

# Modified DO-Sag Equation to Assess River Water Quality

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## NOMENCLATURE-

Following terms have been used to define DO-Sag Equation by alternative algorithm

- a) DO<sub>Ref</sub> – Reference DO Model made by Streeter Phelps equation
- b) DO<sub>New</sub> – DO Model made using new equation (quadratic polynomial)
- c) De-oxidation Coefficient – k<sub>1</sub>
- d) Re-oxygenation Coefficient – k<sub>2</sub>
- e) A- Re-aeration Coefficient in DO<sub>New</sub> Model

**1. Abstract-** The presence of dissolved oxygen (DO) in river water is primary criterion to evince the water quality. The river may receive point and nonpoint pollution from various sources like municipal, industrial and agricultural land on its way to the downstream. Pioneer work to assess river water quality was done by Streeter and Phelps (1925) when they 1<sup>st</sup> time noticed the importance of DO present in the river water. Variation of DO in water depends on de-oxidation coefficient (k<sub>1</sub>), and re-aeration (k<sub>2</sub>) rate of oxygen transfer between the water surface and the atmosphere. Off late many authors derived empirical formulae to determine hydraulic parameters of the stream to compute coefficients (k<sub>1</sub>), and (k<sub>2</sub>) depending upon their place of study. Present research work is planned with two fold objectives: (i) to make use of Streeter and Phelps equation to test Shrivnath River water quality flowing near Durg-Bhilai in C.G. India and (ii) To understand River Water DO Mechanics for best location of water treatment plant and supply of drinking water to the township.

**2. Key words-** Dissolved Oxygen (DO), Molecular Diffusion, Surface Re-aeration, Water Quality Modeling, River Water DO Mechanics, G\_Mat DO software

## 3. INTRODUCTION-

Streeter-Phelps (1925) developed equation (1) to assess river water quality of a river

$$D_t = \frac{k_1 L_a}{k_2 - k_1} [10^{-k_1 t} - 10^{-k_2 t}] + D_a 10^{-k_2 t} \quad \text{----- (1) refer Fig.(1):}$$

Where D<sub>t</sub> = DO saturation deficit downstream, (mg/L)

= (DO<sub>sat</sub> – DO<sub>a</sub>) at time t

t = time of travel from upstream to downstream, ( No. of days)

D<sub>a</sub>= initial DO saturation deficit of upstream, (mg/L)

L<sub>a</sub>= Ultimate upstream BOD at t = 0, (mg/L)

K<sub>1</sub>= De-oxidation Coefficient (per day)

K<sub>2</sub>= Re-aeration Coefficient (per day)

Streeter-Phelps Model equation needs following field data of a river:

T = temperature of water, °C

H= average depth of flow, m

v = mean stream velocity, m/s

Equation (1) simulates the variations of the parameters: DO (Dissolved oxygen), BOD (Biological De-oxygenation and Re-aeration coefficients) over time at each point of a river. DO of river water is required for following:

- Evaluation of surface water quality
- Waste-treatment processes control
- Photosynthetic activity of natural water

Sources of DO in surface waters are primarily atmospheric re-aeration and photosynthetic activity of aquatic plants. DO is an important factor in chemical reactions in water and in the survival of aquatic organisms. In surface water, DO concentrations typically range from 2-10 mg/l. DO saturation decreases as water temperature increases, and increases with increased atmospheric pressure. DO may be depleted in inorganic oxidation reaction or by biological and chemical processes that consume dissolved, suspended or precipitated organic matter.

## 2. DATA OBSERVATION PLANNING-

Shivnath River Route Survey from villages: Chandkhuri, Kolihapuri, Durg city (Ganjpara), Chatagargh is planned for observing hydraulic data Fig.(2). The effluent waste streams from these localities join the main River.

