Multiple Attributes Decision Making Approach by TOPSIS Technique

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Abstract

Under the many conditions, crisp data are inadequate to model real-life situations. Human judgments include preferences, which are often vague and cannot be expressed as an exact numerical value. Decision making problem is the process of finding the best option from all of the feasible alternatives. In this paper, from among multi-criteria models in making complex decisions and multiple attributes for most preferable choice, technique for order preference by similarity to ideal solution (TOPSIS) approach has been dealt with. Finally, implementing TOPSIS algorithm, assessment of projects has been done. The results have been tested in numerical example.

AMS Classification number (2010): 11Y16, 90B50

Keywords: MADM, SAW method, TOPSIS method, PIS, NIS.

1. Introduction

In the last two decades, fuzzy set theory has provided a new research direction of both concepts and methodologies to formulate and solve mathematical programming and objective decision making problems. The evaluations of alternatives with respect to some attributes are uncertain and vague, fuzzy set theory has been used. By merging fuzzy set theory and multiple-attribute decision making a new decision support system (DSS), namely fuzzy decision method (FDM), has been developed to compare different alternatives with respect to the attributes as crisp variables, and linguistic variables.

Bellman and Zadeh [1] provide the first fuzzy set theory in decision making. Chen and Hwang [6] gave a comprehensive state of the art in fuzzy multiple attribute decision making (FMADM). Triantaphyllou and Lin [18] evaluated five FMADM methods: fuzzy SAW model, fuzzy weighted product model, fuzzy AHP, revised fuzzy AHP and fuzzy TOPSIS. There are many works in the literature on application of FMADM methods in various fields.

This paper develops a new systematic approach in order to extend the SAW and TOPSIS to the fuzzy environment when the inputs are not only linguistic but also fuzzy and crisp numbers. This method is suitable for solving the MADM problems in a fuzzy environment. Initially, it is needed to describe some terms in fuzzy sets. Fuzzy sets were introduced by Zadeh [19], as a means of representing and manipulating data that was not precise, but rather fuzzy. In section 2, we introduce MADM, Fuzzy MADM and preliminary definitions, In section 3, we have presented our methodology. In section 4, numerical example has been described. Finally, concluding remarks are provided.

2. Preliminary

2.1 Multiple Attribute Decision Making

Decisions making is part of our daily lives. Almost all decision problems have multiple, usually conflicting, criteria. How to solve such problems has been enormous. Methodologies as well as their applications, appear in professional journals of different disciplines. The problems may be classified into two categories: (i) Multiple Attribute Decision Making (MADM) and (ii) Multiple Objective Decision Making (MODM).

MADM refers to making selections among some courses of action in the presence of multiple, usually conflicting, attributes. For example, one may choose a job depending on salary, work location, promotion opportunity, colleagues, etc. Water resources development plans for a community should be evaluated in terms of cost, possibility of water shortage, energy, flood protection, water quality, etc. We can go forever: individuals, organizations, societies, and even whole nations face many problems of this type. A MADM problem can be concisely expressed in a matrix form as:

	C_1	C_2	C_3	….	C_n
A_1	X_{11}	X_{12}	X_{13}	….	X_{1n}
A_2	X_{21}	X_{22}	X_{23}	….	X_{2n}
…..	…..	…..	…..	….	…..
A_m	X_{m1}	X_{m2}	X_{m3}	….	X_{mn}

where A_i, $(i = 1,2, \ldots\ldots, m)$ are possible courses of actions; C_j , $(j = 1,2, \ldots, n)$ are attributes with which alternative are measured, and X_{ij} is the performance of alternative A_i with respect to attribute C_j. Many of the basic concepts of these classical MADM methods are used in fuzzy MADM methods.

2.2 Fuzzy Multiple Attribute Decision Making

Fuzzy MADM methods basically consist of two phases: (i) aggregation of performance scores with respect to all attributes for each alternative, and (ii) rank ordering of alternatives according to aggregated scores. We will refer to results of the first and second phases using the terms “final rating” and “ranking order” respectively. For a crisp MADM problem, final ratings are expressed as real numbers and ranking order can be easily obtained by comparing these real numbers. The main focus of MADM problem solving is the first phase. In a fuzzy MADM problem, performance scores of an alternative with respect to all attributes may be expressed by linguistic data or fuzzy sets. As a result, the final ratings are expressed by linguistic data or fuzzy sets. Obtaining a ranking order of these fuzzy sets is not a trivial task. In this case, both phase one and phase two are important in solving a fuzzy MADM problem.

