 1], μ: Ø⟶ [0, 1] respectively for the vertices are as follows,

$\sigma(v_i) = \frac{4n-3}{4n-1}$, for all n = 1 to 4

$\rho(v_i) = \frac{3n-1}{4n}$

$\mu(v_i) = \frac{5n-1}{10n}$



$F_{1,3}$ Fan Graph

The vertices of fan graph $F_{1,3}$ is noted as follows

ϕ_1(0.33,0.5,0.4),ϕ_2(0.71,0.63,0.45), ϕ_3(0.82,0.67,0.47), ϕ_4(0.87,0.69,0.48).

Then the corresponding edge values are ψ_1(0.26,0.42,0.44), ψ_2(0.32,0.23,0.46), ψ_3(0.16,0.17,0.42), ψ_4(0.10,0.36,0.41), ψ_5(0.27,0.21,0.47)

Existence of bi magic values are

ϕ_1(0.33,0.5,0.4) + ψ_1(0.26,0.42,0.44) + ϕ_2(0.71,0.63,0.45) is (1.3,1.55,1.29)

,ϕ_2(0.71,0.63,0.45) + ψ_2(0.32,0.23,0.46) + ϕ_3(0.82,0.67,0.47) is (1.85,1.53,1.38)

ϕ_3(0.82,0.67,0.47) +ψ_3(0.16,0.17,0.42) + ϕ_4(0.87,0.69,0.48) is (1.85,1.53,1.37),

ϕ_4(0.87,0.69,0.48) + ψ_4(0.10,0.36,0.41) + ϕ_1(0.33,0.5,0.4) is (1.3,1.55,1.29)

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 15 Issue 04 , April - 2026

ϕ_4(0.87,0.69,0.48) + ψ_5(0.27,0.21,0.47) + ϕ_2(0.71,0.63,0.45) is (1.85,1.53,1.38)

Therefore the Bi magic values are (1.3,1.55,1.29) and (1.85,1.53,1.38)

Hence the Fan graph $F_{1,3}$ admits a Neutrosophic fuzzy bi magic labeling.

Preposition: 3.2

Every fuzzy fan graph $F_{1,n}$ is a Neutrosophic fuzzy bi magic labeling graph for n=5.

Proof:

Let $\phi_1, \phi_2,\ldots\ldots \phi_n$ be the vertices and $\psi_1, \psi_{2\ldots}$ ψ_n be the edges of the fan graph.

The vertices are defined for the truth, indeterminacy and false membership functions σ: Ø⟶ [0, 1], ρ: Ø⟶ [0, 1], μ: Ø⟶ [0, 1] respectively for the vertices are as follows,

$\sigma(v_i) = \frac{3n+1}{p}$, for n = 5,6,8 for some p=31,1≤i≤6

$\frac{3n}{p}$, for n=9

$\frac{3n11}{p}$, for n=6,10

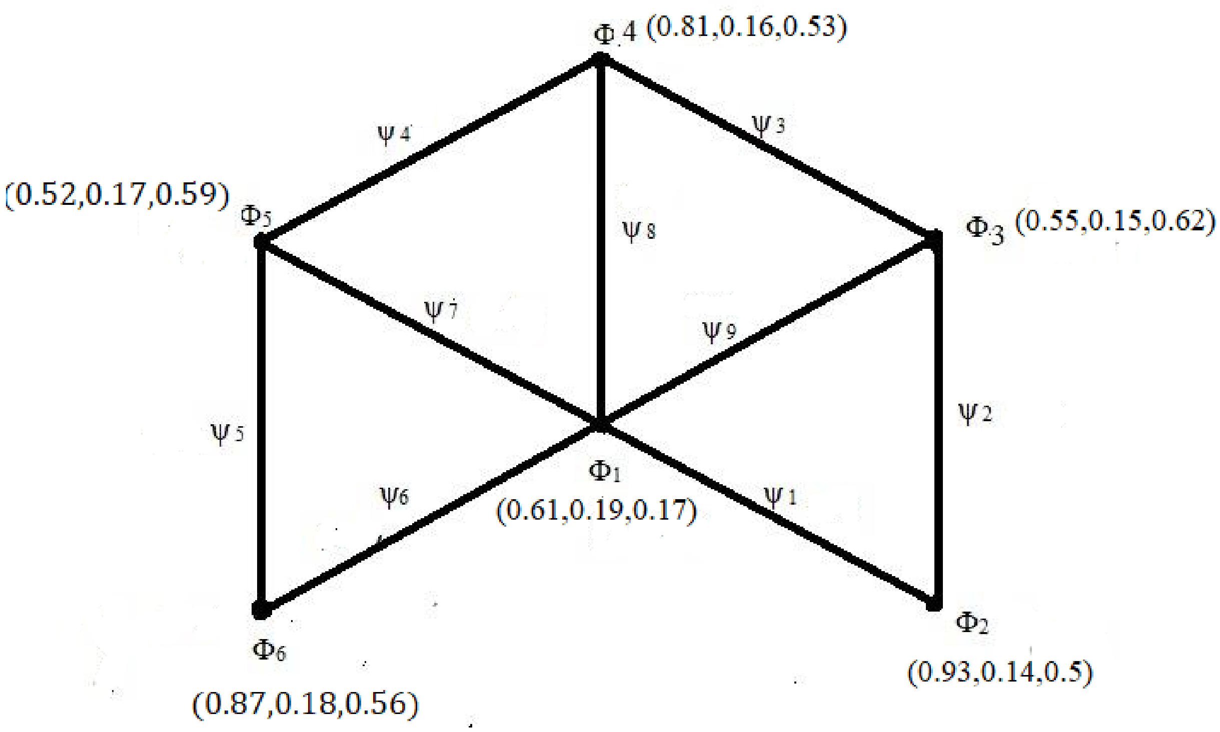
$\rho(v_i) = \frac{n}{10^2}$,for n=1 to n

