# Modified Dual Unscented Kalman Filter Approach for Measuring Vehicle States and Vehicle Parameters

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Abstract— This paper is concerning in improving performance of AVs using a novel approach of Multi Sensor Data Fusion (MSDF) technique. The state estimation theory of data fusion technology is applied to estimate Vehicle state, sideslip angles, lateral tire road forces and tire road friction coefficient. A four wheel vehicle model is used. A vehicle model contains Dugoff tire model in order to satisfy the required accuracy. Due to the nonlinear characteristic of tireroad behavior, a Modified Dual Unscented Kalman Filter (MDUKF) method is presented to estimate both the vehicle states, sideslip angle, lateral tire road forces and tire road friction coefficient. Performances are tested using an experimental car simulation (CarSim simulator) and Matlab environment. Experimental results demonstrate the ability of this approach to provide accurate estimations if compared with other methods discussed in the paper, and show its practical potential as a low-cost solution for calculating lateral tire forces and sideslip angle.

## Keywords—MSDF; MDUKF; CarSim

1.

## INTRODUCTION

Vehicle state, sideslip angles, lateral tire road forces and tire road friction coefficient are of great importance to vehicles control systems in general and to autonomous vehicle as special case this because the effective operation of each of control systems depends not only on an accurate knowledge of the vehicle states but also vehicle parameters. For example, the vehicle sideslip angle is one of the most important variables in AV as the knowledge of such a variable is of paramount importance in order to improve the performance of a control systems designed to guarantee stability of the vehicle motion in emergency situations. It is used to reduce the accidents and to get full vehicle motion knowledge. However, it is too expensive to measure directly the sideslip angle for common vehicles. A low cost solution which still provides accurate estimation results is necessary for the automotive engineering.

Unfortunately, direct measurement of such variables (parameters) requires the use of complex and quite expensive devices and this limits the algorithms that can be incorporated in production systems. The inability to sense vehicle sideslip angle is the primary challenge in the development of stability control systems [1]. Poor real time estimation is a result of using complicated models and ignoring road friction coefficient.

Recently, many estimation methods have been put forward to solve the problem of difficulties to obtain information with high accuracy and low expense. Kalman filter is the most popular techniques used to solve estimation problem. The main purpose of this paper is provide a model based estimator using DUKF to enhance the real time performance of the algorithm a model-based estimator deals with both estimation of vehicle states and vehicle parameters(sideslip angles, lateral tire road forces, tire road friction coefficient and vehicle mass). This technique is called Modified Dual Unscented Kalman Filter(MDUKF) A four wheel vehicle model is designed and coupled with the Highway Safety Research Institute (HSRI) tire model.

The estimation effectiveness is tested by comparing the outputs of the estimator with the responses of the vehicle model in CarSim.

The vehicle model is in section two, the tire road model is in section three, the modified Dual Unscented Kalman Filter is in section four, the simulation of the propose method is in section five, comparison with other works is done in section six, and conclusion and future work is in section seven.

## 2. VEHICLE MODEL

Many of the estimators of the vehicle states work only with a reduced number of states or a reduced vehicle model, such as a bicycle model Such as approaches that use this model for the estimation of side slip angle and yaw rate or for lateral acceleration, yaw rate and tyre slip angles.

Other approaches use of more complicated vehicle models, such as a four-wheel model [2,3] as an example it uses a four-wheel model to estimate lateral velocity, yaw rate and the vehicle's roll motion, but this work has a clear drawback that the implemented estimator requires the use of a whole set of additional costly acceleration sensors.

A four-wheel model is used by [4] to estimate longitudinal and lateral velocity and yaw and roll motion.

All approaches in [2-6] have a common limitation. They all use fixed vehicle parameters, such as mass, moments of inertia and/or position of the centre of gravity (cog), and all these parameters are assumed to be known. The Four Wheel Vehicle Model (FWVM) is chosen for this research because it is simple and corresponds sufficiently to our objectives.

The FWVM is widely used to describe transversal vehicledynamic behavior Fig. 1 shows a simple diagram of the FWVM model in the longitudinal and lateral planes. In order to simplify the lateral and longitudinal dynamics, rolling resistance is neglected. Additionally, the front and rear track widths (E) are assumed to be equal [7].

