MRC Receiver in Correlated Hoyt Fading Channel with Multiple Modulation Technique

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Abstract—In this paper the performance of diversity receivers and M-ary modulation over Hoyt fading channels is analysed. M-ary modulation scheme has been tested on Hoyt fading channel as it is more realistic representation of fading in satellite link. We have tested it for 16-PSK, 4-PSK and 32 PSK scheme for various fading and correlation coefficient parameters of Hoyt channel. Hoyt channel is derived form nakagami-m channel and also known as nakagami-q channel. The classical PDF based approach has been followed to derive the performance measures of basic diversity combiner namely Maximal ratio combining (MRC) receiver over Hoyt. The analysis is carried out for both independent and correlated fading channels for various coherent and noncoherent modulation schemes. For independent diversity receivers the analysis has been carried out for arbitrary number of input branches. The effect of diversity order and fading parameters on performance measures is studied with the help of the numerical evaluation of the obtained expressions. For dual correlated receivers the analysis is carried out for arbitrary correlation, whereas for L diversity receivers it is for most important practical correlation models exponential correlation. Exponential correlation is used to model the system when the receiving antennas are placed in a linear array. The effect of correlation on the receiver performance is studied for all the systems. To validate the derived expressions Monte carlo simulation is performed.

Keywords—M-Array Modulation; Diversity; Nakagami-m fading channel.

I. INTRODUCTION

In past few years, wireless communication has played an important role in information technology as information can be transmitted without the need of dedicated link between transmitter and receiver unlike wired communication, where a dedicated link/channel exist between transmitter and receiver. Compared to wired communication systems, wireless systems introduce a very interesting feature ‘mobility’. In any kind of communication, wired or wireless, there are some parameters like bandwidth, transmitted power, data rate etc. which decide the reliability of a system. The one which optimizes all of them is said to be a perfect system. In recent years, lots of research has been done on both kinds of communication so that a reliable system can be designed with high bandwidth, low transmitted power, high data rates and low bit or symbol error probability.

A. Wireless Communication System

In a wireless communication system, data is transmitted in the form of electromagnetic waves using antennas. When signal propagates through the wireless media, phenomena such as reflection and scattering through buildings, trees etc. and refraction through the edges causes the signal to follow multiple paths having different path loss factors and different delays. Thus, at the receiver, the received signal consists of multiple copies of same information bearing signal having different amplitudes and different phases arising due to different path lengths. Figure 1 shows a typical urban/suburban mobile radio environment. In the figure the direct path between the transmitter and the receiver is called line-of-sight (LOS) path,

![Figure 1 Paths between transmitter and receiver](image)

Whereas, the path corresponding to reflected signal is called on line-of-sight (NLOS) path. These multipaths have different phases corresponding to different path-delays, so that they interfere at the receiver either constructively or destructively resulting in variation in signal-to-noise ratio. In addition, mobility introduces time variation in channel response, i.e. if a very short pulse is transmitted; the received signal appears as a train of pulses due to presence of multipath. Secondly, as a result of time varying response, if same procedure is followed multiple times, a change is observed in the received pulse train over time, which will include changes in the sizes of individual pulses, changes in relative delays among the pulses and often, changes in the number of pulses observed. Hence, the equivalent low-pass time varying impulse response of the channel can be modelled as [1]:

\[ c(\tau; t) = \sum_i \alpha_i(t)e^{-j2\pi f_c \tau_i(t)} \delta[t - \tau_i(t)] \]
Where, $\alpha_i(t)$ and $\tau_i(t)$ are time varying attenuation factor and path delay for ith path respectively. For a transmitted signal $s(t) = 1$ the received signal for the case of discrete multipath is given by [1]:

$$r(t) = \sum_i \alpha_i(t)e^{-j\theta_i(t)}$$

The $\alpha_i(t)$ and $\tau_i(t)$ associated with different signals vary at different rates and in random manner. So, received signal $r(t)$ can be modelled as a random process. For large number of paths, central limit theorem can be applied and $r(t)$ can be modeled as complex-valued Gaussian random process i.e. $c(\tau ;t)$ is also a complex-valued random process in $t$ variable [1].

