

N – Factor Satisfactory Roommates Problem With Three Roommates

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Abstract

The Three Person Satisfactory Roommates Problem (TPSRP) involves three people and a preference list that each of them will receive for their two partners. Each member of the set might be ranked in the order of preference with other $(3n-1)$ members. A set of triples is referred to as a matching. In this paper, we extend the single preference list case into N - Factor TPSRP, which contains multiple factors; each member of the list provides their preference list; a better matching can be achieved by taking into account all the factors in the end; an example is provided to demonstrate the technique. This paper presents a novel sophisticated algorithm for finding perfect triples in rooms.

Keywords: Preference value, N - factors, perfect matching, three-person roommates, modified preference value matrix.

1. INTRODUCTION

One of the stable matching (SM) problems initially presented by Gale and Shapley [1] is the stable roommates' problem (SRP). Since the set of stable matching remains the same if we convert an SM instance into an SRP instance by adding all other members who are of the exact same gender to the very end of each member's preference list, it is widely known that the stable roommates problem is an extension of the stable marriage problem [2, 3]. Irving [4] presented an $O(m)$ technique that determines if a stable matching exists or not given an instance of SRP. The methodology is based on the suppositions that n is even and all preference lists in I are full (that is, $A_i = A \setminus \{a_i\}$ for each $a_i \in A$). A sample of SRP consists of two people, each of whom has a preference list that ranks the other two people in total order based on his preferences. An algorithm for the roommate problem was investigated by Linden et al. [5]. It matched soulmates—individuals who share a preference for each other—first and then iterated through the procedure. With two roommates and explicit preferences, the original roommate problem is the focus of all this research.

Bredereck et al. [6] in 2020 explored a unique variation of multidimensional roommate games with a central list or poset that acts as the source of all agents' preference lists. McKay and Manlove [7] formalized a three-dimensional variant using generic additively separable preferences in which each agent provides an integer evaluation of each successive agent. This variant shows that the related preference issue is NP-complete and that a stable matching may not happen even in situations when the valuations are binary. Park [8] indicated that it is possible to find a stable matching in polynomial

3. SATISFACTORY MATCHING ALGORITHM FOR N- FACTOR ROOMMATES

1. Get the preference lists from each person in each factor.
2. Form a preference value matrix based on their preference lists.
3. Summing all the Preference Value Matrix (PVM) for each factor.
4. Considering the overall preference value matrix as a Maximization assignment problem.
5. Construct a Minimized Preference Value Matrix and applying Hungarian algorithm for that matrix.
6. The Resultant pairs must be the optimum pairs like $(i, j), (k, l), (m, n)$ and so on obtained.
7. Construct the Modified Preference Value matrix by considering the pairs $(i, j), (k, l), (m, n) \dots$ as rows and $1, 2, \dots, 3n$ members as column. By using MPVM definition which is given above.
8. Considering the Modified preference value matrix as a minimized assignment problem and apply Hungarian Algorithm for getting the optimum triples.
9. List out all the triples and let it be $(i, j, k), (l, m, n), (i, l, n) \dots$
10. From the obtained triples, choose one by one and find the satisfactory value of each member of a group.
11. If (i, j, k) be the first triples, find the preference value of i with respect to j and k , then adding the preference values. Now we get the overall preference value and multiply the value by 50. It gives a satisfactory value of i with respect to j and k . similarly find the satisfactory value of j w.r.to i & k , also for k w.r.to i & j .
12. After getting these three satisfactory values, find overall satisfaction for the triple (i, j, k) . In the same manner repeat the process for all the remaining triples.
13. Now choose mutually exclusive and exhaustive triples which achieves the maximum level of satisfaction that be the optimum triples.

4. EXAMPLE

Consider the problem instance of TPSRP based on order of preference, here all the students are considering the Intelligence factor of the remaining students.

1	6	4	3	5	2
2	3	4	5	1	6
3	5	2	6	4	1
4	1	2	6	3	5
5	3	1	2	6	4
6	4	2	1	5	3

SOLUTION:

The given preference list can be constructed as a preference value matrix by considering first person as $\frac{3n-2}{n-2}$, second person as $\frac{3n-2}{n-2}$, third person as $\frac{3n-3}{n-2}$, fourth person as $\frac{3n-4}{n-2}$, fifth person as $\frac{3n-5}{n-2}$. The preference values are presented in the form of matrix which is given below
The preference value matrix for the Intelligence factor

$$PVM = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix} & \begin{pmatrix} - & \frac{1}{4} & \frac{3}{4} & \frac{4}{4} & \frac{2}{4} & \frac{4}{4} \\ \frac{2}{4} & - & \frac{4}{4} & \frac{4}{4} & \frac{3}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{4}{4} & - & \frac{2}{4} & \frac{4}{4} & \frac{3}{4} \\ \frac{4}{4} & \frac{4}{4} & \frac{2}{4} & - & \frac{1}{4} & \frac{3}{4} \\ \frac{4}{4} & \frac{3}{4} & \frac{4}{4} & \frac{1}{4} & - & \frac{2}{4} \\ \frac{3}{4} & \frac{4}{4} & \frac{1}{4} & \frac{4}{4} & \frac{2}{4} & - \end{pmatrix} \end{matrix}$$

Consider the problem instance of TPSRP based on order of preference, here all the students are considering the sports interest factor of the remaining students.