$\mu(v_i) = \frac{7n\pm1}{10^2}$,for n=7,9

$\frac{7n}{10^2}$, for n=8

$\frac{7n\pm3}{10^2}$,for n= 2,8

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 15 Issue 04 , April - 2026



$F_{1,5}$ Fan Graph

The vertices $F_{1,5}$ as follows

The vertices are defined for the truth, indeterminacy and false membership functions σ: Ø⟶ [0, 1], ρ: Ø⟶ [0, 1], µ: Ø⟶ [0, 1] respectively for the vertices are as follows,

ϕ_1(0.61,0.19,0.17),ϕ_2(0.93,0.14,0.5), ϕ_3(0.55,0.15,0.62), ϕ_4(0.81,0.16,0.53), ϕ_5(0.52,0.17,0.59), ϕ_6(0.87,0.18,0.56).

Then the corresponding edge values are ψ_1(0.03,0.13,0.32) , ψ_2(0.07,0.08,0.59), ψ_3(0.19,0.06,0.56), ψ_4(0.22,0.04,0.59), ψ_5(0.16,0.02,0.56), ψ_6(0.09,0.09,0.26), ψ_7(0.44,0.10,0.23), ψ_8(0.15,0.11,0.29), ψ_9(0.41,0.12,0.20)

Existence of bi magic values are

ϕ_1(0.61,0.19,0.17) + ψ_1(0.03,0.13,0.32) + ϕ_2(0.93,0.14,0.5) = (1.57,0.46,0.99)

ϕ_1(0.61,0.19,0.17) + ψ_9(0.41,0.12,0.20) + ϕ_3(0.55,0.15,0.62) = (1.57,0.46,0.99)

ϕ_1(0.61,0.19,0.17) + ψ_8(0.15,0.11,0.29) + ϕ_4(0.81,0.16,0.53) = (1.57,0.46,0.99)

ϕ_1(0.61,0.19,0.17) + ψ_7(0.44,0.10,0.23) + ϕ_5(0.52,0.17,0.59) = (1.57,0.46,0.99)

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 15 Issue 04 , April - 2026

ϕ_1(0.61,0.19,0.17) + ψ_6(0.09,0.09,0.26) + ϕ_6(0.87,0.18,0.56) = (1.57,0.46,0.99)

ϕ_2(0.93,0.14,0.5) + ψ_2(0.07,0.08,0.59) + ϕ_3(0.55,0.15,0.62) = (1.55,0.37,1.71)

ϕ_3(0.55,0.15,0.62) + ψ_3(0.19,0.06,0.56) + ϕ_4(0.81,0.16,0.53) = (1.55,0.37,1.71)

ϕ_4(0.81,0.16,0.53) + ψ_4(0.22,0.04,0.59) + ϕ_5(0.52,0.17,0.59) = (1.55,0.37,1.71)

ϕ_5(0.52,0.17,0.59) + ψ_5(0.16,0.02,0.56) + ϕ_6(0.87,0.18,0.56) = (1.55,0.37,1.71). Hence the Fan graph $F_{1,3}$ admits a Neutrosophic fuzzy bi magic labeling.

CONCLUSION:

The definitions of Neutrosophic fuzzy bi magic labelling for the fan graphs $F_{1,3}$, $F_{1,5}$ have been investigated in this paper.

5. REFERENCES

[1] 1.Zadeh L.A. Fuzzy Sets, Information and Control 8 (3), 338-353 (1965)
[2] 2.Rosenfield. A, Fuzzy Graphs, Fuzzy sets and their applications, Academic Press, New York, 77 - 95 (1975).
[3] 3.A NagoorGani and V. T. Chandrasekaran, A first look at Fuzzy graph theory Allied publishers Pvt. Ltd., 2010.
[4] 4.A NagoorGani and D. Rajalakshmi (a) Subhashini, A note on Fuzzy labeling, International Journal of Fuzzy Mathematical Archive 2014.
[5] 5.A NagoorGani, D. Rajalakshmi & Subhashini, "Properties of fuzzy labeling graph" Applied Mathematical sciences, Vol.6 (2012), 3461 – 3466.
[6] 6. K Ameenal Bibi and M.Devi, Fuzzy Bi-Magic labeling on Cycle Graph
[7] and Star Graph, Global Journal of Pure and Applied Mathematics,
[8] Vol 13(2017), 0973-1768.
[9] 7.D. Ajay and P.Chellamani, Fuzzy magic labeling of Neutrosophic path and star graph,Advances in Mathematics: Scientific Journal 9 (2020), no.8,6059-6070ISSN:1857-8365(printed):1857-8438(electronic).
[10] 8. S. Sudha, M. Lathamaheswari, S. Broumi, F. Samarandache, Neutrosophic fuzzy magic labelling graph with its application in Academic performance of the students, Neutrosophic sets and systems, Vol 60,2023, University of New Mexico.