II. DIVERSITY COMBINING TECHNIQUES

There are different types of diversity combining techniques used in practice [2], which are as follows:

A. Maximal Ratio Combining

In maximal ratio combining technique, the received multiple faded copies of the transmitted signal are co-phased. The co-phased signal copies are weighted individually in proportion to their strength to maximize SNR at the output of the combiner. Assuming the received signal SNR at the input of the combiner is $\gamma_i$, $i = 1, 2, ..N$, the output SNR can be shown to be [5]:

$$\gamma_{MR} = \sum_{i=1}^{N} \gamma_i$$

The MRC operation requires estimation of phase and amplitude of each received input branch signal. Hence, the complexity of implementation is high.

B. Equal Gain Combining

Different weights for each branch may not be convenient as it may increase the complexity of the receiver as in the case of MRC. So it is convenient to set all the gains to unity, while co-phasing all signals before combining [2]. This technique of combining is called Equal Gain Combining.

C. Selection Combining

In selection combining (SC), the system chooses the received signal having maximum SNR out of all copies of signals received. In this scheme the output SNR can be given as [2]:

$$\gamma_{SC} = \max\{\gamma_1, \gamma_2, ..., \gamma_N \}$$

D. Switch and Stay Combining

The switch and stay combining (SSC) technique discussed here is presented in [2] and also shown in Figure 4. In this system, there are only two copies of fading signals are used. The combiner has only two antennas to receive fading signals. The received signal is fed as shown in Figure 4. In this scheme the received SNR $\gamma_1$ at antenna L1 is compared with a predefined threshold $\gamma_T$. Switching occurs to the input branch L2 if $\gamma_1 < \gamma_T$. And it again switches to first branch if $\gamma_1 > \gamma_T$. It may happen that after switching the input SNR $\gamma_2$ at L2 is less than $\gamma_T$ or even less than $\gamma_1$, in such case the switch will still be connected to L2 until the SNR of first branch becomes greater than $\gamma_T$. Switching from branch L2 to branch L1 is done in similar manner.

E. Switch and Examine Combining

Unlike SSC combining scheme, switch and examine combining (SEC) adds the benefit of having multiple branches at the receiver, especially when they are independent and identically distributed (i.i.d.) or equicorrelated and identically distributed. In SSC scheme, receiver switches between the
best two paths, adding a path does not improve the performance unless the added path is better than at least one of the best two ones. In SEC combining scheme, the receiver starts examining from the first path. If first path is acceptable, it continues to receiver from it, else, it switches and examines the next available path. This process continues until an acceptable path is found or all paths have been examined. In the latter case, the receiver stays on the last examined path or selects the best path for reception [7].

III. PROPOSED WORK

In our work we have analyzed the Hoyt fading channel performance in two different systems. One is M-ary PSK simulation and other is MRC diversity scheme in Monte-carlo simulation for correlated Hoyt fading channel. The motivation behind MPSK is to increase the bandwidth efficiency of the PSK modulation schemes. In BPSK, a data bit is represented by a symbol. In MPSK, \( n = \log_2 M \) data bits are represented by a symbol, thus the bandwidth efficiency is increased to \( n \) times. Among all MPSK schemes, QPSK is the most-often-used scheme since it does not suffer from BER degradation while the bandwidth efficiency is increased. Since the description about M-ary PSK modulation scheme is not so important to inherit in this chapter. So we have put that detail in appendix below.