Chan and Hwang [5] and Chen, Hwang and Lai [7] classified fuzzy ranking methods based on two factors: (i) the comparison medium used, and (ii) the technique need to develop the comparison medium. According to Chen, Hwang and Lai, classification of fuzzy MADM involves the following five stages.

Stage 1. Size of a MADM problem is characterized by the number of attributes and number of alternatives. Fuzzy MADM methods are suitable for solving a problem that has either less than ten alternatives and ten attributes, or any number of attributes and less than 350 attributes.

Stage 2. Data type allowed by each method can be: (i) all fuzzy, (ii) all fuzzy singleton, (iii) all crisp, or (iv) a mixture of fuzzy and crisp. Real world MADM problems contain a mixture of fuzzy and crisp data.

Stage 3. Basic concept of fuzzy MADM methods are derived mainly from classical MADM methods whose basic concepts were adopted include simple additive weighing (SAW) method, Technique for Order Performance by Similarity to Ideal Solution (TOPSIS), analytic hierarchical process (AHP) method, conjunctive method, disjunctive method, multiple attribute utility function (MAUF) theory, out ranking method, maxi-min, and general classical MADM methods.

Stage 4. Techniques are required to apply each fuzzy MADM method. They include α-cut, fuzzy arithmetic operations, eigenvector methods, weight assessing method, possibility and necessity measures, human intuition, fuzzy ranking and fuzzy arithmetic, fuzzy out ranking relation, maximum and minimum operators, and semantic modeling (linguistic data $\rightarrow$ fuzzy data $\rightarrow$ crisp number).

Stage 5. Major approaches in any branch formed from the previous four stages are listed.

2.3 Definitions

1. Fuzzy set: A fuzzy set A in X is characterized by a membership function $\mu_A(x)$ which associates with each point x a real number in the interval $[0,1]$ representing the grade of membership of x in A.

Mathematically, $A = \{(x, \mu_A(x)); x \in X\}$, where $\mu_A(x): X \to [0,1]$. If $\mu_A(x) = 1$, then $x \in A$; if $\mu_A(x) = 0$, then $x \in A$. Space X is called the universe of discourse.

2. Fuzzy number: A fuzzy set A is a fuzzy number if the universe of discourse X is R and the fuzzy set A is convex, normal, the membership function of the fuzzy set $\mu_A(x)$ is piecewise continuous, and the core of the fuzzy set consists of one value only ($\mu_A(x) = 1$).

2.4 Decision Support System

Fuzzy decision making (FDM) has been developed as a powerful DSS in a user friendly, interactive decision making environment. FDM merges fuzzy set theory MADM methods to have a FMADM method. The software embodies an expert system whose duty is to choose appropriate methods from SAW, fuzzy SAW, TOPSIS or fuzzy TOPSIS based on the structure of the problem. The corresponding rules used in the expert system are fired depending on the number of attributes, number of alternatives and type of the evaluations in a decision matrix. The rules of expert system are as follows:

- If all the evaluations in decision matrix are crisp, FDM does not use the fuzzy logic and it will use TOPSIS or SAW.

- If the number of alternatives with respect to the number of attributes is less than half, either SAW or fuzzy SAW will be selected, else FDM uses TOPSIS or fuzzy TOPSIS.

In FDM, evaluations of alternatives versus attribute can be crisp variables, linguistic variables. Also the importance weights of the attributes are considered to be linguistic variables or crisp number**.**

3. Proposed Methodology

Hwang and Yoon [9] provide a good survey for state of the art on MADM methods. Here, two methods namely: Simple additive weighting (SAW) and Technique for order performance of similarity to ideal solution (TOPSIS) are introduced.

SAW Method

Suppose the evaluation of any alternative with respect to each attribute is known in the form of a decision matrix $D[X_{ij}]$, $i = 1, \dots, m; j = 1, \dots, n$.

Step 1. Transform the real values of evaluations of alternatives with respect to the attributes into non dimensional units to allow comparisons across them. In this study, the evaluations have been divided by the maximum if the relevant attribute is positive (1a), else the minimum of the evaluations have been divided by the evaluation in the relevant attribute (1b).

$$r_{ij} = \frac{X_j}{Max_i(X)} \quad (1a) \quad \text{and} \quad r_{ij} = \frac{Min_i(X)}{X_j} \quad (1b)$$

Step 2. Accommodate a set of weights $w_j, (j = 1,2, \dots, n)$, obtained by decision maker (DM) for each attribute and calculate the weighted normalized decision matrix V by multiplying each of the matrix R by its associated weight w_j:

$$V = [w_j\, r_j]\,,\ j = 1, \dots, n$$

Step 3. Calculate the combined measure of goodness for each alternative: $S_j = \sum_{j=1}^{n} w_j\, r_j$

Step 4. Rank the alternatives in descending order of S_i .