1	4	5	6	2	3
2	5	1	3	6	4
3	2	6	5	4	1
4	6	5	1	3	2
5	3	4	1	2	6
6	1	2	4	5	3

The preference values are presented in the form of matrix which is given below
The preference value matrix for the sports interest factor

$$PVM = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix} & \begin{pmatrix} - & \frac{2}{4} & \frac{1}{4} & \frac{4}{4} & \frac{4}{4} & \frac{3}{4} \\ \frac{4}{4} & - & \frac{3}{4} & \frac{1}{4} & \frac{4}{4} & \frac{2}{4} \\ \frac{1}{4} & \frac{4}{4} & - & \frac{2}{4} & \frac{3}{4} & \frac{4}{4} \\ \frac{3}{4} & \frac{1}{4} & \frac{2}{4} & - & \frac{4}{4} & \frac{4}{4} \\ \frac{3}{4} & \frac{2}{4} & \frac{4}{4} & \frac{4}{4} & - & \frac{1}{4} \\ \frac{4}{4} & \frac{4}{4} & \frac{1}{4} & \frac{3}{4} & \frac{2}{4} & - \end{pmatrix} \end{matrix}$$

Consider the problem instance of TPSRP based on order of preference, here all the students are considering the Music interest factor of the remaining students.

1 6 2 3 5 4
 2 5 1 3 6 4
 3 1 2 4 5 6
 4 5 3 2 6 1
 5 2 4 1 6 3
 6 4 5 3 2 1

The preference values are presented in the form of matrix which is given below
 The preference value matrix for the Music interest factor

$$PVM = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix} & \begin{pmatrix} - & \frac{4}{4} & \frac{3}{4} & \frac{1}{4} & \frac{2}{4} & \frac{4}{4} \\ \frac{4}{4} & - & \frac{3}{4} & \frac{1}{4} & \frac{4}{4} & \frac{2}{4} \\ \frac{4}{4} & \frac{4}{4} & - & \frac{3}{4} & \frac{2}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{3}{4} & \frac{4}{4} & - & \frac{4}{4} & \frac{2}{4} \\ \frac{3}{4} & \frac{4}{4} & \frac{1}{4} & \frac{4}{4} & - & \frac{2}{4} \\ \frac{1}{4} & \frac{2}{4} & \frac{3}{4} & \frac{4}{4} & \frac{4}{4} & - \end{pmatrix} \end{matrix}$$

Consider the problem instance of TPSRP based on order of preference, here all the students are considering the Spiritual interest factor of the remaining students.

1 4 6 5 3 2
 2 1 5 6 3 4
 3 5 6 2 1 4
 4 1 6 5 2 3
 5 4 1 3 6 2
 6 4 1 3 2 5

The preference values are presented in the form of matrix which is given below

The preference value matrix for the Spiritual interest factor

$$PVM = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix} & \begin{pmatrix} - & \frac{1}{4} & \frac{2}{4} & \frac{4}{4} & \frac{3}{4} & \frac{4}{4} \\ \frac{4}{4} & - & \frac{2}{4} & \frac{1}{4} & \frac{4}{4} & \frac{3}{4} \\ \frac{2}{4} & \frac{3}{4} & - & \frac{1}{4} & \frac{4}{4} & \frac{4}{4} \\ \frac{4}{4} & \frac{2}{4} & \frac{1}{4} & - & \frac{3}{4} & \frac{4}{4} \\ \frac{4}{4} & \frac{1}{4} & \frac{3}{4} & \frac{4}{4} & - & \frac{2}{4} \\ \frac{4}{4} & \frac{2}{4} & \frac{3}{4} & \frac{4}{4} & \frac{1}{4} & - \end{pmatrix} \end{matrix}$$

Consider the problem instance of TPSRP based on order of preference, here all the students are considering the Cultural interest factor of the remaining students.