Correlation among received fading signals cannot be avoided due to reasons discussed in [1, 2]. Analysis of diversity receivers for correlated channels is relatively more complicated compared to the independent fading case. In this section, performance of dual-MRC, receivers are analyzed for correlated Hoyt fading channels. For MRC receiver an analysis for unequal fading parameters is also presented in addition to the equal fading parameter case. Unequal channel fading parameters may be observed in urban fading environments where diversity channels may have different characteristics. In the analysis presented here the PDF based approach is used. Some conditions are followed for MRC simulation which are:

1. We have \( N \) receive antennas and one transmit antenna.
2. The channel is flat fading – In simple terms, it means that the multipath channel has only one tap. So, the convolution operation reduces to a simple multiplication.
3. The channel experienced by each receive antenna is randomly varying in time. For the \( i^\text{th} \) receive antenna, each transmitted symbol gets multiplied by a randomly varying complex number \( h_i \). As the channel under consideration is a Hoyt channel, the real and imaginary parts of \( h_i \) are Gaussian distributed having mean \( \mu_{h_i} \) and variance \( \sigma_{h_i}^2 = 1/2 \).
4. The channel experience by each receive antenna is independent from the channel experienced by other receive antennas.
5. On each receive antenna, the noise \( n \) has the Gaussian probability density function with \( \mu = 0 \) and \( \sigma^2 = \frac{N_0}{2} \).

The noise on each receive antenna is independent from the noise on the other receive antennas.

6. At each receive antenna, the channel \( h_i \) is known at the receiver.

7. In the presence of channel \( h_i \), the instantaneous bit energy to noise ratio at \( i^\text{th} \) receive antenna is \( \frac{|h_i|^2 E_b}{N_o} \). For notational convenience, let us define,

\[
\gamma = \frac{|h_i|^2 E_b}{N_o}
\]

A. Maximal Ratio combining diversity scheme

A signal transmitted at a particular carrier frequency and at a particular instant of time may be received in a multipath null. Diversity reception reduces the probability of occurrence of communication failures (outages) caused by fades by combining several copies of the same message received over different channels. In general, the efficiency of the diversity techniques reduces if the signal fading is correlated at different branches. The most common and efficient diversity scheme is maximal ratio combining (MRC). In Maximum Ratio combining each signal branch is multiplied by a weight factor that is proportional to the signal amplitude. That is, branches with strong signal are further amplified, while weak signals are attenuated as shown in figure 5.

In simple terms, it means that \( M \) data bits are represented by a symbol, thus the bandwidth efficiency is increased to \( n \) times. Among all MPSK schemes, QPSK is the most-often-used scheme since it does not suffer from BER degradation while the bandwidth efficiency is increased. Since the description about M-ary PSK modulation scheme is not so important to inherit in this chapter.

MRC, the attenuation/amplification factor is proportional to the signal amplitude \( a_i = r_i \) for each channel \( i \).

On the \( i^\text{th} \) receive antenna, the received signal is, \( y_i = h_i x + n_i \) Where \( y_i \), the received symbol on the \( i^\text{th} \) receive antenna, \( h_i \) is the channel on the \( i^\text{th} \) receive antenna, \( x \) is the transmitted symbol and \( n_i \) is the noise on \( i^\text{th} \) receive antenna.

\( y_i \) Expressing it in matrix form, the received signal is,

\[
Y = hx + n,
\]

where \( y = [y_1, y_2, ..., y_N]^T \) is the received symbol from all the receive antenna
\( h = [h_1, h_2, ..., h_N]^T \) is the channel on all the receive antenna
\( x \) is the transmitted symbol and
\( n = [n_1, n_2, ..., n_N]^T \) is the noise on all the receive antenna.
Similarly, the joint CF of RVs \( X_1, X_2, \ldots, X_L \) can be written as

\[
\phi_{X_1^2, X_2^2, \ldots, X_L^2}(j\omega_1, j\omega_2, \ldots, j\omega_L) = \frac{1}{(2\sigma_x^2)^{L/2}} \prod_{l=1}^{L} \frac{1}{\sqrt{1 + (2\sigma_x^2 + j\omega_l)}}
\]

Hence, the joint CF in Equation 1.2.3 can be obtained as

\[
\phi_{a_1^2, a_2^2}(j\omega_1, j\omega_2, \ldots, j\omega_L) = \phi_{X_1^2, X_2^2}(j\omega_1, j\omega_2, \ldots, j\omega_L)
\]