Additionally, the front and rear track widths (E) are assumed to be equal. L1 and L2 represent the distance from the vehicle's COG to the front and rear axles, respectively. The sideslip at the vehicle COG ( $\beta$ ) is the difference between the velocity heading (Vg) and the true heading of

the vehicle  $(\psi)$ . The yaw rate  $(\dot{\varphi})$  is the angular velocity of the vehicle about the COG. The forward and lateral

velocities are V and U, respectively. The longitudinal and lateral forces ( $F_x$ , y, i, j) are shown for front and rear tires of the vehicle.

Longitudinal forces should be taken into account to enable accurate lateral forces estimation during vehicle braking or acceleration. While considering their effect is certainly important, its inclusion makes solving the lateral estimation problem considerably more complex. Thus, it may be desirable to solve the lateral estimation problem in the absence of longitudinal forces first and include them in later studies. This can be done by focusing on solving the estimation problem when the vehicle is driven at constant speeds. This study extended the hypothesis of moving in a constant speed and addresses the case of a front-wheel drive, where rear longitudinal forces. Longitudinal front axle forces are considered by assuming that

 $F_{x1} = F_{x11} + F_{x12}$  (1) The longitudinal force evolution is modeled with a random walk model, where its derivative is equal to random noise. This is due to the lack of knowledge on the longitudinal slip and the effective radius of the tires.

The lateral dynamics of the vehicle can be obtained by summing the forces and moments about the vehicle's COG. Consequently, the simplified FWVM is formulated as the following dynamic relationships as next equations:

$$\dot{V}_{g} = \frac{1}{m} \begin{bmatrix} F_{x1} \cos(\beta - \delta) + F_{y11} \sin(\beta - \delta) \\ + F_{y12} \sin(\beta - \delta) + (F_{y21} + F_{y22}) \sin(\beta) \end{bmatrix}$$
(2)

$$\ddot{\psi} = \frac{1}{I_z} \begin{bmatrix} L_1 \begin{bmatrix} F_{y11} & \cos \delta + F_{y12} & \cos \delta + F_{x1} \sin \delta \end{bmatrix} \\ & -L_2 \begin{bmatrix} F_{y12} + F_{y22} \end{bmatrix} \\ & + \frac{E}{2} \begin{bmatrix} F_{y11} & \sin \delta - F_{y12} \sin \delta \end{bmatrix}$$
(3)

$$\dot{\beta} = \frac{1}{m V_g} \begin{bmatrix} -F_{x1} \sin(\beta - \delta) + F_{y11} \cos(\beta - \delta) \\ +F_{y12} \cos(\beta - \delta) + (F_{y11} + F_{y12}) \cos\beta \end{bmatrix} - \dot{\psi}$$
(4)

$$a_{y} = \frac{1}{m} \left[ F_{y11} \cos \delta + F_{y12} \cos \delta + \left( F_{y12} + F_{y22} \right) + F_{x1} \sin \delta \right]$$
(5)

$$a_{x} = \frac{1}{m} \left[ -F_{y11} \sin \delta - F_{y12} \sin \delta + F_{x1} \cos \delta \right]$$
(6)

Where *m* is the vehicle mass and  $I_z$  is the yaw moment of inertia. The tire slip angle ( $\alpha_{ij}$ ), is the difference between the tire's longitudinal axis and the tire's velocity vector. The tire velocity vector can be obtained from the vehicle's velocity (at the COG) and the yaw rate. Assuming that rear steering angles are approximately null, the direction or heading of the rear tires is the same as that of the vehicle. The heading of the front tires includes the steering angle ( $\delta$ ). The formst steering angles are assumed to be equal ( $\delta_{11} = \delta_{12} = \delta$ ). The forward velocity *V*, steering angle  $\delta$ , yaw rate

, and the vehicle body slip angle  $\beta$  are then used to calculate the tire slip angles  $\alpha_{ij}$ , where

$$\alpha_{11} = \delta - \arctan \frac{V_{\beta} + L_1 \psi}{V - E \psi/2}$$
(7)

$$\alpha_{12} = \delta - \arctan \frac{V_{\beta} + L_1 \psi}{V + E \psi/2} \tag{8}$$

$$\alpha_{21} = -\arctan\frac{V_{\beta} + L_2 \psi}{V - E \psi/2} \tag{9}$$

$$\alpha_{22} = -\arctan\frac{V_{\beta} + L_2 \psi}{V + E \psi/2}$$
(10)

