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Navier-Stokes Exact Solutions Morphologies "Behaviour" in Pipes and Coaxial Cylinders using Simple Closed Forms

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***Abstract*—In this article, new morphologies of exact solutions of the full set of Navier–Stokes equations in the presence of porous boundaries of axisymmetric rotating geometries are presented. Such flows have significant industrial applications including filtration and particle separation.**

***Keywords*—Navier-Stokes, porous cylinder, Bessel fumction, rotating pipes**

I. INTRODUCTION

In the present investigation we have considered the flow in pipes and between porous rotating cylinders. The effect of suction and injection is studied on the walls of pipes and of the cylinders. Several authors have analysed [1,2,3,4,5,6,7,8] all these flows for the Navier–Stokes equations for incompressible flow, written in polar-coordinates. In a new attempt to resolve these equations strong analytically with closed forms, according to an Polyanin-Aristov assumption [9], it is assumed that the effect of the body force by mass transfer phenomena is the 'porosity' of the porous boundary in which the fluid moves. The effect of porous boundaries on the viscous flow is examined for two different cases. The first one examines the flow between two rotated porous cylinders and the second one discusses the swirl flow in a rotated porous pipe.The developed solutions are of general application and can be applied to any swirling flow in porous axisymmetric rotating geometries.The exact solutions are obtained by employing the Bessel functions for the case of three dimensional unsteady flow between rotated porous cylinders and for the case of unsteady swirl flow in rotated porous pipes.

II. MATHEMATICAL AND PHYSICAL MODELLING

For the first case study we assume the flow of a Newtonian fluid within a rotating porous cylindrical pipe (Figure 1), and for the second case study the flow through an annulus formed between two rotating porous cylinders (Figure 2). . For the two cases, the basic equations are the mass conservation equation and the equations of motion (Navier–Stokes), in a cylindrical system of coordinates (r,θ,z) where the z– axis lies along the centre of the symmetry, r is the radial distance and θ is the peripheral angle.

II.1. Conditions in a rotating pipe

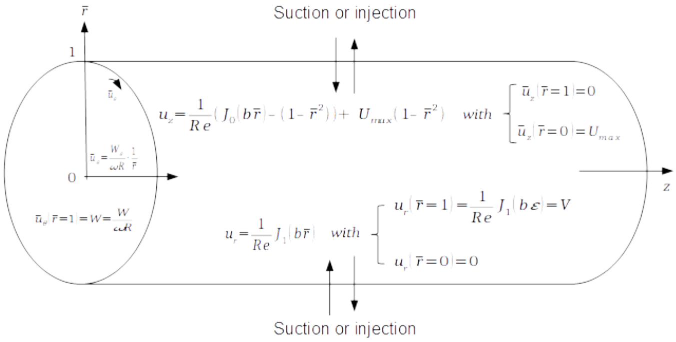

Figure 1. Flow within a rotating porous pipe

II.2. Axial flow conditions in two rotating porous cylinders

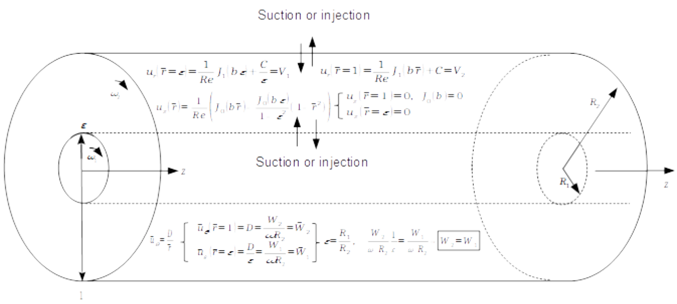

Figure 2: Rotating porous cylinders

III. GOVERNING EQUATIONS

Considering that the flow modelling describes the motion of a homogeneous incompressible Newtonian fluid, the Navier–Stokes equations have as follows:

$$\frac{u_r}{r}+\frac{\partial u_r}{\partial r}+\frac{\partial u_z}{\partial z}+\frac{1}{r}\frac{\partial u_\theta}{\partial \theta}=0 \tag{1}$$