1 2 6 4 3 5
 2 3 1 6 5 4
 3 2 5 4 6 1
 4 5 2 1 3 6
 5 6 1 4 3 2
 6 5 4 3 1 2

The preference values are presented in the form of matrix which is given below

The preference value matrix for the Cultural interest factor

$$PVM = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix} & \begin{pmatrix} - & \frac{4}{4} & \frac{2}{4} & \frac{3}{4} & \frac{1}{4} & \frac{4}{4} \\ \frac{4}{4} & - & \frac{4}{4} & \frac{1}{4} & \frac{2}{4} & \frac{3}{4} \\ \frac{1}{4} & \frac{4}{4} & - & \frac{3}{4} & \frac{4}{4} & \frac{2}{4} \\ \frac{3}{4} & \frac{4}{4} & \frac{2}{4} & - & \frac{4}{4} & \frac{1}{4} \\ \frac{4}{4} & \frac{1}{4} & \frac{2}{4} & \frac{3}{4} & - & \frac{4}{4} \\ \frac{2}{4} & \frac{1}{4} & \frac{3}{4} & \frac{4}{4} & \frac{4}{4} & - \end{pmatrix} \end{matrix}$$

Summing all the Preference Value Matrix,

$$PVM = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix} & \left(\begin{array}{cccccc} - & \frac{12}{4} & \frac{11}{4} & \frac{16}{4} & \frac{12}{4} & \frac{19}{4} \\ \frac{18}{4} & - & \frac{16}{4} & \frac{8}{4} & \frac{17}{4} & \frac{11}{4} \\ \frac{9}{4} & \frac{19}{4} & - & \frac{11}{4} & \frac{17}{4} & \frac{14}{4} \\ \frac{15}{4} & \frac{14}{4} & \frac{11}{4} & - & \frac{16}{4} & \frac{14}{4} \\ \frac{18}{4} & \frac{11}{4} & \frac{14}{4} & \frac{16}{4} & - & \frac{11}{4} \\ \frac{14}{4} & \frac{13}{4} & \frac{11}{4} & \frac{19}{4} & \frac{13}{4} & - \end{array} \right) \end{matrix}$$

From the above preference value matrix to construct the minimized preference value matrix by subtracting all the elements in the matrix from the highest element in the matrix. Then the Minimized preference value matrix given below

The minimized preference value matrix

$$mPVM = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix} & \left(\begin{array}{cccccc} - & \frac{7}{4} & \frac{8}{4} & \frac{3}{4} & \frac{7}{4} & 0 \\ \frac{1}{4} & - & \frac{3}{4} & \frac{11}{4} & \frac{2}{4} & \frac{8}{4} \\ \frac{10}{4} & 0 & - & \frac{8}{4} & \frac{2}{4} & \frac{5}{4} \\ \frac{4}{4} & \frac{5}{4} & \frac{8}{4} & - & \frac{3}{4} & \frac{5}{4} \\ \frac{1}{4} & \frac{8}{4} & \frac{5}{4} & \frac{3}{4} & - & \frac{8}{4} \\ \frac{5}{4} & \frac{6}{4} & \frac{8}{4} & 0 & \frac{6}{4} & - \end{array} \right) \end{matrix}$$

Table 1. Individual and overall satisfaction of possible triples

S.NO		INDIVIDUAL SATISFACTION		
1	(1,4,6)	1 → 4 & 6 = 87.5%.	80.83%	79.5
		4 → 1 & 6 = 72.5%.		
		6 → 1 & 4 = 82.5%.		
2	(2,3,5)	2 → 3 & 5 = 82.5%.	78.3%	
		3 → 2 & 5 = 90%.		
		5 → 2 & 3 = 62.5%.		
3	(1,2,3)	1 → 2 & 3 = 57.5%.	70.83%	72.4
		2 → 1 & 3 = 85%.		
		3 → 1 & 2 = 70%.		
4	(4,5,6)	4 → 5 & 6 = 75%.	74.16%	
		5 → 4 & 6 = 67.5%.		
		6 → 4 & 5 = 80%.		
5	(1,3,5)	1 → 3 & 5 = 57.5%.	67.5%	66.65
		3 → 1 & 5 = 65%.		
		5 → 1 & 3 = 80%.		
6	(2,4,6)	2 → 4 & 6 = 47.5%.	65.8%	
		4 → 2 & 6 = 70%.		
		6 → 2 & 4 = 80%.		

Table 1 depicts the results that the roommates 1,4 and 6 having the highest percentage as the roommates. As well as 2,3 and 5 also having the highest percentage as the roommates. So we conclude that the triples (1,4,6) and (2,3,5) are the satisfactory roommates.

5. CONCLUSION

In this research, we defined a preference value for the preference lists of roommate instances with three-person rooms, and we developed an algorithm for N factors TPSMA. First, this method produces a perfectly satisfactory pair matching; second, it produces a triple matching. Examining the data, we find that the acquired matching produces excellent triple matching. Upon analysis of the data, we deduce that the optimal level of satisfaction is reached by perfect triple matching. We have demonstrated the rapidness and effectiveness of TPSMA. Thus, the roommate problem's N-Factor Satisfactory matching with all other extended situations is an outcome of TPSMA.

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