Assuming small tire slip angles, the wheel–ground contact point velocities  $V w_{ij}$  depend on the vehicle COG velocity  $V_e$  according to the following equations:

$$V_{w_{11}} = V_g - \dot{\psi} (E_{2} - \frac{L_1}{\beta})$$
 (11)

$$V w_{12} = V_g + \dot{\psi} \left( \frac{E}{2} - \frac{L_1}{\beta} \right)$$
(12)

$$V w_{21} = V_g - \dot{\psi} \left( \frac{E}{2} - \frac{L_2}{\beta} \right)$$
(13)

$$V w_{22} = V_g + \dot{\psi} \left( \frac{E}{2} - \frac{L_2}{\beta} \right)$$
(14)





Fig. 1 2D schema representation of a FWVM

## 3. TIRE ROAD MODEL

Modeling tire road forces is complex because of the interaction of many physical phenomena in a multitude of environmental and tires characteristics (applied load, tire pressure, road surface). Several models have been used in the literature to model these forces. We distinguish two types, physical models that can characterize the tire/road contact surface and empirical models that are derived from identification of parameters curves from experimental readings obtained on test [8].

Many different tire models are based on the physical nature of the tire. These models include the Burckhardt, Dugoff, and Pacejka models [8].

One of the most commonly used model is the Pacejka's "magic formula." It does an excellent job of predicting real-tire behavior. However, it requires a large number of tire-specific parameters that are usually unknown.

The exact model of the tire-road contact forces is complex, This is because when the sideslip angle is small, the lateral force is linear with the wheel slip angle. Fyij = C ij , i, j = 1, 2, where Cij is the cornering stiffness of each wheel. When the sideslip angle increases, the lateral force increases non-linearly with it. After a threshold, the road tire forces begin to saturate For the reason of simplicity, the linear models are popular used in [8, 9]. Another commonly used model is the Dugoff tire model. It synthesizes all the tire property parameters into two constants Cx and Cy, referred to as the longitudinal and

## 4. MODIFIED DUAL UNSCENTED KALMAN FILTER (MDUKF)

From our study we find that improving performance of autonomous vehicle has five essential elements which are: the real time performance, the accuracy of estimation, the effect of road friction and wheel slip angle, and cost. These elements have to be emphasized within the estimation of vehicle dynamics.

For the previous purpose, the implementation of a modelbased vehicle estimator is proposed, which combined estimation of vehicle states and vehicle parameters. The estimator is realized using the dual unscented Kalman filter (DUKF) technique, using two unscented Kalman filters running in parallel, Which means 'splitting' the state and parameter estimation problems. This technique proposed several advantages, such as the possibility of switching off the parameter estimator, when a sufficiently good set of estimates is obtained.

The objective of the nonlinear observer is to simultaneously estimate the sideslip angle, lateral tire road forces and the tire road friction coefficient. A state space model can be used to achieve this purpose, where the sideslip angle and lateral tire road forces are taken as the state variable and the tire road friction coefficient is the unknown system parameter. Then the objective is to simultaneously estimate the state and the parameter. In order to use the UKF, we use an Euler discretization for all continuous variables in the vehicle model and the tire model to obtain a discrete time system. A general discrete state space model is given by the next two equations:

$$x_{t} = f(x_{t-1}, u_{t}, \theta_{t}) + w_{t}$$
(15)

$$=h(x_t, u_t, \theta_t) + v_t \tag{16}$$

Where  $x_t$  is the state variable,  $y_t$  is the observation variable, is the input variable and the  $\Theta_t$  is the unknown parameter,  $w_t$  and  $v_t$  are the state and the observation noises. Normally, UKF can estimate the state  $x_t$  when  $\Theta_t$  is known.

Уt

The state vector  $x_t$  contains vehicle velocity, yaw rate, sideslip angle and lateral forces as in equation17:

$$x_t = [V_g, \dot{\varphi}, \beta, F_{y11}, F_{y12}, F_{y21}, F_{y22}]$$
<sup>(17)</sup>

The parameter  $\theta$  is the tire road friction coefficient  $\mu$ . The evolution of the state vector is given by the Four-Wheel Vehicle Model and the evolution of the lateral tire forces are described by the dynamic Dugoff model

The observation vector  $y_t$  comprise vehicle velocity, yaw rate, longitudinal acceleration and lateral acceleration as in equation 18:

$$y_t = [V_g, \dot{\varphi}, a_x, a_y]$$
 (18)

We suppose that the steer angles, the vertical normal loads and the longitudinal forces are known. The input vector is given in equation 19:

$$u_{t} = \begin{bmatrix} \delta 11, \delta 12, F_{z11}, F_{z12}, F_{z21}, F_{z22}, F_{z11}, F_{z12} \end{bmatrix}$$
(19)

The evolution of the state vector and the observation vector is given by the vehicle model. All the lateral tire-road forces are calculated by the Dugoff model.