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$$\frac{\partial u_r}{\partial t}+u_r\frac{\partial u_r}{\partial r}-\frac{u_\theta^2}{r}+u_z\frac{\partial u_r}{\partial z}=-\frac{1}{\rho}\cdot\frac{\partial P}{\partial r}+\frac{\mu}{\rho}\left[\frac{\partial^2 u_r}{\partial r^2}+\frac{1}{r}\cdot\frac{\partial u_r}{\partial r}-\frac{u_r}{r^2}+\frac{\partial^2 u_r}{\partial z^2}\right] \quad (2)$$

$$\frac{\partial u_\theta}{\partial t}+u_r\frac{\partial u_\theta}{\partial r}+\frac{u_r\cdot u_\theta}{r}+u_z\frac{\partial u_\theta}{\partial z}=-\frac{1}{\rho}\cdot\frac{1}{r}\cdot\frac{\partial P}{\partial r}+\frac{\mu}{\rho}\left[\frac{\partial^2 u_\theta}{\partial r^2}+\frac{1}{r}\cdot\frac{\partial u_\theta}{\partial r}-\frac{u_\theta}{r^2}+\frac{\partial^2 u_\theta}{\partial z^2}\right] \quad (3)$$

$$\frac{\partial u_z}{\partial t}+u_r\frac{\partial u_z}{\partial r}+u_z\frac{\partial u_z}{\partial z}=-\frac{1}{\rho}\cdot\frac{\partial P}{\partial z}+\frac{\mu}{\rho}\left[\frac{\partial^2 u_z}{\partial r^2}+\frac{1}{r}\cdot\frac{\partial u_z}{\partial r}+\frac{\partial^2 u_z}{\partial z^2}\right] \quad (4)$$

IV. EXACT SOLUTIONS SETS AND THEIR OWN BOYNDARY CONDITIONS .

IV.1.Time-dependent swirling flow in a rotated porous pipe

The exact solutions	*The boundary conditions*	
$u_z=A J_0 e^{-bz}e^{kt}+B(1-\bar{r}^2)$	$u_z(\bar{r}=0)=U_{max}, u_z(\bar{r}=1)=0$	(5)
$u_r=A J_1 e^{-bz}e^{kt}$	$u_r(\bar{r}=0)=0, u_r(\bar{r}=1)=V$	(6)
$u_\theta=\frac{C e^{kt}}{\bar{r}}$	$u_\theta(\bar{r}=1)=\frac{W}{\omega R}$	(7)

IV.2. Time-dependent axial flow between two coaxial porous rotating cylinders

The exact solutions	*The boundary conditions*	
$u_z=A J_0 e^{-bz}e^{kt}+B(1-\bar{r}^2)$	$u_z(\bar{r}=\varepsilon)=0 \; , \; u_z(\bar{r}=1)=0$	(8)
$u_r=A J_1 e^{-bz}e^{kt}+\frac{C}{\bar{r}}$	$u_r(\bar{r}=\varepsilon)=V_1, u_r(\bar{r}=1)=V_2$	(9)
$u_\theta=\frac{C e^{kt}}{\bar{r}}$	$u_\theta(\bar{r}=\varepsilon)=\frac{W_1}{\omega R_2}, u_\theta(\bar{r}=1)=\frac{W_2}{\omega R_2}$	(10)

IV.3a. Time-dependent flow between rotating coaxial cylinders with suction-injection at the porous walls. (Version 1)

The exact solutions	*The boundary conditions*	
$u_r=\frac{A}{\bar{r}}+C e^{kt}\sin\theta$	$u_r(\bar{r}=\varepsilon)=V_1, u_r(\bar{r}=1)=V_2$	(11)
$u_\theta=C e^{kt}\cos\theta$	$u_\theta(\bar{r}=\varepsilon)=\frac{W_1}{\omega R_1}, \; u_\theta(\bar{r}=1)=\frac{W_2}{\omega R_2}$	(12)
$u_z=0$		(13)

IV.3b. Time-dependent flow between rotating coaxial cylinders with suction-injection at the porous walls. (Version 2)

The exact solutions	*The boundary conditions*	
$u_r=\frac{A}{\bar{r}}+C e^{kt}\sin\theta$	$u_r(\bar{r}=\varepsilon)=V_1, u_r(\bar{r}=1)=V_2$	(14)
$u_\theta=\frac{A}{\bar{r}}+C e^{kt}\cos\theta$	$u_\theta(\bar{r}=\varepsilon)=\frac{W_1}{\omega R_1}, u_\theta(\bar{r}=1)=\frac{W_2}{\omega R_2}$	(15)
$u_z=0$		(16)