TOPSIS Method

TOPSIS is based upon the concept that the chosen alternative should have the shortest distance from the ideal solution and farthest from the negative ideal solution [9]. Assume that each alternative takes the monotonically increasing (or decreasing) utility. It is then easy to locate the ideal solution, which is a combination of all the best attribute value attainable, while the negative ideal solution is a combination of all the worst attribute values attainable. One approach is to take an alternative that has the minimum (weighted) Euclidean distance to the ideal solution of the TOPSIS method consists of the following steps:

Step 1 Calculate the normalized decision matrix. The normalized value N_{ij} is calculated as

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 2 Issue 11, November - 2013

$N_{ij} = x_{ij} / \sqrt{\sum_{i=1}^{m} x_{ij}^2}\, , i = 1, \ldots, m\, , j = 1, \ldots, n.$ (2)

Step 2 Calculate the weighted normalized decision matrix. The weighted normalized value V_{ij} is calculated as $V_{ij} = W_j N_{ij}$, $i = 1, \ldots, m$, $j = 1, \ldots, n$, where W_j is the weight of the i th attribute and $\sum_{j=1}^{n} W_j = 1$.

Step 3 Actually A^+, A^- are not absolute values but represent the best or worst evaluation among the different alternatives analyzed in the matrix V.

$$A^+ = \{V_1^+, V_2^+, \ldots, V_n^+\}, \text{ where } V_j^+ = \left(max_i V_{ij} \middle| j \in J\right) \quad (3)$$

and

$$A^- = \{V_1^-, V_2^-, \ldots, V_n^-\}, \text{ where } V_j^- = \left(min_i V_{ij} \middle| j \in J\right) \quad (4)$$

where J is associated with benefit attributes.

Step 4 Compute the distance of each alternative from the positive ideal solution by

$$D_i^+ = \sqrt{\sum_{j=1}^{n} \left(V_{ij} - V_j^+\right)^2}, \text{ for } i = 1, \ldots, m \quad (5)$$

and the distance of each alternative from the negative ideal solution by

$$D_i^- = \sqrt{\sum_{j=1}^{n} \left(V_{ij} - V_j^-\right)^2}, \text{ for } i = 1, \ldots, m \quad (6)$$

Step 5 Compute the relative combined measure of goodness for the alternative A_i with respect to A^+ as: $R_i = D_i^- / (D_i^+ + D_i^-)$, $i = 1, \ldots, m$

Since $D_i^- \geq 0$ and $D_i^+ \geq 0$, then $R_i \in [0,1]$.

Step 6 Rank the preference order.

For ranking alternatives using this index, we can rank alternatives in decreasing order. The basic principle of the TOPSIS technique, chosen the alternative should have the "shortest distance" from the positive ideal solution and the "farthest distance" from the negative ideal solution.

4. Numerical Example

The economic and financial analysis of the project is based on the comparison of the case flow of all costs and benefit resulting from the activities. There are four methods of comparing alternative investments i.e. net present value (NPV), rate of return (ROR), benefit cost analysis and payback period (PBP). Each of these is dependent on a selected interest rate or discount rate to adjust cash flows at different points in time [10]. Assume that the management wants to choose the best project amongst all proposed projects.

Table-1
Decision Matrix

Weight	0.1	0.4	0.3	0.2
	ROR	PBP	NPV	BCA
Project 1	18	7	9500	2.0
Project 2	19	7	9700	2.8
Project 3	16	8	8500	2.9
Project 4	17	9	9000	3.1
Project 5	15	6	8800	3.0
Project 6	18	7	9150	2.7
Project 7	19	8	9450	2.6
Project 8	16	6	9300	2.4

Table-2
S.M. D_i^+ of each alternative

D_1^+	0.0482
D_2^+	0.0395
D_3^+	0.0211
D_4^+	0.0048
D_5^+	0.0587
D_6^+	0.0402
D_7^+	0.0233
D_8^+	0.0611

Table-3
S.M. D_i^- of each alternative

D_1^-	0.0204
D_2^-	0.0298
D_3^-	0.0453
D_4^-	0.0648
D_5^-	0.0261
D_6^-	0.0273
D_7^-	0.0426
D_8^-	0.0109

Table-4
Score of each project

Sl. No.	Score	Rank
Project 1	0.2974	7
Project 2	0.4300	4
Project 3	0.6822	2
Project 4	0.9310	1
Project 5	0.3078	6
Project 6	0.4044	5
Project 7	0.6464	3
Project 8	0.1514	8

The ranking order of all the alternatives by calculating the distance of both the “positive ideal solution” and the “negative ideal solution” are given in Table-2 and Table-3. As it is shown in Table-4, project 4 can gain the best score among all projects.