- (i) To determine coefficients  $k_1$ , and  $k_2$  (hydraulic parameters): mean velocity of stream and depth are required.
- (ii) **De-oxidation Rate Constant** – ( $K_1$ ) from river water samples  $BOD_5$  value is determined by standard method of incubation in the laboratory.

### (iii) **Route Reconnaissance Survey with Google Map-**

The sampling points  $S_5$  to  $S_7$  are situated on Shivnath river reach = 24 Km. Pollution through streams and sewers from villages on the route join main river. The municipal, agricultural and industrial waste makes the river water unhealthy, therefore sampling point near to these places are selected for the study.

## 3. METHODOLOGY OF DATA OBSERVATION-

Water Samples of the river have been collected from points marked Fig.(2). All samples have been collected at about 20 cm depth from the river mid-point, as per Hydro-Bios standard water sampler by dip/grab method. The samples were stored in pre-cleaned polythene bottles. Sample collection is done on date 15<sup>th</sup> each of each month- Jan, Feb, Mar, Apr, and May 2017. Study of temperature and other physiochemical property effect on de-oxidation and re-aeration is the part of research work.

### 3.1 **River Velocity & Depth-**

To determine river-traverse lengths from A to D sampling points, GPS device (smart mobile phone) is used. Mean Velocity of river is found = 0.36 m/sec and mean depth = 2.53 m

## 4. RESULTS & ANALYSIS: DO AND BOD TEST RESULTS:

obtained from laboratory test for the month Jan. 2017, to May 2017 is shown in Table (1).

Distance in Km	Sample Stn.	<-----DO (mg/L)-----→				
		Jan	Feb	Mar	Apr	May
0	$S_5$	<b>5.4</b>	5.1	5	4.9	4.8
5	$S_6$	5.2	4.8	4.5	4.2	4.7
17	$S_7$	4.1	3.5	3.4	3.6	3.8
24	$S_8$	4	3.1	2.8	3.2	3.3

Table (1) – DO variation from Jan to May 2017 at Station  $S_7$

24 Km. It is apparent that DO on upstream points  $S_5$  and  $S_6$  are higher than DO at  $S_7$ , near to the intake well on Shivnath River (near NH-6). It is therefore concluded that waste water stream sources meet the river at points  $S_5$  and  $S_6$ . Study of DO variation in a river over the distances is useful in correspondence with Streeter-Phelps equation. It shows De-oxidation ( $k_1$ ) in Jan – Feb useful in determining the point sources of waste water streams joining the main river.

The DO value at station  $S_7$  ranges from 4.1 to 3.8 from Jan to May months justify the location of present intake well/ waste water treatment plant because at no point of time DO level goes below 2.4 mg/L. Further confirmation of DO presence at  $S_7$ , Lab test for  $BOD_5$  determination is performed for waste sample collected at  $S_7$  in the month May 2017. The results are shown below:

Table (2) – DO/BOD<sub>5</sub> Lab Test on water sample Station S7 May 2017

Sample Stn.	Days	DO	BOD (mg/L)
S <sub>7</sub>	0	6.40	0
1	4.64	5.4	
2	4.47	12.1	
3	4.40	13.9	
4	4.37	10.2	
5	4.35	14.5	

It is evident that the treatment plant located on Shivnath river to supply drinking water to the township meets the requirement of DO present.

#### 4.1 DO predictive Model of Streeter-Phelps equation:

Determination of  $k_1$ .

$L_A = 14.2 \text{ mg/L}$ ;  $L_B = 5.4 \text{ mg/L}$  from Table (1)

$\Delta t = (24 \times 10^3) / [(0.36)(8.64 \times 10^4)] = 0.77 \text{ days}$

$k_1 = (1/0.77) \log_{10}(14.2/5.4) = 0.56/\text{day}$

$$k_2 = 0.5 \times 3.9 v^{0.5} [(1.025)^{T-20}] (H)^{-1.5}$$

Substituting  $v = 0.36 \text{ m/sec}$  and  $h = 2.53 \text{ m}$ , mean temp.  $t = 30^\circ\text{C}$