IV.4. One-dimentional flow in centrifugal pumps impellers

The exact solutions	*The boundary conditions*	
$u_\theta=J_1 e^{-bz}$	$u_\theta(\bar{r}=\varepsilon)=\frac{W_1}{\omega R_1}, u_\theta(\bar{r}=1)=\frac{W_2}{\omega R_2}$	(17)
$u_r=0$		(18)
$u_z=0$		(19)

$$0.5\cdot(J_0^2+J_1^2)\cdot e^{-2bz}=-p \quad (20)$$

$$\frac{\partial p}{\partial\theta}=0 \; , \; \frac{\partial p}{\partial z}=0 \; , \; \frac{\partial p}{\partial r}=\frac{J_1^2 e^{-2bz}}{\bar{r}} \quad (21)$$

From the pressure field described by (20) and (21) for *z=0*, one can derive the manometer head for any centrifugal pump. This derivation will appear in the next publication of the authors.

V. REFERENCES

[1] Basant K. Jha and Dauda Gambo, 2021, "Hydrodynamic effect of slip boundaries and exponentially decaying/growing time-dependent pressure gradient on Dean flow", Journal of the Egyptian Mathematical Society 2021.

[2] U. K. Sarkar and Nirmalendu Biswas, 2021, "Exact and limiting solutions of fluid flow for axially oscillating cylindrical pipe and annulus", Applied Sciences 2021, https://doi.org/10.1007/s42452-021-04192-5

[3] Bchara Sidnawi, Sridhar Santhanam and Qianhong Wu, 2019, "Analytical and Numerical Study of a Pulsatile Flow in a Porous Tube", Journal of Fluids Engineering, ASME DECEMBER 2019, Vol. 141, https://fluidsengineering.asmedigitalcollection.asme.org .

[4] Alexander V.Koptev, 2020, "Exact Solution of 3D Navier–Stokes Equations", Journal of Siberian Federal University. Mathematics &and Physics 2020, 13(3), 306–313.

[5] Dyck, Nolan J. and Straatman, Anthony G., 2020, "Exact solutions to the three-dimensional Navier-Stokes equations using the extended Beltrami method". Transactions of the ASME , Vol. 87, JANUARY 2020. https://ir.lib.uwo.ca/mechanicalpub/4

[6] Irene Daprà , Giambattista Scarpi, 2015, "Unsteady flow of fluids with arbitrarily time-dependent rheological behaviour", Journal of Fluids Engineering. Received December 03, 2015.

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[7] Konstantin Ilin and Andrey Morgulis, 2019, “On the stability of the Couette-Taylor flow between rotating porous cylinders with radial flow”, European Journal of Mechanics. B/Fluids.December 2, 2019.

[8] Eric C. Johnson and Richard M. Lueptow, 1997, “Hydrodynamic stability of flow between rotating porous cylinders with radial and axial flow”, Phys. Fluids 9 (12), December 1997.

[9] A. D. Polyanin and S. N. Aristov, 2011, “A New Method for Constructing Exact Solutions to Three_Dimensional Navier–Stokes and Euler Equations”, Theoretical Foundations of Chemical Engineering, 2011, Vol. 45, No. 6, pp. 885-890.

APPENDIX

FLOW FIELD BETWEEN ROTATING COAXIAL CYLINDERS WITH SUCTION-INJECTION AT THE POROUS WALLS (Figure 3).

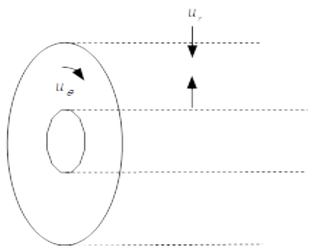

Figure 3. Rotating coaxial cylinders

Version 1.