The objective of the non-linear observer is to simultaneously estimate the state (sideslip angle, lateral tire road forces) and parameter (tire road friction coefficient). Normally, the nonlinear observer is designed as dual filter structure.

At a fixed time t, we use the first filter in the dual filter structure to derive the state variables  $\hat{x}_t$  assuming that the parameter

 $\hat{\Theta}_{t-1}$  is known as its previous estimated value. After getting the state  $\hat{x}_t$ , the second filter is applied to estimate the  $\hat{\Theta}_t$ 

assuming that  $x_t$  is known as  $\hat{x}_t$ . The non-linear observer based on the dual UKF has a structure shown in Fig. 2.



Fig. 2 Dual Unscented Kalman Filter

The two Kalman filters are running in parallel, thus 'splitting' the state and parameter estimation problems but the two problems cannot be entirely separated due to their inherent interdependencies. In this technique there is a possibility to switch off the parameter estimator, once a sufficiently good set of estimates has been obtained.

## 5. SIMULATION USING CARSIM

Results from CarSim simulator demonstrates that the estimator proposed is capable of estimating the vehicle states and sideslip angle, lateral tire road forces and the tire road friction coefficient with high accuracy. Performance Experimental results demonstrate the ability of this approach to provide accurate estimations if compared with other methods and show its practical potential as a low-cost solution for calculating vehicle parameters. A nonlinear observer has been proposed to simultaneously estimate sideslip angle, lateral tire road forces and tire road friction coefficient. A state space model is established where the sideslip angle and the lateral tire road forces are the state variable and the tire road friction coefficient is the unknown parameter. The performances of this nonlinear observer are demonstrated by the simulation results. The physical parameters of the simulated vehicle in CarSim are in table 1.

Table 1 Physical parameters of the simulated vehicle in CarSim

Parameter	Physical parameters of the simulated vehicle in CarSim		
	Physical signification	Value	
m	Mass of the vehicle (kg) for example Sedan car model	1296	
μ	Friction coefficient	1	
LI	Front axle distance from center of gravity(m)	1.25	
L2	Rear axle distance from center of gravity(m)	1.32	
Iz	Moment of inertia (kg/m <sup>2</sup> )	1750	
a <sub>x</sub>	Longitudinal accelerations (m/ s <sup>2</sup> )		
a <sub>y</sub>	Lateral accelerations (m/s <sup>2</sup> )		
V <sub>g</sub>	Velocity of the center of gravity (m/s)		
β	Sideslip angle (rad)		
δ	Front steering angle(rad)		
$F_{y11}, F_{y12}$	Front lateral tire forces(N)		
$F_{y_{21}}, F_{y_{22}}$	Rear lateral tire forces (N)		
$F_{x11}, F_{x12}$	Longitudinal tire forces (N)		

There are two different scenarios are done:

First scenario

After defining the required data of the vehicle during the simulation run and set the vehicle speed with 60 km/h on a flat surface. The generated matrices can be loaded with MATLAB and analyzed with a frequency-domain bode plot. The vehicle of the simulation is shown in fig. 3



Fig. 3 Simulated environment after sitting vehicle data





The results of the first scenario is done under normal condition so the effect of the vehicle parameters is in linear region, only the first Unscented Kalman Filter is ON while the Second one is OFF.



Fig. 10 The steering wheel angle







Fig. 12 The longitudinal tire forces



Fig. 13 The vehicle slip angle

In the second scenario the speed of the vehicle is changed from 30 to 180, this makes the vehicle parameters are under the saturated region so the two unscented kalman filter are running in parallel.

#### 6. COMPARISON WITH OTHER WORKS

To measure the performance of state estimation  $x_t$  for t = 1,..., T (where T is the number of the total sampling number), we introduce the root mean squared error as in equation 20.

RMSE 
$$(x_t) = \sqrt{\frac{1}{T} \sum_{t=1}^{T} (x_t - \hat{x}_t)^2}$$

(20)

For all the works done bellow the road condition is set to be dry( $\mu = 1$ ) and the initial velocity of the vehicle is 60 km/h. the sampling interval is 0.01 second. The table in 2 shows the comparison of the other authors with our work.