C. Characteristic Function of Sum of Hoyt Square RVs

In the mathematical model of Hoyt RVs, the expression for the joint CF of RVs \( a_1^2 = x_1^2 + y_1^2 \) and \( a_2^2 = x_2^2 + y_2^2 \) is

\[
f_{a_1, a_2}(a_1, a_2) = \frac{1}{(2\pi \sigma_x^2)^{1/2}} \int_{0}^{\infty} \int_{0}^{\infty} e^{-\frac{1}{2\sigma_x^2}[(a_1^2 + j\omega_1)(a_2^2 + j\omega_2)]} \, dx_1 \, dx_2
\]

An expression for the PDF of the sum of Hoyt RVs can be shown as

\[
f_{X_1^2}(X_1^2) = \frac{1}{\sqrt{2\pi \sigma_x^2 X_1^2}} e^{-\frac{X_1^2}{2\sigma_x^2}}
\]

From above equation \( X_1^2 \) can be obtained as

\[
\phi_{X_1^2}(j\omega_1) = E[e^{j\omega_1 X_1^2}] = \frac{1}{(2\pi \sigma_x^2)^{1/2}} \int_{0}^{\infty} e^{-\frac{1}{2\sigma_x^2} X_1^2} e^{j\omega_1 X_1^2} \, dx_1
\]

Performing the integration we obtain

\[
\phi_{X_1^2}(j\omega_1) = \frac{1}{\sqrt{2\sigma_x^2 X_1^2}} e^{-\frac{1}{2\sigma_x^2} X_1^2}
\]

Since \( X_1 \) s are independent their joint CF is the product of independent CFs, hence

\[
\phi_{X_1^2, X_2^2, \ldots, X_L^2}(j\omega_1, j\omega_2, \ldots, j\omega_L) = \frac{1}{(2\sigma_x^2)^{L/2}} \prod_{l=1}^{L} \frac{1}{\sqrt{1 + (2\sigma_x^2 + j\omega_l)}}
\]

Similarly the joint CF of RVs \( Y_1^2, \ldots, Y_L^2 \) can be obtained as

\[
\phi_{Y_1^2, Y_2^2, \ldots, Y_L^2}(j\omega_1, j\omega_2, \ldots, j\omega_L) = \frac{1}{(2\sigma_y^2)^{L/2}} \prod_{l=1}^{L} \frac{1}{\sqrt{1 + (2\sigma_y^2 + j\omega_l)}}
\]

D. Probability Distribution Analysis

Receiver

In this analysis correlation between the fading envelopes as \( l = 1, 2 \) is assumed. A general expression for the combined output SNR \( \gamma_{mrc} \) is given in below Equation. It can be expressed for the dual diversity case as

\[
\gamma_{mrc} = \frac{\epsilon_{mrc}(\alpha_1^2 + \alpha_2^2)}{2\sigma_x^2(1-\rho^2)}
\]

An expression for the PDF of \( \gamma_{mrc} \) i.e. \( f_{\gamma_{mrc}}(\gamma_{mrc}) \), when \( \alpha_1 \) and \( \alpha_2 \) are correlated with correlation coefficient \( \rho \) can be obtained using the complex Gaussian model of Hoyt RV in [13]. Using the PDF \( f_{\gamma_{mrc}}(\gamma_{mrc}) \), performance measures such as average output SNR, outage probability and ABER for binary, coherent and non-coherent modulations are derived.

The combined output SNR can be given as

\[
\gamma_{mrc} = \frac{\epsilon_{mrc}}{2\sigma_x^2(1-\rho^2)} \alpha_1^2
\]