$K_2 = 0.37/\text{day}$

Oxygen Saturation & DO deficits-

For fresh water the saturation DO level  $C_s$  is given by }  
 $C_s = 14.61 - 0.394 \times T + 0.007714 \times T^2 - 0.0000646 \times T^3 \text{ (mg/L)}$  }-----(2)

$C_s$  the saturated DO level at  $10^\circ\text{C}$  is given by =  $11.38 \text{ mg/l}$  }

Therefore DO of Sample Collected = mean vel.  $\times C_s = 4.44 \text{ mg/l}$

DO Variation for 5 days due to BOD demand

Days	DO Value	BOD
0	6.0	7.22
1	4.40	8.20
2	4.16	9.14
3	3.18	9.88
4	2.23	9.88
5	1.50	10.39

Applying Streeter-Phelps equation, it is apparent from above graph, that  $DO_3 = 3.18$ ,  $DO_4 = 2.23$ , and  $DO_5 = 1.50 \text{ mg/L}$ . The shape of the curve of DO/BOD<sub>5</sub> versus time Appendix –Graph (3) clearly indicates a minimum DO concentration occurs in less than 2.4 Mg/L after 3 days. It implies that the water collected at the intake well must be subject to re-aeration for sufficient number of days to improve DO value.

#### 4.2 DO mechanics of River water & determination of ( $K_1$ & $K_2$ ):

Basics of river water DO mechanics can be derived from Streeter and Phelps equation (1925). The equation (1), Fig.(1) gives DO deficit over particular distance of river reach. The DO Sag curve when expressed in Cartesian coordinates, where:

X- Axis represents No. of days or distances

Y- Axis represents DO values in mg/L

Experimentally the extreme coordinates of points on ideal DO-Sag curve are expressed as follows:

- Point O- Origin (0, 0)  $\rightarrow$  (d=0, DO=0)
- Point A- Coordinates (0, 12)  $\rightarrow$  fully DO saturated river water in contact with atmosphere
- Point B- Coordinates (0, 6)  $\rightarrow$  DO of river water before waste water stream joins
- Point C- Coordinates (2, 2.4)  $\rightarrow$  DO reaches minimum level due to de-oxidation process and become dangerous for aquatic life. The process takes 2 days time for completion. Thereafter re-oxygenation begins.
- Point D- Coordinates (5, 6)  $\rightarrow$  Re-aeration continues and DO becomes equal to point C.
- Point E- Coordinates (12, 12)  $\rightarrow$  Re-aeration continues to wards super saturation zone and DO finally after 12 days becomes 12 mg/L.

#### MODIFIED STREETER PHELPS DO-SAG EQUATION-

In the classical equation, the rate of de-oxidation is taken =  $e^{-k_1 t}$  and re-aeration is taken =  $e^{-k_2 t}$  rate coefficients  $k_1$  and  $k_2$  are determined from river hydraulic parameters:  $v$  (velocity) and  $d$  (depth). Also it is evident from above DO mechanics that the

two processes de-oxidation and re-aeration for 5 days over distances in km of river reach. It is proportional to river mean velocity and depth of river bed. The classical equation assimilates a theoretical DO Model from river site data as well as lab tests conducted on water samples collected over a period of completion of de-oxidation and re-aeration process after a waste water stream meets the main river. This period is 5 days cycle apparent from Streeter Phelps equation application takes place in following steps:

i) Initially atmospheric surface at DO level is 12 mg/L, when comes in contact with river surface becomes 6 mg/L (0-day to begin BOD test).

ii) As soon as polluted stream meets the river, de-oxidation process ( $e^{-k_1 t}$  – exponential function) begins and DO level starts falling. De-oxidation coefficient  $k_1$  depends upon river velocity and depth. DO-level becomes 2.4 mg/L (minimum -danger for aquatic life) with 2 days time.