All of the methods mentioned in the table assume that the mass vehicle parameter is preknown and is constant except the work done in [7]. In the real world, there is no reason to restrict this kind of parameter as the mass of vehicle, even the weight of empty vehicle is well measured, it will change as the total number of passengers, the weight of baggage and the fuel tank varies sensibly during different journeys. And getting the accuracy information of vehicle mass undoubtedly leads to effective vehicle state estimation. This considered as a limitation of our work but it can be added in the future.

It is notable that the dual UKF always provides the best estimation of the vehicle parameter (sideslip angle is taken as an example). The results of the joint UKF observer are better than those of the dual EKF filter. The EKF performs obviously worse than the Joint UKF and DUKF. In our work the MDUKF gives better sideslip angle estimation than the traditional DUKF.

Moreover, in MDUKF, it combines both the state and parameter states of the vehicle; it is possible to switch off the parameter estimator, when we get a sufficiently good set of estimates for the parameters. This leads to increase

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the performance of the state estimator, as it reduces the parameter uncertainties and also the disturbances arising from the varying model parameters. Better parameter estimation gives smaller model uncertainties.

The paper demonstrates the effectiveness of the MDUKF. Results indicate that this approach is considered to be of benefit to the autonomous vehicle. Comparison is presented in table 2.

## 7. CONCLUSION AND FUTURE WORK

Measuring tire forces, sideslip angle and tire coefficient friction is very difficult for technical, physical, and economic reasons. Therefore, these important data must be observed or estimated. Accurate data about tire forces and sideslip angle leads to a better evaluation of the road friction and the vehicle's possible trajectories, and to a better vehicle control performance. Moreover, it makes possible the development of a diagnostic tool for evaluating the potential risks of accidents related to poor adherence or dangerous maneuvers. In this research, a MDUKF estimator designed based on a three-DOF vehicle model coupled with Dugoff tire model is presented to realize the simultaneous estimation of simulating the lateral tire forces, sideslip angle and road friction coefficient.

For the proposed MDUKF, the vehicle mass is assumed to be constant In the real world, there is no reason to restrict this kind of parameter. When the weight of empty vehicle is well measured, it will change as the total number of passengers, the weight of baggage and the fuel tank varies sensibly during different journeys. And getting the accuracy information of vehicle mass undoubtedly leads to effective vehicle state estimation. In the future this parameter will be taken in consideration.

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# Table 2 Comparison with other works

Author of work	Comparison with other works				
	Control parameters	Filtering Method	Limitations	Performance RMSE (degree)	
[9]	forward velocity, yaw rate and measured lateral acceleration	Dual extended Kalman filter (DEKF)	-Vehicle mass is constant - sideslip angle is not included	0.51	
[7]	sideslip angle, vehicle mass	Dual unscented Kalman filter (DUKF)	Lateral tire road forces, tire road friction coefficient are not included	0.23	
[7]	sideslip angle, vehicle mass	Joint unscented Kalman filter	Lateral tire road forces, tire road friction coefficient are not included	0.25	
[7]	sideslip angle, vehicle mass	Dual extended Kalman filter	Lateral tire road forces, tire road friction coefficient are not included	0.38	
[10]	Vehicle Lateral Tire–Road Forces and Sideslip Angle	Extended Kalman filter	-limitation of using EKF -Vehicle mass is constant		
[11]	Sideslip angle, lateral tire force and road friction	Extended Kalman filter	-Using single-track model of vehicle -Vehicle mass is constant		
[12]	Sideslip angle and tire road friction coefficient	Unscented Kalman filter	-Vehicle mass is contant		
[13]	the lateral tire force and the sideslip angle	Extended Kalman filter	-steering angles are assumed to be equal -Vehicle mass is constant		
[14]	sideslip angle and Lateral friction coefficient	Extended Kalman filter	-Non observable when steering angles are null, vehicle is at rest. -limitation of EKF -Vehicle mass is constant		
[14]	sideslip angle and Lateral friction coefficient	Unscented Kalman filter	Non observable when steering angles are null, vehicle is at rest. -Vehicle mass is constant		
Us	sideslip angle, lateral tire road forces and tire road friction coefficient	Modified Dual Unscented Kalman Filter (MDUKF)	-Vehicle mass is constant	0.21	