Thus, the PDF of \( \gamma_{mrc} \) can be obtained by scaling equation corresponding to the multiplying factor \( Eb/N0 \), applying the concept of transformation of RVs. For identical branch average power i.e. \( \Omega_1 = \Omega_2 = \Omega \) (equivalently, \( \gamma_1 = \gamma_2 = \gamma_3 = \cdots = \gamma \)), it can be shown that \( Eb/N0 = \gamma/(1 + q^2) \). Substituting this relation, subsequent to the transformation of RV, an expression for \( f_{\gamma_{mrc}}(\gamma_{mrc}) \) can be obtained as

\[
\times \sqrt{\frac{\rho}{2\pi \sigma_y^2(1-\rho^2)}} e^{\frac{\rho}{2(1-\rho^2)}} (2k-1)!! (2t-1)!! \]
The integral in Equation 2 cannot be solved in the given form. Above equation can be rewritten as

\[ P_{\text{out}}(\gamma) = \int_{0}^{\gamma_{\text{th}}} f_{\text{mrc}}(\gamma) d\gamma \]

Putting \( f_{\text{mrc}}(\gamma) \) from previous equation into this equation, an expression for the outage probability for correlated dual-MRC receiver can be expressed as

\[ P_{\text{out}}(\gamma) = \frac{(1 + q^2)}{2q^2(1 - \rho^2)^2} \sum_{k=0}^{\infty} \sum_{t=0}^{\infty} \frac{(2k-1)!!(2t-1)!!}{k!t!(2k+t+1)} \times \frac{\rho(1 + q^2)}{\sqrt{\beta} \Gamma(1 - \rho^2)^{2(k+t)}} \]

IV. RESULT

We have earlier noticed that nakagami-q channel or hoyt fading channel is the mathematical formulation of fading in satellite link or other fading which is more similar to actual signal losses. In previous chapter we have described mathematically the hoyt fading channel and its derivation for outage probability for correlated hoyt fading channels. Results have been analysed by outage probability and bit error rate. We have observed the performance of hoyt fading channel considering M-ary modulation and MRC.

MATLAB R2013a has been used as a simulation tool as it provides a wide range of designed mathematical functions which proved to be useful in calculation of channel response. For example the complex calculation of outage probability for MRC is made easier by MATLAB’s hypergeometric function, zeroth order Bessel function and gamma function.

CASE 1- M-ARY MODULATION IN HOYT FADING CHANNEL

We have tested the performance of nakagami-q channel for 16-PSK modulation. The input data for a short interval is shown in figure 5. The constellation diagram for it is shown in figure 6.

Constellation diagram provides a graphical representation of the complex envelope of each possible Symbol state. The x-axis of the constellation diagram represents the in-phase component of the complex envelope and the y-axis represents the quadrature component of the complex envelope. The distance between the signals on the constellation diagram relates to how different the modulation waveform are, and how well a receiver can differentiate between all possible symbols when random noise is present.

In the previous chapter the channel response of hoyt fading channel depends upon the hoyt fading parameter. Variation in this value results in change in pdf of channel. Fading parameter \( q \) is the ratio of unequal variances \( \sigma_x \) and \( \sigma_y \). A channel response for hoyt channel is shown in figure 8. As per central limit theorem if there is sufficiently much scatter, the channel impulse response will be well-modelled as a Gaussian process irrespective of the distribution of the individual components. If there is no dominant component to the scatter, then such a process will have zero mean and phase evenly distributed between 0 and 2\( \pi \) radians. The envelope of the channel response will therefore be Hoyt distributed. In this case for \( q=0.2 \) is more similar to Gaussian distribution.
response curve is not ideal which is in case when random variable alpha is 1. We have tested this on different value of alpha which are:

| RV alpha | 0.053990966533 | 0.247921068062695 | 0.342198280312217 |

From the figure shown in figure 7 the outage occurs if signal drops below the noise power level. From the figure it is clear that with variation of correlation coefficient ρ, higher values provides less outage probability which means less loss of signal whereas for it is highest for combination of fading coefficient value 1 and correlation coeff =0. From this simulation curve 16-ary PSK modulation it is proved that if fading coeff (q) has range in between 0.4-0.5 and correlation coefficient ρ is 1 then outage in signal will be least. A bit error rate curve for this case is shown in figure 10. It must be kept in consideration here that the M-ary simulation has been checked for single transmitter and receiver antenna. The bit error rate curve in 4.5 shows that minimum value is for q=0.5, which is in accordance with outage probability. To validate the simulation results we have tested the M-ary results for 4 and 32 PSK as shown in table 4.1. These simulation curves of outage probability also proves that for q=1 and ρ = 0, outage probability is highest in hoyt fading channel. Here ρ = 0 represents the un-correlation case as it is correlation coefficient. So in other words for uncorrelated case the hoyt fading channel performs least.