iii) There-after re-oxygenation begins. Re-aeration process ( $e^{-k_2 t}$  – exponential function) continues till DO-level is restored to 6 mg/L within 4 days. Re-aeration coefficient  $k_2$  also depends upon river velocity and depth.

iv) DO Sag curve formation by combining de-oxidation and re-aeration process can be better represented by a quadratic polynomial than the exponential function adopted by Streeter Phelps. Hence modified Streeter Phelps equation as a quadratic polynomial can be stated as:

$$DO_n = k_1 d^2 - k_2 d + DO_0 \quad \dots \dots \dots (3)$$

Where,  $n$  – number of days 1 to 5

$k_1$ - De-oxidation coefficient

$k_2$ - Re-aeration coefficient

Substituting  $d$  as 0, 2, 6 we have following equations from (3)

$$k_1(0)^2 - k_2(0) + DO_0 = 6, \quad \{ \text{Therefore initial Coordinate (0, 6) i.e. (d, DO)}$$

$$k_1(2)^2 - k_2(2) + 6 = 2.4 \quad \{ \dots \dots \dots (4)$$

$$k_1(5)^2 - k_2(5) + 6 = 6 \quad \}$$

Solving set of equations (4) we have  $k_1 = 0.6$   $k_2 = 3$

Hence  $DO_5$  computation of theoretical model is:  $DO_5 = 0.6 d^2 - 3 d + 6 \quad \dots \dots \dots (5)$

Therefore  $DO_1$ ,  $DO_2$  etc. can be computed from eqn. (3)

v) It takes 5 days to complete de-oxidation and re-aeration process and DO mass balance is achieved. Thereafter DO is further computed from  $DO_5$  to super saturation level given by equation-

$$DO = DO_5 + k_1 A^d \quad \dots \dots \dots (6) \text{ On substitution } A = 1.46$$

Where  $A$  = Super aeration coefficient

Therefore  $DO_6$ ,  $DO_7$  etc. can be computed from eqn. (4)

#### 4.3 Shivnath River DO Model from Lab Tests-

(i) Saturated DO of fresh river water at Atmospheric level is = 10 Mg/L and Surface level DO value is 6.0 Mg/L

(ii) DO at Sample Station S<sub>7</sub> on 15<sup>th</sup> May 2017. The sample was tested for DO Value in the lab to determine DO for 5 successive days. Lab Model & Ideal model DO values are shown below.

Day	DO(Ideal)	DO (Field)	BOD(Lab)
0	6.0	6.0	9.76
1	3.6	3.4	9.24
2	2.4	2.2	8.99
3	3.6	4.1	8.74
4	6	6.3	8.51
5	6.9	8.4	
6	7.8	9.1	
7	8.6	9.2	
8	10.4	9.4	
9	9.5	9.9	
10	10.4	10	

Graph (3)- DO ideal curve Vs DO variation at Sampling Station S<sub>7</sub> (G\_Mat Result) are shown. The DO ideal curve can be treated as universal DO model curve because the results are in close resemblance with Shivnath field DO values for river reach of 24 km.

##### 5. CONCLUSION:

Until recently, the classical Streeter-Phelps equation was widely used in predicting the dissolved oxygen deficit to organic waste load discharged into a stream. However the wide deviation of predicted values of oxygen deficit from the observed value lead several investigators to modify the Streeter-Phelps formulation by taking into account other factors like benthal demand, photosynthesis, algal respiration etc., which affect DO in the stream, in order to narrow the gap between the observed and the predicted values. Other investigators postulated statistical models for predicting critical oxygen concentration using stream flow, temperature, and 5-days BOD as independent variable mainly to avoid the errors involved experimental determination of K<sub>1</sub> and K<sub>2</sub> coefficients. This work attempts to develop a predictive model by employing DO mechanics that De-oxidation and Re-aeration vectors produced due to river velocity and depth follow a quadratic polynomial instead of using the exponential variation law. This property is apparent from the Ideal DO Sag Curve given by Streeter – Phelps equation. It simplifies the determination of K<sub>1</sub>, K<sub>2</sub> river velocity (v) and depth (h). Field measurement and analysis given in this paper fully support the use of lab DO model technique to determine the water quality.