$$u_r = \frac{A}{\bar r} + C\cdot e^{kt}\sin\theta \quad (22)$$

$$u_r(\bar r=\varepsilon) = V_1 \quad (23)$$

$$u_r(\bar r=1) = V_2 \quad (24)$$

$t=0$

$$\frac{A}{\varepsilon} + C\sin\theta = V_1 \quad (25)$$

$$A - \frac{A}{\varepsilon} = V_2 - V_1 \rightarrow A = \frac{V_2 - V_1}{1-\frac{1}{\varepsilon}} \quad (26)$$

$$A + C\sin\theta = V_2 \quad (27)$$

$$C = \frac{V_2 - A}{\sin\theta} = \frac{1}{\sin\theta}\left(V_2 - \frac{V_2 - V_1}{1-\frac{1}{\varepsilon}}\right) = \frac{1}{\sin\theta}\,\frac{V_1 - \frac{V_2}{\varepsilon}}{1-\frac{1}{\varepsilon}} \quad (28)$$

$$u_\theta = C e^{kt}\cos\theta \quad (29)$$

$$u_\theta(r=\varepsilon) = \frac{w_1}{\omega R_1} = C\cos\theta, \quad t=0 \quad (30)$$

$$u_\theta(r=1) = \frac{w_2}{\omega R_2} = C\cos\theta, \quad t=0 \quad (31)$$

$$W_1 = \frac{V_1 - \frac{V_2}{\varepsilon}}{1-\frac{1}{\varepsilon}} \cdot \frac{\omega R_1}{\tan\theta} \quad (32)$$

$$W_2 = \frac{V_1 - \frac{V_2}{\varepsilon}}{1-\frac{1}{\varepsilon}} \cdot \frac{\omega R_2}{\tan\theta} \quad (33)$$

$$C K \bar r e^{kt}\sin\theta + \frac{A^2}{2\bar r^2} + \frac{A C e^{kt}\sin\theta}{\bar r} = -p \quad (34)$$

$$C K e^{kt}\sin\theta - \frac{A^2}{\bar r^3} - \frac{A C e^{kt}\sin\theta}{\bar r^2} = \frac{-\partial p}{\partial \bar r} \quad (35)$$

$$C K \bar r e^{kt}\cos\theta + \frac{A C e^{kt}\cos\theta}{\bar r} = \frac{-\partial p}{\partial \theta} \quad (36)$$

Version 2.

$$u_r = \frac{A}{\bar r} + C\cdot e^{kt}\sin\theta \quad (37)$$

$$u_r(\bar r=\varepsilon) = V_1 \quad (38)$$

$$u_r(\bar r=1) = V_2 \quad (39)$$

$t=0$

$$\frac{A}{\varepsilon} + C\sin\theta = V_1 \quad (40)$$

$$A - \frac{A}{\varepsilon} = V_2 - V_1 \rightarrow A = \frac{V_2 - V_1}{1-\frac{1}{\varepsilon}} \quad (41)$$

$$A + C\sin\theta = V_2 \quad (42)$$

$$C = \frac{V_2 - A}{\sin\theta} = \frac{1}{\sin\theta}\left(V_2 - \frac{V_2 - V_1}{1-\frac{1}{\varepsilon}}\right) = \frac{1}{\sin\theta}\,\frac{V_1 - \frac{V_2}{\varepsilon}}{1-\frac{1}{\varepsilon}} \quad (43)$$

$$u_\theta = \frac{A}{\bar r} + C e^{kt}\cos\theta \quad (44)$$

$$u_\theta(\bar r=\varepsilon) = \frac{w_1}{\omega R_1} = \frac{A}{\varepsilon} + C\cos\theta, \quad t=0 \quad (45)$$

$$u_\theta(\bar r=1) = \frac{w_2}{\omega R_2} = A + C\cos\theta, \quad t=0 \quad (46)$$

$$W_1 = \left(\frac{A}{\varepsilon} + C\cos\theta\right)\omega R_1, \quad t=0 \quad (47)$$

$$W_2 = (A + C\cos\theta)\,\omega R_2, \quad t=0 \quad (48)$$

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$$C K \bar{r} e^{kt} \sin\theta + \frac{A^2}{\bar{r}^2} + \frac{A C e^{kt} (\cos\theta + \sin\theta)}{\bar{r}} = -p \tag{49}$$

$$C K e^{kt} \sin\theta - \frac{2A^2}{\bar{r}^3} - \frac{A C e^{kt} (\cos\theta + \sin\theta)}{\bar{r}^2} = \frac{-\partial p}{\partial \bar{r}} \tag{50}$$

$$C K \bar{r} e^{kt} \cos\theta + \frac{A C e^{kt} (\sin\theta + \cos\theta)}{\bar{r}} = \frac{-\partial p}{\partial \theta} \tag{51}$$

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