Case II: MRC monte carlo simulation

The next case considered is MRC scheme with monte carlo simulation. In this case we have considered 2 receivers with BPSK modulation. Results have been shown in figure 11 and 1.7 for outage probability and bit error rate. These are checked for different values of L, q and ρ. For now only 2 receivers case is being analysed, but the script developed is dynamic and can be used for more number of receivers.

The outage probability curve for 16-ary PSK modulation is shown in figure 9. The outage occurs if signal drops below the noise power level. From the figure it is clear that with variation of correlation coefficient ρ, higher values provides less outage probability which means less loss of signal whereas for it is highest for combination of fading coefficient value 1 and correlation coeff =0. From this simulation curve 16-ary PSK modulation it is proved that if fading coeff (q) has range in between 0.4-0.5 and correlation coefficient ρ is 1 then outage in signal will be least. A bit error rate curve for this case is shown in figure 10. It must be kept in consideration here that the M-ary simulation has been checked for single transmitter and receiver antenna. The bit error rate curve in 4.5 shows that minimum value is for q=0.5, which is in accordance with outage probability. To validate the simulation results we have tested the M-ary results for 4 and 32 PSK as shown in table 4.1. These simulation curves of outage probability also proves that for q=1 and ρ = 0, outage probability is highest in hoyt fading channel. Here ρ = 0 represents the un-correlation case as it is correlation coefficient. So in other words for uncorrelated case the hoyt fading channel performs least.

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The effect of branch correlation on the outage can be observed by comparing the outage values for $\rho = 0.8$ against the values for $\rho = 0$ (uncorrelated case). Clearly, with the increase in $\rho$ the receiver suffers more outage, for a fixed value of $q$ and $L$. Again as expected, increase in input branches reduces the probability of outage. The least outage probability is observed at $q=0.5$, $\rho = 0.8$ and $L=2$ & maximum outage probability is at $q=0.4$, $\rho = 0$ and $L=1$. Similar is the case with BER, for $q=0.4$ and $L=1$, it is highest and $q=1$ and $L=2$, it is lowest as shown in figure 12. So increasing the number of receivers lower the effect of fading in hoyt channel which fulfil the MRC purpose since MRC is used to reduce the noise effect in transmitted signal.

![Outage probability curve for MRC in Hoyt fading channel](image1)

![BER for MRC in Hoyt fading channel](image2)

V. CONCLUSION

Hoyt fading channel is more realistic satellite link channel. In this work, Performance of M-ary modulation scheme and MRC diversity receivers are analyzed over Hoyt fading channels. Focusing on the analytical approach, mathematical expressions for various performance measures such as outage probability and BER of diversity receivers have been obtained. The PDF based analytical approach has been preferred in all analyses for these performance measures, wherever possible. It is stressed to analyze diversity receivers with arbitrary order of diversity with correlated fading channels since these cases are encountered frequently in the field deployment of diversity receivers. Results are compared with various correlation coefficients value of hoyt fading channel and different number of receiver antennas and different set of fading coefficients. It has been observed that in case of M-ary simulation with $M=16$, fading coeff $(q)$ has range in between 0.4-0.5 and correlation coefficient $\rho$ is 1 for least outage probability. These values are validated by checking on 4-ary and 32-ary simulation too. In case of MRC scheme, 2 receivers are compared for different values of $q$ and $\rho$, it is observed that with increase in number of antennas, outage probability and BER decreases. The least outage probability is observed at $q=0.5$, $\rho = 0.8$ and $L=2$ & maximum outage probability is at $q=0.4$, $\rho = 0$ and $L=1$. These results are successfully simulated using MATLAB’s communication toolbox.

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