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Appendix- Figs. & Graphs

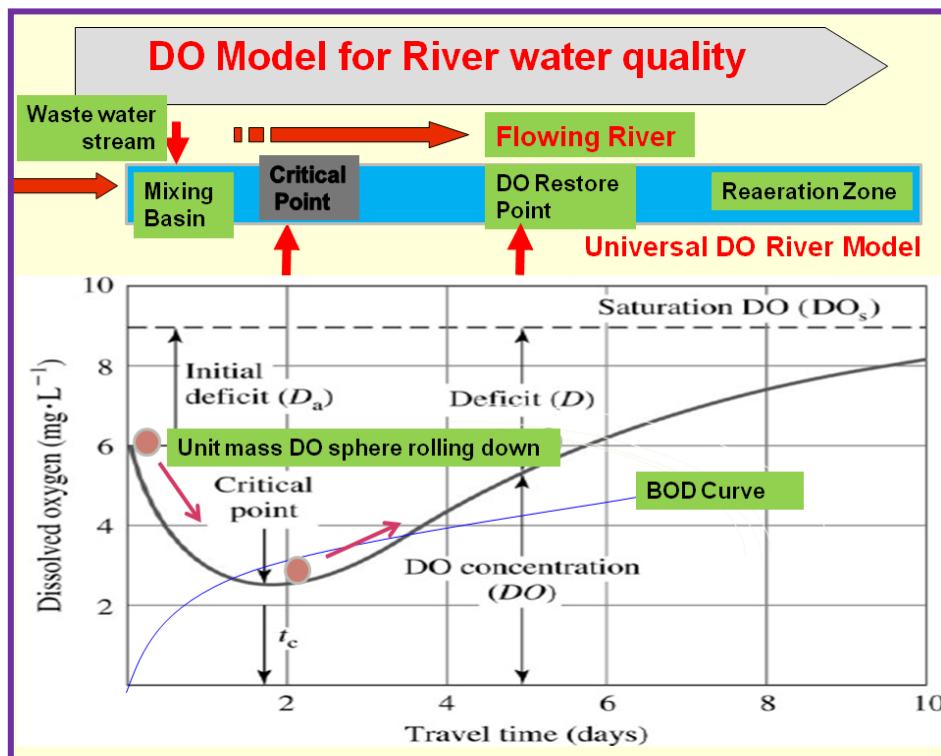


Fig.(1)- Streeter and Phelps ideal DO-Sag curve & DO Mechanics

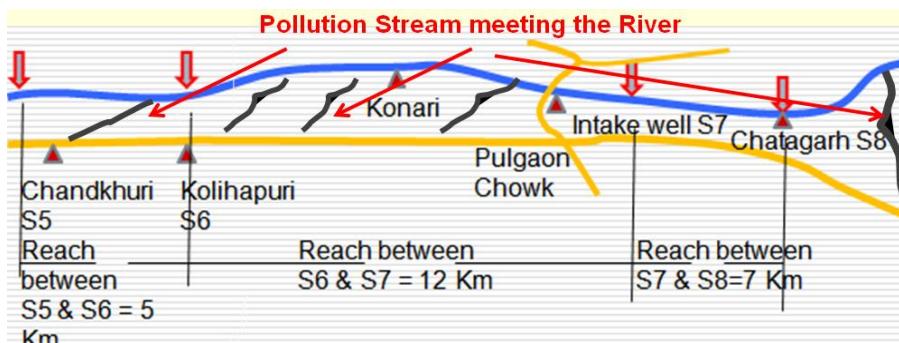
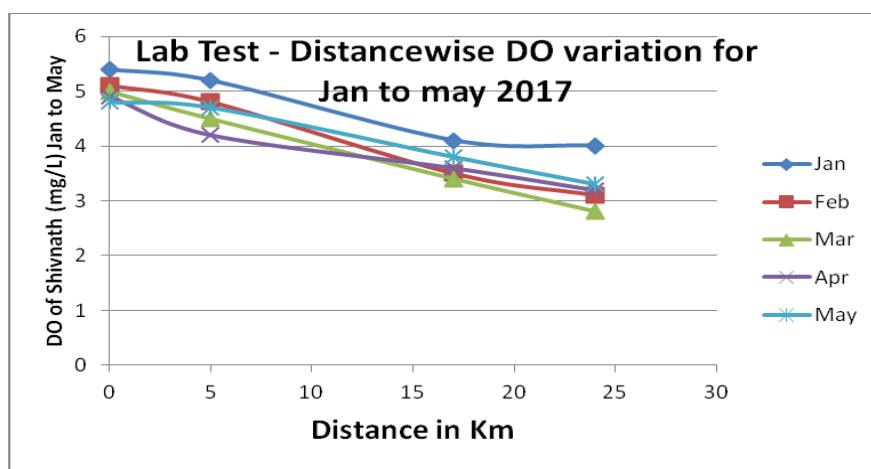
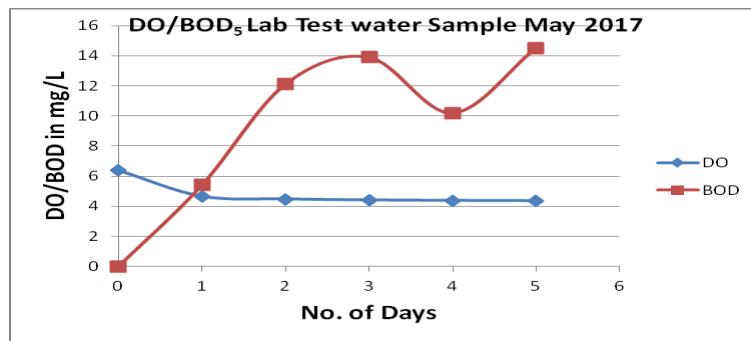


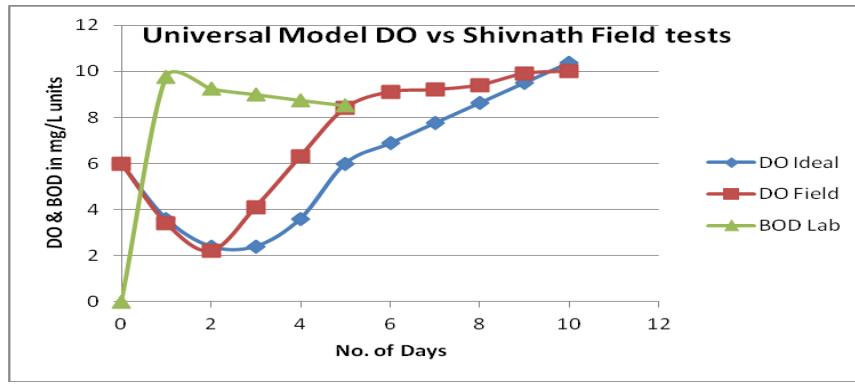
Fig.(2)- Sampling Points S<sub>5</sub> to S<sub>6</sub> = 5 km, S<sub>6</sub> to S<sub>7</sub> = 12 Km and S<sub>7</sub> & S<sub>8</sub> = 7 Km



Graph (1)- DO variation Jan to May 2017 at Sampling points S<sub>5</sub> to S<sub>8</sub> on River Reach



Graph (2)- DO/BOD<sub>5</sub> Lab Test at Sampling Station S<sub>7</sub> (May 2017)



Graph (3)- DO ideal curve Vs DO variation at Sampling Station S<sub>7</sub>