5. Conclusion

The evaluation and selection of industrial projects before investment decision is customarily done using technical and financial information. In this paper, Author proposed a new methodology to provide a simple approach to assess alternative projects and help decision maker to select the best one. In this approach, we considering the distance of an alternative from the positive ideal solution its distance from the negative ideal solution is also considered. The less distance of the alternative under evaluation from positive ideal solution and more its distance from the negative ideal solution, the better its ranking.

References

[1] Bellman, R.A. and Zadeh, L.A., (1970) Decision Making in Fuzzy Environment, Management Science, 17, pp. 141-164.

[2] Bender, M.J., and Simonovic (2000) A Fuzzy Compromise Approach to Water Resource Systems Planning under Uncertainty, Fuzzy Sets and Systems, 115, pp. 35-44.

[3] Bonissone, P.P. (1982) A Fuzzy Sets Based Linguistic Approach: Theory and Applications, in Gupta, M.M. and Sanchez, E. (Eds.) Approximate Reasoning in Decision Making Analysis (North-Halland) pp. 329-339.

[4] Chen, C.T. (2000) Extensions of the TOPSIS for group decision-making under fuzzy environment, Fuzzy Sets and Systems, 114, pp. 01-09.

[5] Chen, S.J. and Hwang, C.L., Fuzzy Mathematical Attribute Decision Making: Methods and Applications (Springer-Verlag, Heidelberg, 1992).

[6] Chen, S.J. and Hwang, C.L., (1991) Fuzzy Multiple Attribute Decision Making (Berlin, Springer-Verlag).

[7] Chen, S.J., Hwang, C.L. and Lai, Y.J., Fuzzy multiple attribute decision making, Fuzzy Sets and Systems, 1993.

[8] Compos, L., Fuzzy linear programming models to solve fuzzy matrix games, Fuzzy Sets and systems, 32, 1989, 271-289.

[9] Hwang, C.L. and Yoon, K., (1981) Multiple Attribute Decision Making: Methods and Applications (Berlin, Springer-Verlag).

[10] Hwang, C.L. and Lin, M.J., Group Decision Making under Multiple Criteria: Methods and Applications (Springer-Verlag, Heidelberg, 1987).

[11] Kacprzyk, J. and Yager, R.R., Management Decision Support Systems using Fuzzy Sets and Possibility Theory (Verlag TUV Rheinland, New York, 1985).

[12] Jahanshahloo, G.R., Hosseinzadeh Lotfi, F. and Izadikhah, M., (2006) Extension of the TOPSIS method for decision making problems with fuzzy data, Applied Mathematics and Computation. 181, pp 1544-1551.

[13] Lai, Y.J. and Hwang, C.L., Fuzzy Mathematical Programming: Methods and Applications (Springer-Verlag, Heidelberg, 1992) LNEMS 394.

[14] Leung, Y., Spatial Analysis and Planning under Imprecision (North-Holland, Amsterdam, 1988).

[15] Lockett, G. and Stratford, M., Ranking of research projects, experiments with two methods, Omega 15, pp. 395-400, 1987.

[16] Prakash, T.N., (2003) Land Suitability Analysis for Agricultural Crops: A Fuzzy Multicriteria Decision Making Approach, M.Sc. Thesis, International Institute for Geo-Information Science and Earth Observation, Enschede, Netherlands, December.

[17] Slowinski, R. and Teghem, J., Stochastic Versus Fuzzy Approaches in Multi-Objective Mathematical Programming under Uncertainty (Kluwer, Dordrecht, 1990).

[18] Triantaphyllou, E. and Lin, C.T. (1996) Development and Evaluation of Five Fuzzy Multi-attribute Decision Making Method, International Journal of Approximate Reasoning, 14, pp. 281-310.

[19] Zadeh, L.A., (1965) Fuzzy Sets, Information and Control, 8, pp. 338-353.

[20] Zarghaami, M., Ardakanian, R. and Memariani, A., (2007) Fuzzy Multiple Attribute Decision Making on Water Resources Projects Case Study: Ranking Water Transformations to Zayanderud Association Water International, Volume 32, No. 2, pp. 281-293, June 2007.

[21] Zimmermann, H.J., Zadeh, L.A. and Gaines, B.R., Fuzzy Sets and Decision Analysis (Elsevie Science, 1984).